Value at Risk Estimation using the Kappa Distribution with Application to Insurance Data

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ABSTRACT

The heavy tailed distributions have mostly been used for modeling the financial data. The kappa distribution has higher peak and heavier tail than the normal distribution. In this paper, we consider the estimation of the three unknown parameters of a Kappa distribution for evaluating the value at risk measure. The value at risk (*VaR*) as a quantile of a distribution is one of the important criteria for financial institution risk management. The maximum likelihood, moment, percentiles and maximum product of spacing methods are considered to estimate the unknown parameters. The data of the insurance stock prices is analyzed for comparing the proposed methods in *VaR* evaluation. An important implication of the present study is that the Kappa distribution can be considered as a loss distribution for the *VaR* estimation. Also, it is observed that the maximum likelihood estimator, in contrast to other estimators, provides smallest *VaR* in the proposed stock prices data.

Keywords:

Insurance Stock Price; Kappa distribution; Maximum Product of Spacing; Percentile Estimator; Value at Risk.

1. Introduction

The Kappa distribution was introduced in the literature by Singh and Maddala (1976) as the member of generalized Beta distribution of second kind. The Kappa distribution is considered to have useful properties, such as flexibility, skewness and heavy tail; also, it has explicit forms for percentiles and moments. The distribution function of the Kappa distribution is given by

$$F(x;\alpha,\beta,\gamma)=(\frac{1+(x/\beta)^{\alpha}}{(x/\beta)^{\alpha}})^{-\gamma};\quad x>0,\ \alpha,\beta,\lambda>0, \tag{1}$$

and the corresponding quantile function

$$X(F)=(F^{-1/\gamma}-1)^{-1/\alpha}\beta. \tag{2}$$

The probability density function is given by

$$f(x;\alpha,\beta,\gamma)=\alpha\beta^{-\gamma\alpha}\gamma x^{\alpha\gamma-1}(1+(x/\beta)^{\alpha})^{-(\gamma+1)};$$

$$x>0,\ \alpha,\beta,\lambda>0 \tag{3}$$

Here β is a scale parameter and α, γ are the shape parameters; γ only affects the right tail, whereas α affects both tails. Figure 1 gives different shapes of the Kappa density function for various parameter specifications. The r^{th} central moment of the Kappa distribution can be written as

$$E(X^{r})=\frac{\beta^{r}\Gamma(\gamma+r/\alpha)\Gamma(1-r/\alpha)}{\Gamma(\gamma)},$$

where $\Gamma(.)$ denotes the gamma function. Furthermore, the variance, skewness and kurtosis are obtained as follows:

$$Var(X)=\frac{\beta^{2}\ \left(\Gamma(\gamma)\Gamma(\gamma+2/\alpha)\Gamma(1-2/\alpha)-\Gamma^{2}(\gamma+1/\alpha)\Gamma^{2}(1-1/\alpha)\right)}{\Gamma^{2}(\gamma)}$$

$$Skewness(X)=\frac{\Gamma^{2}(\gamma)\Im_{3}-3\Gamma(\gamma)\Im_{2}\Im_{1}+2\Im_{1}^{3}}{\left(\Gamma(\gamma)\Im_{2}-\Im_{1}^{2}\right)^{3/2}},$$

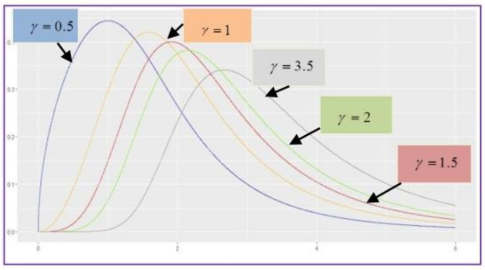
$$Kurtosis(X)=\frac{\Gamma^{3}(\gamma)\Im_{4}-4\Gamma^{2}(\gamma)\Im_{3}\Im_{1}+6\Gamma(\gamma)\Im_{2}\Im_{1}^{2}-3\Im_{1}^{4}}{\left(\Gamma(\gamma)\Im_{2}-\Im_{1}^{2}\right)^{2}}$$

Where, $\Im_{i}=\Gamma(\gamma+i/\alpha)\Gamma(1-i/\alpha);\ \ i=1,2,3,4$.

