



Improved Profitability and Competition in Two Level Supply Chain by Non-Cooperative Games

Nakisa Araghi

Department of Business Management, Science and Research Branch, Islamic Azad University, Tehran, Iran
Nakisa.a.64@gmail.com

Hashem Nikoomaram

Department of Business Management, Science and Research Branch, Islamic Azad University, Tehran, Iran
(Corresponding Author)
Nikoomaram@sbiau.ac.ir

Farhad Ghaffari

Department of Economics, Science and Research Branch, Islamic Azad University, Tehran, Iran
farhad.ghaffari@icloud.com

ABSTRACT

This article by modeling a non-cooperative dynamic game tries to improve profitability and competition. This paper has considered how the manufacturer interacts with multiple competitor distributors. Each distributor also determines the optimal distribution price and inventory replenishment policies to maximize their profits. The issue form a non-cooperative dynamic game. Distributors formulate sub-games and finally, have formed the main game with the manufacturer. After designing the game, we determined the Nash equilibrium. We use the concept of "Nash equilibrium" to analyze supplier, manufacturer and distributor strategies in the overall game with the manufacturer. In order to achieve the Nash equilibrium, we use decisions as input parameters. In this case, each player, in addition to making the right decision, can make decisions in order to prepare for possible changes in the decisions of other actors and thereby maximize their profit. As long as actors are reluctant to change their decisions, the process continues. For this purpose, we used analytical method and solution procedure. The results indicate that by increasing the market scale, increasing price sensitivity, increasing the degree of replacement of products, as well as increasing production costs, distributor's profit increases. In this paper, Lingo software was used for calculations.

Keywords:

Pricing, Inventory, Non-cooperative games, Nash equilibrium, Supply chain



1. Introduction

Supply Chain Management covers a wide range of issues Such as location, logistics, inventory and forecasting, marketing, product design and new product introduction, support and after-sales services, strategic alliances and outsourcing. There are different flows in the supply chain which must coordinate to effectively manage the chain. This article emphasizes in the optimal approach. Optimization can only check some subnets or optimize the whole network. This paper tries to optimize pricing and inventory control decisions.

Game theory is a powerful tool for analyzing situations in which the decisions of multiple agents affect each agent's payoff. As such, game theory deals with interactive optimization problems. (Cachon, G and Netessine, S, 2006).

Non-cooperative games seeks a rational prediction of how the game will be played in practice.

The solution concept for these games was formally introduced by John Nash (1950), (Cachon, G and Netessine, S, 2006).

One of the practical and efficient tools that improve the performance of companies and their market power is the market price. Therefore, pricing policies have been recognized by companies for decades as an important tool that can solve this problem. In fact, companies try to optimize their prices in order to get more market demand.

Previous studies have reviewed Pricing and inventory control decisions. In many industries, pricing and inventory control decisions reviewed to improve efficiency both in the individual firm level and supply chain levels (Weng 1995; Chen et al., 2004).

The supply chain examined in this paper consists of one manufacturer and several distributors.

2. Literature Review

In many manufacturing industries, the coordination between pricing decisions and inventory control, as well as production decisions, improve the efficiency of firms both in the supply chain and at individual levels.

Some studies have introduced the supplier-retailer relationship model and confirm that the coordination of pricing and inventory control decisions could be beneficial both at the individual level of firms and the

supply chain level (Weng and Wong, 1993; Weng, 1995).

Another study introduced the model of coordination of pricing decisions and order quantities in a supply chain composed of one producer and one retailer and then examined. It also addresses the advantages and disadvantages of supply chain coordination (Pratfulla et al., 2006).

Zamarripa et al. (2012) address in their study the supply chain planning in competitive environments with the help of the cooperative and non-cooperative game theoretical approach

Kim and lee (1998) studied pricing and ordering strategies for a single item with fixed or variable capacity to maximize the profit of firm faced to price-sensitive and deterministic demand over a planning horizon.

Sinha and sarmah (2010) reviewed pricing decisions at distribution level under coordination and competition issues, in which two competitive vendors sell their products to a typical retailer on the same market. Choi (1991), studies a pricing problem for a supply chain with two competing manufacturers and one common retailer who sells both manufacturers' products.

In the supply chain management domain, recent research has studied buyer-supplier relationship asymmetries (e.g., Villena & Craighead, 2017) and used these differences to explore relationships across firms, allowing the development of important new insights.

An increasing number of authors devoted their work to emphasise the usefulness and advantages of the game theoretic approach in the purchasing and supply chain sector (Jalali Naini et al., 2011, p. 594).

None of the studies conducted in this regard have focused on the interaction and competition among retailers. However, competition among retailers can affect retail prices for a particular brand (Chintagunta, 2002).

A study has also conducted in which simultaneously the ordering and pricing intervals considered as decision variables. These variables used to design the Stackelberg game in a supply chain with a manufacturer and several retailers (Yu et al., 2009). Different aspects of supply chain management like collaborative relationships on supply chain and supply chain integration are important subjects of the firm competitiveness research agenda, considering the

influence of multiple dimensions of supply chain collaboration (Mandal, 2015). In this research, the game theory approach used to create a coordination between pricing policies and inventory control. In research, there is usually less attention to market structure in terms of equal power between producer and retailer. In research, the concept of Nash equilibrium has used to examine and understand the individual behavior of supply chain members.