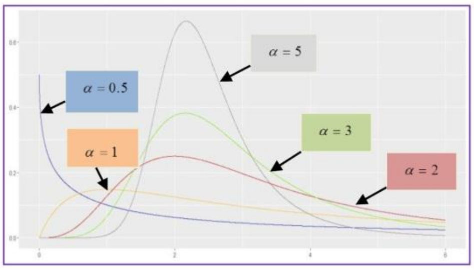
Moreover, the risk management has been intensively used in finance and insurance business. The value at risk is an important measure in risk management. Value at risk is defined as the worst expected loss over a given period at the specified confidence level (δ). Mathematically, $P(X\leq VaR)=\delta$, where *X* is the profit (loss) of the investment over the given time horizon. Also, the statistical loss distribution plays a key role in evaluating the value at risk measure. The main aim of this paper is to evaluate the *VaR* using the Kappa distribution. Based on the Equation (2), the *VaR* can be written as:

$$VaR_{\delta}(X)=F^{-1}(x)=(\delta^{-1/\hat{\gamma}}-1)^{-1/\hat{\alpha}}\hat{\beta}.$$

We proposed different methods, namely, maximum likelihood estimator (MLE), moment estimator (ME), percentiles estimator (PE) and maximum product of spacing estimator (MPS) for estimating the three unknown parameters. In view of above considerations, the rest of the article is organized as follows. In Section 2, the literature review is presented. The different estimation procedures for estimating the three unknown parameters are considered in Section 3. A real life example with data from the stock prices is presented in Section 4 for evaluating the *VaR*. The conclusions are made in Section 5.



$\beta = 2, \alpha = 3$



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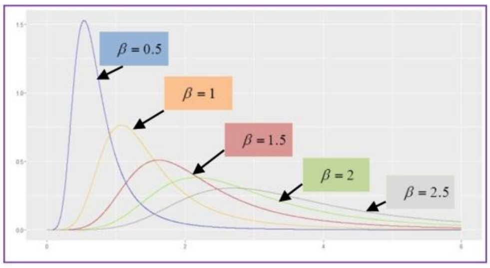

$\gamma = 2, \alpha = 3$

Figure 1. Different shapes of the Kappa density function for various parameter specifications

2. Literature Review

The value at risk is one of the oldest risk measures, which is basically defined as the maximum expected loss for a given probability. In the recent years, this measure has gained some attention among researchers and interesting results have been obtained. For example, Swami et al. (2016) estimated the value at risk for foreign exchange rate risk in India. They used the parametric variance-covariance and non-parametric historical simulation methods and showed that the t-students model can be considered as an adequate model for value at risk estimation. Dupuis et al. (2015) considered a new bias-robust conditional variance estimators based on weighted likelihood at heavy-tailed models for computing the value at risk measure. Mentel (2013) studied the parametric, nonparametric and semi-parametric models for estimating the value at risk. Results of this research indicate that the fat tail distributions present a good performance. Gebizlioglu et al. (2011) considered the Weibull distribution and its quantiles in the value at risk estimation. They used different estimation methods and showed that the maximum likelihood estimators have the best results for predicting the value at risk. Čorkalo (2011) compared the main approaches of calculating *VaR* and implements variance-covariance, historical and bootstrapping approach on stock portfolio. Huang (2009) proposed a process in value at risk estimation with methods of quantile regression and kernel estimator which applies the nonparametric technique with extreme quantile forecasts to realize a tail distribution and locate the value at risk estimates. His results indicate that the proposed approach outperforms others and provides highly reliable estimates. Brandolin and Colucci (2012) compared the *VaR* estimation obtained by two risk models: historical simulation and Monte Carlo filtered bootstrap using unconditional coverage, independence and conditional coverage tests. Abada et al. (2014) studied a theoretical review of the existing literature on *VaR* specifically focusing on the development of new approaches for its estimation. They also considered the backtesting procedures used to evaluate *VaR* approach performance. Sinha and Agnihotri (2015) investigated the effect of non-normality in returns and market capitalization of stock portfolios and stock indices on value at risk and conditional *VaR* estimation. They fitted the return series using Logistic, Weibull and Laplace distributions and observed that *VaR* violations are increasing with decreasing market capitalization. Braione and Scholtes (2016) studied the VaR estimation using the three symmetric and three skewed distributional assumptions. Wong et al. (2016) provided extensive comparison of out-of-sample volatility and *VaR* forecast performance on different equity market indices using 13 risk models. Their results indicated that realized volatility models outperform GARCH models for volatility forecasts, but a simple EGARCH model outperforms the rest models for most of the *VaR* forecasts. Moreover, the heavy tailed distributions play an important role in value at risk estimation. The Kappa distribution is one of the heavy tailed distributions, which has useful properties. Due to its practicality, several authors have considered its properties, inferential methods and applications; for example, Kjeldsen et al. (2017) used the Kappa distribution in regional frequency analysis. Kim (2015) studied the Kappa distribution to analyze the effects of globalization by understanding differences and similarities among Asian countries and developed countries before and after the crisis. Jeong et al. (2014) proposed this distribution for hydrologic application. Livadiotis and McComas (2013) examined the physical foundations and theoretical development of the Kappa distribution. Kumphon (2012) studied the maximum entropy and maximum likelihood estimation methods for evaluating the parameters of the Kappa distribution. Pierrard and Lazar (2010) analyzed the various theories proposed for the Kappa distributions and their valuable applications in coronal and space plasmas. Ashour et al. (2009) and Dupuis and Winchester (2007) considered the different estimation methods for this distribution under complete and censored samples. Considering that so far any research has not used the Kappa distribution to evaluate the value at risk. Therefore, we want to present certain estimation methods for value at risk as a quantile of a Kappa distribution.

3. Methodology

3.1. Different Estimation Methods

Under classical paradigm, a number of estimation methods are available in statistical literature. But we shall be providing here only four of such methods, namely MLE, ME, PE and MPS.

3.1.1. Maximum Likelihood Estimator

This section deals with deriving MLEs of the unknown parameters of a $Kappa(\alpha,\beta,\gamma)$ distribution. Suppose that $X=(X_1,...,X_n)$ is a sample of size *n* from a $Kappa(\alpha,\beta,\gamma)$ distribution. Based on the observation, the likelihood function can be given as follows:

$$L(\alpha,\beta,\gamma)=\prod_{i=1}^{n}\alpha\beta^{-\alpha\gamma}\gamma x_i^{\alpha\gamma-1}(1+(x_i/\beta)^{\alpha})^{-(\gamma+1)}\ . \quad (4)$$

Then, the log-likelihood function in (4) can be written:

$$l(\alpha,\beta,\gamma)=\log(L(\alpha,\beta,\gamma))=n\log\alpha-n\alpha\gamma\log\beta+\log\gamma$$
$$+(\alpha\gamma-1)\sum_{i=1}^{n}\log x_i-(\gamma+1)\sum_{i=1}^{n}\log(1+(x_i/\beta)^{\alpha})\ . \quad (5)$$

Here, we assume that the parameters α, β and γ are unknown. To obtain the normal equations for the unknown parameters, we differentiate (5) partially with respect to α, β and γ and equate to zero as:

$$\frac{\partial l(\alpha,\beta,\gamma)}{\partial\alpha}=\frac{n}{\alpha}-n\gamma\log\beta+\gamma\sum_{i=1}^{n}\log x_i$$
$$-(\gamma+1)\sum_{i=1}^{n}\frac{(x_i/\beta)^{\alpha}\log(x_i/\beta)}{1+(x_i/\beta)^{\alpha}}=0$$

$$\frac{\partial l(\alpha,\beta,\gamma)}{\partial\beta}=-\frac{n\alpha\gamma}{\beta}+(\gamma+1)\sum_{i=1}^{n}\frac{\alpha x_i^{\alpha}\beta^{-\alpha-1}}{1+(x_i/\beta)^{\alpha}}=0\ ,$$

And

$$\frac{\partial l(\alpha,\beta,\gamma)}{\partial\gamma}=\frac{n}{\gamma}-n\alpha\log\beta+\alpha\sum_{i=1}^{n}\log x_i\ .$$
$$-\sum_{i=1}^{n}\log(1+(x_i/\beta)^{\alpha})=0$$

It is observed that the estimations cannot be obtained in closed forms. Numerical methods, such as the Newton-Raphson method, can be used here to solve the above non-linear equations.