To examine the coordination problem of a supply chain with different qualities and uncertain demand, Yin et al (2016) utilized a game-theoretical approach between a manufacturer and multi-suppliers as the chain partners.

Turki et al. (2017), further transformed the hub-and-spoke supply chain into a closed-loop supply chain, and proposed optimization model to enhance its sustainability.

Hjaila et al. (2015), represents the individual objectives as non-zero-sum non-cooperative Stackelberg game considering the risk associated with the uncertain nature of the third parties.

One of the researches develop a framework on the basis of the game theory in combination with a multi-objective optimization to improve tactical decision making in supply chains of interest and to allow managers to specify objectives (Zamarripa et al., 2013).

In this paper, decision-making issues will examine in a two-level supply chain where different manufacturers and distributors located. In this supply chain, there is equal power for all members, and members decide on non-participation.

3. Methodology

In general, the purpose of this paper is to optimize pricing decisions and inventory control, as well as maximize the individual profit of each member, i.e., producer, and distributor.

This paper answers two levels of questions:

The first part relates to questions raised at the distributor's level, and part two of the questions relate to issues that the manufacturer must decide on.

To answer these questions, a framework for game theory developed and designed. In this game, coordinating issues in decision making form a non-cooperative game. We also assume that the information is available in the correct and complete way. The competition between distributors designed as

sub-games that shape the original game with the manufacturer. The game deals with the establishment of a non-cooperative equilibrium, such as that found in Nash equilibrium. In this situation, the chain members alone can upgrade their own profits without weakening the performance of others. Meanwhile, analytical and computational methods implemented to achieve equilibrium in this type of game.

Finally, to justify the game model and the solution algorithm, the implementation process presented in numerical form. Sensitivity analysis has used to test the effect of market and production-related parameters on decision making and profitability of supply chain members.

3.1. Assumptions

3.1.1. Explaining the shape and assumptions

The chain studied in this paper includes the buyer and seller. As manufacturer purchases raw materials for the production and after the production, it delivers to various distributors. Products have the ability to replace and this is the reason for competition between distributors. Non-cooperative distributors agree to negotiate with the manufacturer in general terms about pricing and inventory control decisions and thereby increase their profits. After the equilibrium achieved, the manufacturer adjusts its production process, and distributors will buy products and distribute it to customers.

The following assumptions can consider:

First assumption: Every distributor sells only one type of product. Given this, a production- inventory model created (Lu, 1995).

Second assumption: The time to replenish the inventory of raw materials is an integral part of the time period set by the manufacturer. Also, it is the multiple integer of replenishment time of distributors (Goyal, 1997; Moutaz, 2002).

Third assumption: The raw materials used in the products are different.

Fourth hypothesis: When the annual production capacity is greater than or equal to the annual market demand, it is not permissible for the manufacturer to inventory deficit (Esmaili et al., 2009).

Fifth hypothesis: Both distributors and manufacturer are rational decision makers and have equal market power.

3.2. Model formation

3.2.1. Game map

In this game, the competition between the distributors designed as sub-games (DS) and the main games formed between the distributors and the manufacturer (MD). There is a kind of dynamics in this game. Manufacturer strategies can influence the strategies of distributors in their subcontracted and competitive games. Distribution strategies can also affect overall game strategies with the manufacturer (Kim et al., 2003).

In the course of these interactions, which occur between subsidiary game distributors (DS) and general game with the manufacturer (MD), Both manufacturers and distributors can further optimize their pricing and inventory control decisions, as well as maximize their desired outcomes. As stated above, there will be no agreement or commitment. Sub-games between rival distributors occur in the main game with the manufacturer, and both forms of play are concurrent and non-cooperative.

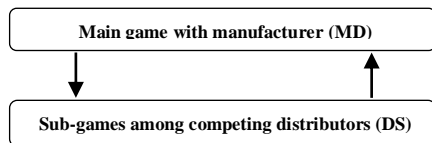


Figure1: The overall structure of the game

3.2.2. Distributors Model

Distributors aim to maximize the profit network by optimizing the strategy (X_{d_i}). Payoff function for distributors (Z_{d_i}), X_{d_i} consisted of profit margin (G_{d_i}) and replenishment decision (K_{d_i}).

The linear demand function widely used in marketing literature. In this research, the MC Guaier and Staelin (1983) and Choi (1996) demand functions used:

$$D(P_{d_i}) = A_{d_i} - e_{d_{il}} P_{d_i} + \sum_{j=1,2,\dots,L,j \neq i} e_{d_{ij}} P_{d_j}$$

Because the distributor products have the ability to replace, so $e_{d_{ij}} \geq 0$ (Samuelson, 1947).

After calculations, it is clear that the demand function is a convex function with a downward slope.