3.1.2. Moment Estimators

The moment estimator of the kappa distribution can be evaluated by equating the first three theoretical moments with the sample moments as:

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{\beta\,\Gamma(\frac{1}{\alpha}+\gamma)\Gamma(1-\frac{1}{\alpha})}{\Gamma(\gamma)},$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{\beta^2\,\Gamma(\frac{2}{\alpha}+\gamma)\Gamma(1-\frac{2}{\alpha})}{\Gamma(\gamma)},$$

and

$$\frac{1}{n}\sum_{i=1}^{n} x_i^3 = \frac{\beta^3\,\Gamma(\frac{3}{\alpha}+\gamma)\Gamma(1-\frac{3}{\alpha})}{\Gamma(\gamma)}.$$

3.1.3. Percentile Estimators

Estimation based on percentiles was originally explored by Kao (1958). Let $X_{1:n},\ldots,X_{n:n}$ be the order statistics of a random sample of size n from $Kappa(\alpha,\beta,\gamma)$ distribution. If p_i denotes an estimate of $F(x_{i:n},\alpha,\beta,\gamma)$, then the percentile estimators of the parameters α, β and γ can be obtained by minimizing, with respect to α, β and γ the function:

$$\sum_{i=1}^{n}\left[x_{i:n}-F^{-1}(p_i)\right]=\sum_{i=1}^{n}\left[x_{i:n}-(p_i^{-1/\hat{\gamma}}-1)^{-1/\hat{\alpha}}\hat{\beta}\right], \quad (6)$$

with respect to α, β and γ. Here, p_i is taken as $\frac{i}{n+1}$, So based on the Kappa distribution the Equation (6) can be rewritten as:

$$\sum_{i=1}^{n}\left[x_{i:n}-F^{-1}(p_i)\right]=\sum_{i=1}^{n}\left[x_{i:n}-\left((\frac{i}{n+1})^{-1/\hat{\gamma}}-1\right)^{-1/\hat{\alpha}}\hat{\beta}\right].$$

3.1.4. Maximum Product of Spacings

Cheng and Amin (1983) suggest a simple method for obtaining the estimation of the parameters of continuous distributions. Based on the Equation (1), the product spacings can be written as:

$$\Upsilon(\alpha,\beta,\gamma)=\prod_{i=1}^{n+1}\left[F(x_{i:n},\alpha,\beta,\gamma)-F(x_{i-1:n},\alpha,\beta,\gamma)\right]$$

$$=\prod_{i=1}^{n+1}\left[(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}\right]$$

and the log likelihood is:

$$\log\Upsilon(\alpha,\beta,\gamma)=\sum_{i=1}^{n+1}\log\left[\begin{array}{l}(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}\\ -(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}\end{array}\right] \quad (7)$$

The maximum product of spacing estimates of the unknown parameters can be obtained using the following non-linear equations:

$$\frac{\partial\log\Upsilon(\alpha,\beta,\gamma)}{\partial\alpha}=\sum_{i=1}^{n+1}\frac{\gamma(x_{i:n}/\beta)^{-\alpha}\log(x_{i:n}/\beta)(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma-1}}{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}}-\frac{\gamma(x_{i-1:n}/\beta)^{-\alpha}\log(x_{i-1:n}/\beta)(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma-1}}{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}}=0,$$

$$\frac{\partial\log\Upsilon(\alpha,\beta,\gamma)}{\partial\beta}=\sum_{i=1}^{n+1}-\frac{\alpha\gamma\beta^{\alpha-1}(x_{i:n})^{-\alpha}(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma-1}}{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}}+\frac{\alpha\gamma\beta^{-\alpha-1}(x_{i-1:n})^{-\alpha}(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma-1}}{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}}=0,$$

And

$$\frac{\partial\log\Upsilon(\alpha,\beta,\gamma)}{\partial\gamma}=\sum_{i=1}^{n}-\frac{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}\log(1+(x_{i:n}/\beta)^{-\alpha})}{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}}+\frac{(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}\log(1+(x_{i-1:n}/\beta)^{-\alpha})}{(1+(x_{i:n}/\beta)^{-\alpha})^{-\gamma}-(1+(x_{i-1:n}/\beta)^{-\alpha})^{-\gamma}}=0.$$