The annual cost of the asset is:

$$\frac{T D d_l}{2 K d_l} h_{d_l}$$

The cost of the order process is:

$$\frac{O_{d_l} K_{d_l}}{T}$$

Finally, we can determine the objective function of the distributor 1 as follows:

$$\text{Max } Z_{d_1} = G_{d_1} D d_1 - \frac{T D d_l}{2 K d_l} h_{d_l} - \frac{O_{d_l} K_{d_l}}{T}$$

The annual demand limits do not exceed the annual production capacity.

$$0 \leq D d_1 \leq P d_1$$

3.2.3. Manufacturer model

The manufacturer's goal is to set up a strategy (x_m) which the profit margins (G_{m_i}), Production launch times (T) and raw material procurement decisions (n_{m_i}) lead to maximizing manufacturer payoff (Z_m).

The production period for each product 1 is:

$$\frac{T D d_l}{P_l}$$

The annual cost of inventory of raw materials is:

$$\frac{h_{m s_{lv_l}}}{2 P_l} (\delta_{s_{lv_l}} D d_l)^2 T + \frac{(N_{m_i} - 1) h_{m s_{lv_l}}}{2} \delta_{s_{lv_l}} D d_l T$$

The inventory level for product 1 shown in Figure 3-2-A. Raw material cost per product 1 is:

$$\frac{O_{m_i}}{n_{m_i} T}$$

So we can easily extract the manufacturer's payoff function:

$$\begin{aligned} \text{Max}_{G_{m_i}, n_{m_i}, T_{n=1,2,\dots,L}} Z_m = & \sum_L G_{m_i} D d_l - \frac{T}{2} \sum_L [h_{m p_l} D d_l [1 + \frac{1}{K d_l} - \frac{D d_l}{P_l}]] \\ & - \sum_L \sum V_l [\frac{h_{m s_{lv_l}}}{2 P_l} (\delta_{s_{lv_l}} D d_l)^2 T \\ & + \frac{(n_{m_i} - 1) h_{m s_{lv_l}}}{2} \delta_{s_{lv_l}} D d_l T] - \frac{S_m}{T} \\ & - \sum_L \end{aligned}$$

$$\frac{O_{m_i}}{n_{m_i} T} - D_m$$

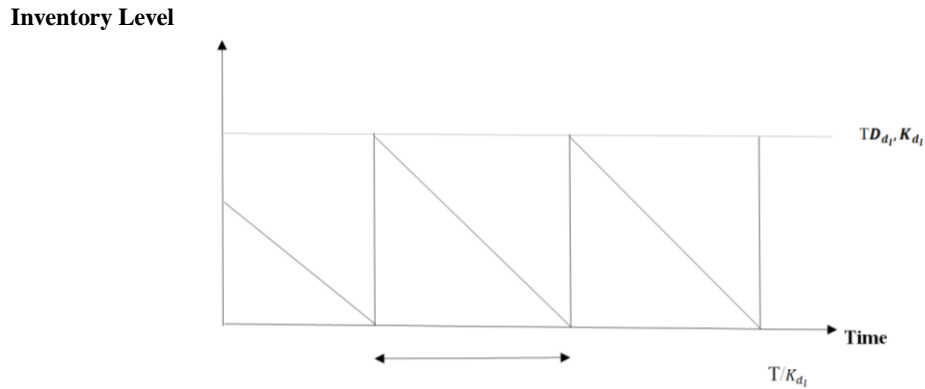


Figure 3-2-A: Final inventory level for distributor 1

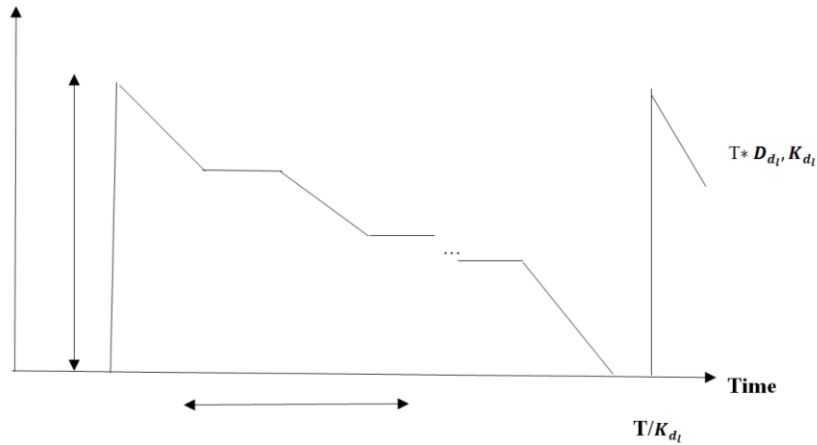


Figure 3-2B: Launch period for product 1

3.2.4. The overall game model with the manufacturer (MD)

Given the game structure shown in Figure 1-3 as well as the payoff function of all the players identified in the previous section, we can now formulate the overall gaming model of the manufacturer's distributors (MDs) as follows:

$$Max_{G_{m_1, n_{m_1, T}}} Z_m$$

$$Max_{G_{d_l, K_{d_l}}} Z_{d_l} \quad l=1,2,\dots,L$$

Distributors have the same cost structure. Therefore, we can use a fixed form to represent their payoff function.

Z_m , Is the manufacturer's model and shows the overall game between distributors and manufacturer (MD).

3.3. Solution Algorithm

The Nash equilibrium concept is the most well-known non-cooperative solution in game theory and widely used in non-cooperative dynamic games (Basar and Olsder, 1982). We also use the concept of "Nash equilibrium" to analyze supplier, manufacturer and distributor strategies in the overall game with the manufacturer (MD). In order to achieve the Nash equilibrium, we use decisions as input parameters. In this case, each player, in addition to making the right decision, can make decisions in order to prepare for possible changes in the decisions of other actors and thereby maximize their profit. As long as actors are

reluctant to change their decisions, the process continues. If any players unilaterally make changes to their decisions, they will cause damage to the manufacturer. In order to determine the equilibrium in the overall game between the distributors and the manufacturer (MD), we first use the analytic method to calculate the best reaction function of each player, and then, by applying the algorithm of the solution, create the Nash equilibrium we do.

3.3.1. Reaction function

3.3.1.1. Distributors Reaction

So, the distributor's reaction function corresponds to its profit margins. Thus:

$$D_{d_1} = A_{d_1} e_{r_{11}} \left[G_{d_1} + G_{m_1} + \sum V_l \delta_{s_{lv_1}} C_{s_{lv_1}} + C_{m_1} \right] + \sum_{j \neq 1}^j e_{d_{1j}} [G_{d_j} + G_{m_j}] + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j}]$$

If we assume that the decision variables for the supplier and the manufacturer are constant, the optimal value for replenishment cycle by the distributor is as follows:

$$Min_{K_{d_1}}^{U_{d_1}} = \frac{TD_{d_1} h_{d_1}}{2K_{d_1}} + \frac{K_{d_1} O_{d_1}}{T}$$

The optimum K_{d_1} can expressed as:

$$K_{d_1}^* = \left[\frac{(\sqrt{1 + 2T^2 h_{d_1} D_{d_1}})}{O_{d_1}} \right] / 2]$$

The optimal value of θ_{d_1} is:

$$G_{d_1} = \frac{1}{e_{d_{1l}}} (A_{d_1} + \sum_{j \neq l}^j = 1, 2, \dots, L e_{d_{1j}} (G_{d_j} + G_{m_j} + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j})) (G_{m_l} + \sum V_l \delta_{s_{lv_l}} C_{s_{lv_l}} + C_{m_l})$$

It can be seen that Z_{d_1} is a quadratic function of G_{d_1} function:

$$\frac{\partial^2 Z_{d_1}}{\partial G_{d_1}^2} = -2e_{1l} < 0$$

So we conclude that Z_{d_1} function is a concave function of G_{d_1} function.

G_{d_1} , is available as follows:

$$G_{d_1} = \frac{C_l}{2e_{d_{1l}}} + \frac{h_{d_1} T}{4K_{d_1}}$$

So the optimal sales reaction will be within the G_{d_1} lower and upper limit.

Case A1:

When the retail market is less sensitive to price, there is a good opportunity for distributors to maximize profits by raising the price.

Case B1:

When a product has fewer substitutes, retailers can maximize their profits by reducing their prices.

If the optimal G_{d_1} obtained at the lower limit and the upper limit of the G_{d_1} function, then:

$$\frac{\partial G_{d_1}}{\partial e_{d_{1j}}} = \frac{(G_{d_j} + G_{m_j} + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j})}{2e_{d_{1l}}} > 0$$

Otherwise if $G_{d_1}^*$ is not in the allowed range:

$$\frac{\partial G_{d_1}}{\partial e_{d_{1l}}} = 0, -\frac{1}{e_{d_{1l}}^2} (A_{d_1} - P_l + \sum_{j \neq l}^j = 1, 2, \dots, L e_{d_{1j}} (G_{d_j} + G_{m_j} + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j})) \leq 0$$

$$\frac{\partial G_{d_1}}{\partial e_{d_{1l}}} = -\frac{1}{e_{d_{1l}}^2} (A_{d_1} + \sum_{j \neq l}^j = 1, 2, \dots, L e_{d_{1j}} (G_{d_j} + G_{m_j} + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j})) < 0$$

$$\frac{\partial G_{d_1}}{\partial e_{d_{1j}}} = 0, \frac{1}{e_{d_{1l}}} (G_{d_j} + G_{m_j} + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j}) \geq 0$$

$$\frac{\partial G_{d_1}}{\partial e_{d_{1j}}} = \frac{1}{e_{d_{1l}}} (G_{d_j} + G_{m_j} + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j}) > 0$$

When P_{m_l} is fixed, then:

$$\frac{\partial P_{d_l}}{\partial e_{d_{ll}}} = \frac{\partial G_{d_l}}{\partial e_{d_{ll}}}$$

So, the case completed.