4. Results

Application in Insurance Stock Prices

In this section, a case analysis of the observed data is provided to demonstrate some applications of the developed methods. The data set is taken from Nwobi and Ugomma (2014) consisting of 100 observations of the weekly stock prices collected from Cornerstone insurance company. First, we consider the model selection for choosing a sparse model that adequately explains the data. Recent past, different aspects of the model selection have been focus of investigation for many authors, see for example, Basu at al. (2009), Ouarda et al. (2015), Johnson et al. (2016) and Panahi (2016; 2017). We compared the Kappa distribution with the other three distributions such as Gamma, Weibull and Burr distributions. The proposed distributions have heavy tail properties and so can be considered in financial data. We obtained the different criteria as:

- **Akaike Information Criterion:**

$$Akaike = 2\xi - 2\ln\Re.$$

- **Bayesian Information Criterion:**

$$Bayesian = \xi\ln(n) - 2\ln\Re.$$

- **Likelihood Criterion:**

$$Likelihood = \ln\Re = \ln(\prod_{i=1}^{n} f(x_i)) \cdot$$

- **Kolmogrov-Smirnov Distance:**

$$KS = \sup|F_n - F|; \quad F_n = \frac{1}{n}\sum_{i=1}^{n} I_{(-\infty,x]} X_i \cdot$$

- **Anderson-Darling Distance:**

$$AD = n\int_{-\infty}^{\infty} \frac{(F_n - F)^2}{F(1-F)} dF; \quad F_n = \frac{1}{n}\sum_{i=1}^{n} I_{(-\infty,x]} X_i \cdot$$

Where, $I_{(-\infty,x]} = \begin{cases} 1; & X \le x \\ 0; & otherwise \end{cases}$ and ξ & $\Re$ are the number of parameters and likelihood function respectively. The results for different distributions are:

- **Kappa distribution:**

$$Akaike(Kappa) = 2(3) - 2(100\log(7.466043)$$
$$-(100\times 7.466043\times 0.228924)\log(5.158308)$$
$$+\log(0.228924) + (7.466043\times 0.228924 - 1)\sum_{i=1}^{100}\log x_i$$
$$-(0.228924+1)\sum_{i=1}^{100}\log(1+(x_i / 5.158308)^{7.466043})) = 384.6223,$$

$$Bayesian(Kappa) = 13.8155 - 2(100\log(7.466043)$$
$$-(100\times 7.466043\times 0.228924)\log(5.158308)$$
$$+\log(0.228924) + (7.466043\times 0.228924 - 1)\sum_{i=1}^{100}\log x_i$$
$$-(0.228924+1)\sum_{i=1}^{100}\log(1+(x_i / 5.158308)^{7.466043})) = 392.0378,$$

$$Likelihood(Kappa) = 100\log(7.466043)$$
$$-(100\times 7.466043\times 0.228924)\log(5.158308)$$
$$+\log(0.228924) + (7.466043\times 0.228924 - 1)\sum_{i=1}^{100}\log x_i$$
$$-(0.228924+1)\sum_{i=1}^{100}\log(1+(x_i / 5.158308)^{7.466043})) = \text{-}189.3111,$$

$KS(Kappa)$=0.1130957 and $AD(Kappa) = 2.0302055$

- **Gamma distribution:**

$$Akaike(Gamma) = 4 - 2\log\left[\left((1.037421)^{3.380695}\Gamma(3.380695)\right)^{-100} \times\prod_{i=1}^{100} x_i^{3.380695-1}\times e^{-(x_i/1.037421)}\right] = 395.7132,$$

$$Bayesian(Gamma) = 9.21 - 2\log\left[\left((1.037421)^{3.380695}\Gamma(3.380695)\right)^{-100} \times\prod_{i=1}^{100} x_i^{3.380695-1}\times e^{-(x_i/1.037421)}\right] = 400.9235,$$

$$Likelihood(Gamma) = \log\left[\left((1.037421)^{3.380695}\Gamma(3.380695)\right)^{-100} \times\prod_{i=1}^{100} x_i^{3.380695-1}\times e^{-(x_i/1.037421)}\right] = \text{-}195.857,$$