3.3.1.2. Manufacturer's reaction:

We assume that the decision variables are constant for distributors. The manufacturer's problem is to find the optimal Production launch courses:

$$\begin{aligned} \text{Min}_T U_m = & \frac{T}{2} \sum L \left(D_{d_l} \left(1 + \frac{1}{K_{d_l}} - \frac{D_{d_l}}{P_l} \right) h_{mp_l} \right) + \frac{S_m}{T} \\ & + \sum L \frac{O_{m_l}}{N_{m_l} T} \end{aligned}$$

By calculating the second derivative of the above relation, the optimum can extract from the following equation:

$$T^* = \sqrt{\frac{S_m + \sum L \frac{O_{m_l}}{N_{m_l}}}{\frac{1}{2} \sum L \left(D_{d_l} \left(1 + \frac{1}{K_{d_l}} - \frac{D_{d_l}}{P_l} \right) h_{mp_l} \right)}}$$

The manufacturer's problem is to find the optimal conditions for replenishment of raw materials for the production of product l as follows:

$$\text{Min}_{n_{m_l}}^{U_{sv}} = \sum L \left(\sum v_l \frac{(n_{m_l} - 1) h_{ms_{lv_l}}}{2} \delta_{s_{lv_l}} D_l T + \frac{O_{m_l}}{n_{m_l}} \right)$$

Z_m , is a quadratic function of G_{m_l} function. So, we'll calculate the lower band and the upper band of G_{m_l} function.

The second derivative of π_m for G_{m_l} is:

$$\begin{aligned} \frac{\partial^2 Z_m}{\partial G_{m_l}^2} = & -2e_{d_{ll}} + \sum_{i=1,2,\dots,L} (h_{mp_i} \\ & - \sum_{v_i} h_{ms_{jv_j}} \delta_{s_{lv_l}}^2) \frac{e_{d_{ll}}^2 T}{P_i} \end{aligned}$$

If, $\frac{\partial^2 Z_m}{\partial G_{m_l}^2} \leq 0$, so optimal G_{m_l} is:

$$G_{m_l} = \frac{B_{l1} + B_{l2} Q_l + B_{l3} + X_l}{e_{d_{ll}} + e_{d_{ll}} B_{l2} - B_{l3}}$$

However:

$$\begin{aligned} B_{l1} = & \sum_{j=1,2,\dots,L; j \neq l} \frac{e_{r_{lj}} T}{P_j} \left(h_{mp_j} \sum V_j h_{ms_{jv_j}} \delta_{s_{jv_j}}^2 \right) \left(A_{d_j} \right. \\ & \left. - e_{d_{jj}} P_j + \sum_{i \neq j, l} e_{d_{ji}} P_i \right); \end{aligned}$$

$$B_{l2} = 1 - \frac{e_{d_{ll}} T}{P_j} \left(h_{mp_l} - \sum_{v_l} h_{ms_{lv_l}} \delta_{s_{jv_j}}^2 \right);$$

$$B_{l3} = \sum_{j=1,2,\dots,L; j \neq l} \frac{e_{d_{lj}}^2 T}{P_j} \left(h_{mp_j} \sum V_j h_{ms_{jv_j}} \delta_{s_{jv_j}}^2 \right);$$

$$\begin{aligned} Q_l = & A_{d_l} - e_{d_{ll}} \left(G_{d_l} \sum_{v_l} \delta_{s_{lv_l}} C_{s_{lv_l}} + C_{m_l} \right) \\ & + \sum_{j=1,2,\dots,L; j \neq l} e_{d_{lj}} \left(G_{d_j} + G_{m_j} \right. \\ & \left. + \sum V_j \delta_{s_{jv_j}} C_{s_{jv_j}} + C_{m_j} \right); \end{aligned}$$

$$\begin{aligned} X_l = & \sum_{j=1,2,\dots,L; j \neq l} e_{d_{lj}} G_{m_j} \left(\frac{h_{mp_i}}{P_i} \left(1 \right. \right. \\ & \left. \left. + \frac{1}{K_{d_l}} \right) \sum_{v_i} \frac{(n_{m_l} - 1)}{2} h_{ms_{jv_j}} \delta_{s_{jv_j}} \right) e_{d_{ll}} T, e_{d_{ll}}; i = 1 - e_{d_{ll}}; i \\ & \neq 1 \end{aligned}$$

If the calculated G_{m_l} is within the lower band and the upper band, G_{m_l} function will be optimal.

In other words; when G_{m_l} located on its boundaries, Z_m reaches its highest value.

3.3.2. Numerical calculation of equilibrium (An algorithm for equilibrium)

When the overall game formed consecutively between the distributors and the manufacturer, the backward analysis technique can help to understand sub-games trends for equilibrium (Olsder and Basar, 1982).

So, to understand the Nash equilibrium, we first analyze the games formed between the distributors (DS) and then the overall game between the distributors and the manufacturer. The numerical calculation solution is as follows:

Preparation stage: First, enter the initial values into the strategy set X , which is the following:

$$X^0 = (X_m^0, X_d^0)$$

Step 1: X_{-m}^0 means the strategy set for all members of the supply chain other than the manufacturer. After the value of X_{-m}^0 determined, we compute the optimal manufacturer response (X_m^*);

$$X_m^* = (G_{m_1}^*, \dots, G_{m_L}^*, T, n_{m_1}^*, n_{m_L}^*)$$

To this end, we optimize the Z_m payoff function in the X_m strategy set.

Step Two: X_{d-l} means the entire strategy of all chain members except distributor l . When X_{d-l}^* determined, we calculate the optimal retail reaction l ($X_{d_l}^*$), by calculating (Z_{d_l}); the payoff function will be optimized.

Step Three: $X^* = (X_m^*, X_d^*)$ if $|X^* - X^0| \leq \epsilon_2$, X^* will be the overall game equilibrium between the distributors with the manufacturer.