$KS(Gamma) = 0.169257$ and $AD(Gamma) = 4.353757 \cdot$

- **Weibull distribution**

$$Akaike(Weibull) = 4 - 2\log\Big[(2.216181/3.956465)^{100}$$

$$\times\prod_{i=1}^{100}(x_i / 3.956465)^{2.216181-1} \times e^{-(x_i/3.956465)^{2.216181}}\Big] = 386.8964,$$

$$Bayesian(Weibull) = 9.21 - 2\log\Big[(2.216181/3.956465)^{100}$$

$$\times\prod_{i=1}^{100}(x_i / 3.956465)^{2.216181-1} \times e^{-(x_i/3.956465)^{2.216181}}\Big] = 392.1068,$$

$$Likelihood(Weibull) = \Big[(2.216181/3.956465)^{100}$$

$$\times\prod_{i=1}^{100}(x_i / 3.956465)^{2.216181-1} \times e^{-(x_i/3.956465)^{2.216181}}\Big] = -191.4482,$$

$KS(Weibull) = 0.127438$ and $AD(Weibull) = 2.977904$.

- **Burr distribution:**

$$Akaike(Burr) = 4 - 2\log\Big[(0.111670\times\ 8.006705)^{100}\prod_{i=1}^{100} x_i^{8.006705-1}$$

$$\times\prod_{i=1}^{100}(1 + x_i^{8.006705})^{-0.111670-1}\ \Big] = 476.0898,$$

$$Bayesian(Burr) = 9.21 - 2\log\Big[(0.111670\times\ 8.006705)^{100}\prod_{i=1}^{100} x_i^{8.006705-1}$$

$$\times\prod_{i=1}^{100}(1 + x_i^{8.006705})^{-0.111670-1}\ \Big] = 481.3002,$$

$$Likelihood(Burr) = \log\Big[(0.111670\times\ 8.006705)^{100}\prod_{i=1}^{100} x_i^{8.006705-1}$$

$$\times\prod_{i=1}^{100}(1 + x_i^{8.006705})^{-0.111670-1}\ \Big] = -236.044,$$

$KS(Burr) = 0.328569$ and $AD(Burr) = 13.13082$.

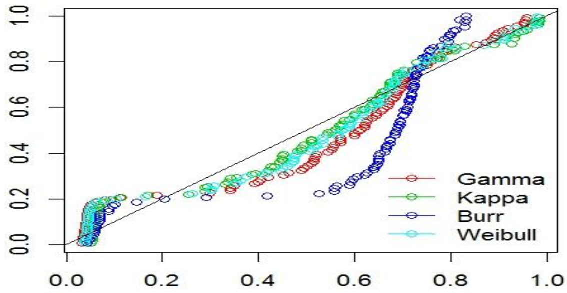

Figure 2. The P-P plot for the different istributions.

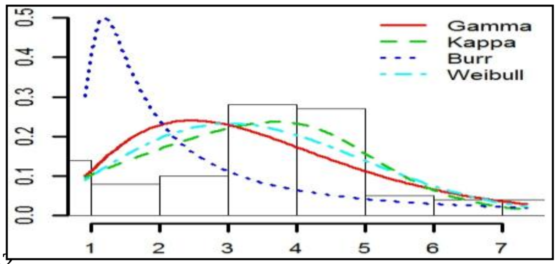

Figure3. The density plot for the different distributions

Based on different criteria, it is observed that the Kappa distribution has smallest Akaike, Bayesian, KS and AD values. Also, it has largest likelihood value, so, the Kappa distribution can be considered as an adequate model for analyzing the proposed data. Figures 2 and 3 show the P-P plot and the density plots for the four different proposed distributions. The figures show that the Kappa distribution provides the best fit. Now, we obtained the unknown parameters of the Kappa distribution using maximum likelihood, maximum product of spacing, moment and percentile methods. The results for different methods are given by:

- **Maximum Likelihood Estimator (MLE)**

$$\alpha = 7.466043,\ \beta = 5.158308,$$
$$\gamma = 0.228924.$$

- **Maximum Product of Spacings (MPS)**

$$\alpha = 7.422851,\ \beta = 5.136978,$$
$$\gamma = 0.208943.$$

- **Moment estimator (ME)**

$$\alpha = 7.52634,\ \beta = 6.06546,$$
$$\gamma = 0.237385.$$

- **Percentile Estimator (PE)**

$$\alpha = 7.44983,\ \beta = 5.14997,$$
$$\gamma = 0.213265.$$

Based on the Hannan–Quinn information criterion (HQC), we compared the above estimators as:

$$HQ_{MLE} = -2(100\log(7.466043) - 170.915642\log(5.158308)$$
$$+100\log(0.228924) + 0.7091564\sum_{i=1}^{100}\log x_i$$
$$-(1.228924)\sum_{i=1}^{100}\log(1+(x_i/5.158308)^{7.466043})$$
$$+6\log(\log(100)) = 387.78535,$$

$$HQ_{MSP} = -2(100\log(7.422851) - 155.095275\log(5.136978)$$
$$+100\log(0.208943) + 0.5509527\sum_{i=1}^{100}\log x_i$$
$$-(1.208943)\sum_{i=1}^{100}\log(1+(x_i/5.136978)^{7.422851})$$
$$+6\log(\log(100)) = 388.8093,$$

$$HQ_{ME} = -2(100\log(7.52634) - 178.664022\log(6.06546)$$
$$+100\log(0.237385) + 0.7866402\sum_{i=1}^{100}\log x_i$$
$$-(1.237385)\sum_{i=1}^{100}\log(1+(x_i/6.06546)^{7.52634})$$
$$+6\log(\log(100)) = 402.9454,$$

And

$$HQ_{PE} = -2(100\log(7.44983) - 158.878799\log(5.14997)$$
$$+100\log(0.213265) + 0.5887879\sum_{i=1}^{100}\log x_i$$
$$-(1.213265)\sum_{i=1}^{100}\log(1+(x_i/5.14997)^{7.44983})$$
$$+6\log(\log(100)) = 388.339.$$

Now, we obtain the value at risk using the maximum likelihood, maximum product of spacing, moment and percentile methods as:

$$VaR_{0.95}(MLE) = (0.95^{-1/\hat{\gamma}} - 1)^{-1/\hat{\alpha}}\,\hat{\beta}$$
$$= (0.95^{-1/0.228924} - 1)^{-1/7.466043}(5.158308) = 4.286811,$$

$$VaR_{0.95}(MPS) = (0.95^{-1/\hat{\gamma}} - 1)^{-1/\hat{\alpha}}(\hat{\beta})$$
$$= (0.95^{-1/0.208943} - 1)^{-1/7.422851}(5.136978) = 4.323762,$$

$$VaR_{0.95}(ME) = (0.95^{-1/\hat{\gamma}} - 1)^{-1/\hat{\alpha}}\,\hat{\beta}$$
$$= (0.95^{-1/0.237385} - 1)^{-1/7.52634}(6.06546) = 5.021131,$$

And

$$VaR_{0.95}(PE) = (0.95^{-1/\hat{\gamma}} - 1)^{-1/\hat{\alpha}}\,\hat{\beta}$$
$$= (0.95^{-1/0.213265} - 1)^{-1/7.44983}(5.14997) = 4.323997.$$

respectively. We finally compare the relative performances of the four estimators using the Hannan–Quinn information criterion. The maximum likelihood estimator gives the best performance. However, the percentiles and the maximum product of spacing estimators perform equally well and also the moment estimator gives the worst performance. For obtaining the *VaR* estimation, the different proposed estimators of the parameters have been inserted into the inverse cumulative distribution function of the Kappa loss function ($F^{-1}(x)$). It is observed that the results of the *VaR* estimation using the proposed methods are satisfactory.

5. Discussion and Conclusions

Value at risk is one of the oldest risk measures that have been intensively used in finance and insurance business. It is a point estimate based upon the assumed probability distribution. The heavy tailed distribution is one of the important distributions for modeling the financial data. In this paper, the Kappa distribution which has the heavy tail property is considered for evaluating the value at risk. Different estimation methods, namely maximum likelihood, maximum product of spacing, moment and percentile have been introduced for estimating the three unknown parameters. Based on different tests and criteria, it is observed that the Kappa distribution can be considered for modeling the data of the stock prices insurance well. We also observed that the value at risk is smallest using the maximum likelihood estimation method. The implication of our study is important and that is the value at risk can be evaluated using different estimation methods and based on the heavy tail distributions such as Kappa distribution, as adopted in the present work.

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