4. Results

Numerical results are shown in table 1, part A and B:

In this paper, a model of the game theory for a two-level supply chain developed in relation to problems derived from analytic statements and the implementation of the solution algorithm. The most important achievement of this research is to optimize the competition between distributors.

This article has considered how the manufacturer interacts with multiple competitor distributors. Each distributor also determines the optimal distribution price and inventory replenishment policies to maximize their profits. The issue forms a non-cooperative dynamic game. At first, sub-games formed between competing distributors, and then the distribution departments form the original game as a set with the manufacturer. To determine the Nash equilibrium in this game, the analytical method and solution procedure have been used.

A: Manufacturer results

Table 1. Results for producer and distributor

Example Data		P_{m_1}	P_{m_2}	P_{m_3}	T	n_{m_1}	n_{m_2}	n_{m_3}	$Z_m (\times 10^7)$
A_{d_1}	180,000	77.69	104.79	175.27	0.0714	1	1	1	1.2554
	220,000	72.85	99.22	156.13	0.0741	1	1	1	1.1400
$e_{d_{11}}$	1500	81.34	105.96	179.17	0.0711	1	1	1	1.2946
	1700	74.47	103.64	171.41	0.0717	1	1	1	1.2193
$e_{d_{12}}$	80200	76.91	103.63	172.19	0.0716	1	1	1	1.2365
	5	81.69	110.91	191.14	0.0702	1	1	1	1.3475
C_{m_1}	25	71.54	109.23	190.52	0.0702	1	1	1	1.3341
	200	83.83	100.35	160.02	0.0733	1	1	1	1.1689
S_m	1000	77.69	104.79	175.27	0.0447	1	1	2	1.2582
	150,000	77.69	104.80	175.28	0.1029	1	1	1	1.2500
P_1	800,000	77.67	104.79	175.26	0.0782	1	1	1	1.2542
	1	77.69	104.79	175.27	0.0707	1	1	1	1.2554
h_{mp_1}	7	77.69	104.79	175.28	0.0857	1	1	1	1.2553
	2/5	77.67	104.78	175.25	0.0434	1	1	2	1.2552
h_{d_1}	18	77.70	104.80	175.31	0.0691	1	1	1	1.2565
		77.67	104.78	175.22	0.0733	1	1	1	1.2538

B: Distributor results

		P_{d_1}	P_{d_2}	P_{d_3}	K_{d_1}	K_{d_2}	K_{d_3}	$D_{d_1} (\times 10^4)$	$D_{d_2} (\times 10^4)$	$D_{d_3} (\times 10^4)$
Example data		111.69	158.00	287.24	5	6	3	5.4323	7.4441	4.4770
A_{d_1}	180,000	102.73	154.46	277.16	5	7	4	4.7712	7.7291	4.8394
	220,000	120.66	161.54	297.33	6	6	3	6.0942	7.1595	4.1150
$e_{d_{11}}$	1500	118.45	158.92	289.50	5	6	3	5.5587	7.4093	4.4109
	1700	105.74	157.11	285.06	5	6	3	5.3061	7.4820	4.5435
$e_{d_{12}}$	80200	110.27	157.30	285.62	5	6	3	5.3288	7.5087	4.5350
	5	119.02	161.68	295.65	5	6	3	5.9642	7.1028	4.1779
C_{m_1}	25	108.84	160.44	294.89	5	6	3	5.9591	7.1649	4.1726
	200	114.55	155.55	279.61	5	6	4	4.9055	7.7233	4.7816
S_m	1000	111.70	158.00	287.25	3	4	2	5.4320	7.4445	4.4771
	150,000	111.69	158.00	287.25	8	9	5	5.4328	7.4439	4.4770
P_1	800,000	111.68	158.00	287.34	6	7	4	5.4342	7.4443	4.4774
	1	111.70	158.00	287.25	5	6	3	5.4321	7.4441	4.4769
h_{mp_1}	7	111.70	158.00	287.25	6	7	4	5.4319	7.4439	4.4770
	2/5	111.69	157.99	287.24	3	4	2	5.4334	7.4447	4.4774
h_{d_1}	18	111.69	157.01	287.27	3	6	3	5.4338	7.4434	4.4761
		111.70	157.99	287.23	8	6	3	5.4311	7.4449	4.4778

Continue Table B

$Z_{d_1} (\times 10^6)$	$Z_{d_2} (\times 10^6)$	$Z_{d_3} (\times 10^6)$
1.7688	3.8641	4.9883
1.3538	4.1482	5.8203
2.2208	3.5727	4.2127
1.9830	3.8281	4.8419
1.5819	3.9036	5.1379
1.7003	3.9320	5.1189
2.1417	3.5151	4.3430
2.1380	3.5778	4.3320
1.4338	4.1605	5.6820
1.8170	3.9200	5.0022
1.6675	3.7527	4.9544
1.7454	3.8376	4.9779
1.7694	3.8650	4.9884
1.7343	3.8256	4.9737
1.8186	3.9213	5.0031
1.8317	3.8664	4.9872
1.5603	3.8625	4.9899

5. Discussion and Conclusions

Interesting findings summarized in more detail:

First: manufacturer and distributors can gain more profit by increasing market scale, product replacement or decreasing price elasticity. While this happens to one side, it creates conflicting conditions for others. Reducing the value of $e_{d_{1i}}$ indicates that market demand is less sensitive to retailer 1. The larger " $e_{d_{12}}$ ", indicates more replacement capacity for product 1 and 2. According to Table 1, the manufacturer is more

likely to produce product 1. Because there is a bigger demand market for product 1. In this case, the manufacturer's desire to produce other products reduced. The rise in prices and the drop in market demand for other distributors will reduce their profits.

Second: when the cost of producing a product by the manufacturer increases, the profit of the product will clearly decrease, but the profit of other distributors will increase. A larger C_{m_1} represents higher costs for product 1 production. Therefore, the

distributor of this product will notice a significant decreased in profits. In this case, the manufacturer tries to increase his profit through the production of other products, thereby covering the decreased liquidity caused by the production of the product 1. So the manufacturer will reduce the wholesale price to stimulate market demand. In this process, other distributors will benefit from lower prices.

Along with all this, rising product prices and rising market demand for other products could lead to competition between products and will benefit distributors as well.

Third, the manufacturer's production capacity, launch costs, and implementation costs are the parameters that have a definite effect on the start of production periods. However, these parameters have a significant impact on the pricing decisions of the supply chain members. Pricing decisions are less sensitive to parameters of " $h_{mp1}, h_{d1}, P_1, S_m$ ". Because these parameters affect most of the inventory decisions.

Fourth: Any increase in the cost of a finished product by the manufacturer may increase the profit of the distributor.

If the distributor's inventory costs increase, the manufacturer's launch period will be long. On the other hand, the inventory refill period will shorten. The reason for this is the distributor's action to reduce inventory, and this will done through the frequency of more orders. However, the manufacturer is reluctant to share more stock with distributors. In this case, the manufacturer would prefer to have longer launch intervals.

Research limitations and suggestions for future research

In this paper, the issue of coordination in a two-level supply chain examined. We assume that the production rate is greater than or equal to the demand rate. In this case, there will be no shortage of costs. Without this assumption, the cost should consider in the model.

Other types of demand functions such as exponential demand function can used to analyze pricing and inventory decisions.

To gain more efficiency, it is necessary to use a more sophisticated solution algorithm to solve this model. For example, the use of an exploratory solution algorithm suggested.

The present study and comparisons conducted for different game structures can be a good guide for business owners to take appropriate policies.

References

- 1) Başar, T. and Olsder, G.J., (1998). Dynamic noncooperative game theory. Society for Industrial and Applied Mathematics.
- 2) Boyacı, T. and Gallego, G., (2002). Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. *International Journal of Production Economics*, 77(2), pp.95-111.
- 3) Cachon, G.P. and Netessine, S., (2006). Game theory in supply chain analysis. In *Models, Methods, and Applications for Innovative Decision Making* (pp. 200-233). INFORMS.
- 4) Chen, F.Y., Yan, H. and Yao, L., (2004). A newsvendor pricing game. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 34(4), pp.450-456.
- 5) Chintagunta, P.K., (2002). Investigating category pricing behavior at a retail chain. *Journal of Marketing Research*, 39(2), pp.141-154.
- 6) Choi, S.C., (1991). Price competition in a channel structure with a common retailer. *Marketing Science*, 10(4), pp.271-296.
- 7) Choi, S.C., (1996). Price competition in a duopoly common retailer channel. *Journal of Retailing*, 72(2), pp.117-134.
- 8) Esmaeili, M., Aryanezhad, M.B. and Zeephongsekul, P., (2009). A game theory approach in seller-buyer supply chain. *European Journal of Operational Research*, 195(2), pp.442-448.
- 9) Goyal, S.K., (1977). An integrated inventory model for a single supplier-single customer problem. *The International Journal of Production Research*, 15(1), pp.107-111.
- 10) Hjalila, K., Puigjaner, L. and Espuña, A., (2015). Scenario-based price negotiations vs. game theory in the optimization of coordinated supply chains. In *Computer Aided Chemical Engineering* (Vol. 37, pp. 1859-1864). Elsevier.
- 11) Naini, S.G.J., Aliahmadi, A.R. and Jafari-Eskandari, M., (2011). Designing a mixed performance measurement system for environmental supply chain management using

- evolutionary game theory and balanced scorecard: A case study of an auto industry supply chain. *Resources, Conservation and Recycling*, 55(6), pp.593-603.
- 12) Jeuland, A.P. and Shugan, S.M., (1988). Note—Channel of Distribution Profits When Channel Members Form Conjectures. *Marketing Science*, 7(2), pp.202-210.
 - 13) Kim, D. and Lee, W.J., (1998). Optimal joint pricing and lot sizing with fixed and variable capacity. *European Journal of Operational Research*, 109(1), pp.212-227.
 - 14) Kim, J.H., Park, J.B., Park, J.K. and Kim, B.H., (2003). A new game-theoretic framework for maintenance strategy analysis. *IEEE Transactions on Power Systems*, 18(2), pp.698-706.
 - 15) Liu, B., (1998). Stackelberg-Nash equilibrium for multilevel programming with multiple followers using genetic algorithms. *Computers & Mathematics with Applications*, 36(7), pp.79-89.
 - 16) Lu, L., (1995). A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research*, 81(2), pp.312-323.
 - 17) Mandal, S., (2015). An empirical-collaborative model of supply chain agility. *International Journal of Logistics Systems and Management*, 21(4), pp.465-502.
 - 18) McGuire, T.W. and Staelin, R., (1983). An industry equilibrium analysis of downstream vertical integration. *Marketing science*, 2(2), pp.161-191.
 - 19) Khouja, M., (2003). Optimizing inventory decisions in a multi-stage multi-customer supply chain. *Transportation Research Part E: Logistics and Transportation Review*, 39(3), pp.193-208.
 - 20) Parlar, M. and Wang, Q., (1994). Discounting decisions in a supplier-buyer relationship with a linear buyer's demand. *IIE transactions*, 26(2), pp.34-41.
 - 21) Pashigian, B.P., (1961). The Distribution of Automobiles. An Economic Analysis of the Franchise System.
 - 22) Joglekar, P., Tavana, M. and Rappaport, J., (2006). A Comprehensive Set of Models of Intra and Inter-Organizational Coordination in a Supply Chain.
 - 23) Samuelson, P.A., (1947) *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard University Press, 1947). Samuelson Foundations of Economic Analysis
 - 24) Sinha, S. and Sarmah, S.P., (2010). Coordination and price competition in a duopoly common retailer supply chain. *Computers & Industrial Engineering*, 59(2), pp.280-295.
 - 25) Turki, S., Didukh, S., Sauvey, C. and Rezg, N., (2017). Optimization and analysis of a manufacturing–remanufacturing–transport–warehousing system within a closed-loop supply chain. *Sustainability*, 9(4), p.561.
 - 26) Villena, V.H. and Craighead, C.W., (2017). On the same page? How asymmetric buyer–supplier relationships affect opportunism and performance. *Production and Operations Management*, 26(3), pp.491-508.
 - 27) Weng, Z.K., (1995). Modeling quantity discounts under general price-sensitive demand functions: optimal policies and relationships. *European Journal of Operational Research*, 86(2), pp.300-314.
 - 28) Weng, Z.K. and Wong, R.T., (1993). General models for the supplier's all-unit quantity discount policy. *Naval Research Logistics (NRL)*, 40(7), pp.971-991.
 - 29) Woo, Y.Y., Hsu, S.L. and Wu, S., (2001). An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction. *International Journal of Production Economics*, 73(3), pp.203-215.
 - 30) Yin, S., Nishi, T. and Zhang, G., (2016). A game theoretic model for coordination of single manufacturer and multiple suppliers with quality variations under uncertain demands. *International Journal of Systems Science: Operations & Logistics*, 3(2), pp.79-91.
 - 31) Yu, Y., Chu, F. and Chen, H., (2009). A Stackelberg game and its improvement in a VMI system with a manufacturing vendor. *European Journal of Operational Research*, 192(3), pp.929-948.
 - 32) Zamarripa, M.A., Aguirre, A.M., Méndez, C.A. and Espuña, A., (2012). Improving supply chain planning in a competitive environment. *Computers & Chemical Engineering*, 42, pp.178-188.
 - 33) Zamarripa, M.A., Aguirre, A.M., Méndez, C.A. and Espuña, A., (2013). Mathematical programming and game theory optimization-based tool for supply chain planning in cooperative/competitive environments. *Chemical*

Engineering Research and Design, 91(8), pp.1588-1600.

Appendix A: Symbols

All input parameters and variables used in the model are specified as follows:

Distributor's Section:

L	Number of Distributors
d_l	Distributor Index l
R_{d_l}	Annual fixed cost of the distributor d_l
A_{d_l}	The stability of the distributor's demand function also shows the market scale.
e_{d_l}	Price elasticity of products for the distributor's demand l.
$e_{d_{jl}}$	The coefficient associated with the replacement degree of product j by product l.
h_{d_l}	Cost per unit of product for distributor l.
P_{d_l}	The designated distributor price for customers by the distributor l.
D_{d_l}	Annual demand for Distributor l
G_{d_l}	The profit margin of Distributor l
K_{d_l}	The integer divisor used to determine the inventory replenishment of the distributor's l

Manufacturer's Section:

m	Manufacturer index
V_l	Total raw material required for product l
S_{lv_l}	The raw material indicator used in each v_l
h_{mvl}	Cost of each unit product l
h_{mslv_l}	The cost of having the raw material in each unit v_l used for product l
δ_{slv_l}	The raw materials contained in v_l used for per unit product l.
S_m	Setup cost per production
O_{m_l}	The cost of the procurement process for raw materials used in per order of product l
p_l	The annual production capacity of l is considered constant.
R_m	Producer fixed annual costs for equipping and organizing product production
P_{m_l}	The fixed price of the products is mainly for supply to the retailer l
C_{m_l}	Production cost per unit product l.
C_{slv_l}	The cost of purchasing the raw materials in each v_l used in the product l
G_{m_l}	Profit Margin of manufacturer for product l
T	Time to set up cycles by the manufacturer
n_{m_l}	Integer multipliers used to determine the supply cycle of raw materials required in product l