A Novel Selection Model of Optimal Portfolio based on Fuzzy Goal Planning, Considering Types of Investors

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ABSTRACT
Assessing risk assets is one of the most important research issues in the financial field. There are various pricing models of capital assets in financial. In many models, it is not possible to consider a lot of restrictions on portfolio selection. In this paper, for choosing optimal portfolios, taking into account the prosperity and recession periods, and the types of investors in terms of risk taking and risk aversion as a limitation, fuzzy goal models have beed used. And finally, it has been compared to the results of the Markowitz pricing model.

Keywords:
Multiple objective programming, Fuzzy goal programming, Portfolio selection model, Risk preferences of investors, Capital Asset Pricing Model

1. Introduction

The concept of portfolio analysis is derived from financial areas. except some cases, Financial investments generally do not have fixed Profitability, but they change with specific variables. This Variability is the determinant of investment risk, Hence, in most cases, higher risk investments have a greater profitability potential. The optimal choice for investing is the goals of any investor. In the past, investors were getting help their experience and business intelligence to realize their dream of achieving expected returns. With the progression of financial management, investors' selection became systematic and by applying different models and integrating their results with their experiences, they were able to achieve optimal choice.

The variety of investment methods and the complexity of decision making has strongly developed in recent decades, and due to this widespread growth, there has been created need to inclusive and integrative models to meet this need, Financial modeling is created from the connection between financial approach and mathematical planning.

2. Literature Review

In order to ensure more realistic investment performance, the decision makers can assign the preemptive priorities to some objectives which have higher importance or priorities than the others. In this study, to take into account the hierarchy among the objectives of decision makers, the fuzzy goal programming approach proposed by Chen and Tsai (2001), based on the preemptive priority framework, is used. According to Chen and Tsai (2001), the fuzzy goals are ranked into the desired priority levels as follows:

$$\begin{array}{l} \text{Priority Level } 1 : \{G_{1j}\} \quad j \in I = \{1, 2, \ldots, p\} \\ \text{Priority Level } 2 : \{G_{2j}\} \quad j \in I = \{1, 2, \ldots, p\} \\ \quad \vdots \\ \text{Priority Level } m : \{G_{mj}\} \quad j \in I = \{1, 2, \ldots, p\} \end{array} \tag{1}$$

where $\{G_{1j}\}$ ‹$\{G_{2j}\}$ and $\{G_{mj}\}$ are the disjoint sets of fuzz goals ($m \leq p$). According to above preemptive priority structure, the relationship among the fuzzy goals can be arranged as follows:

$$\begin{array}{l} \mu_{1j} \geqslant \mu_{2j} \\ \quad \vdots \\ \mu_{m-1j} \geqslant \mu_{mj} \quad j \in I = \{1, 2, \ldots, p\} \end{array} \tag{2}$$

Where μij is the fuzzy membership function that corresponds to jth fuzzy goal in the ith priority level. In order to find a set of solutions that satisfies inequality system in Eq. (2) under the system constraints $Ax \geq 0$, the sum of each fuzzy goal's achievement degrees can be maximized as follows:

Model 1

$$\begin{array}{l} Max \sum_{i=1}^{m}\sum_{j=1}^{p} \mu_{ij} \\ \text{s.t.} \\ \quad \mu_{1j} \geqslant \mu_{2j} \\ \quad \vdots \\ \quad \mu_{m-1j} \geqslant \mu_{mj} \\ \quad Ax \geqslant 0 \\ j \in I = \{1, 2, \ldots, p\} \end{array} \tag{3}$$

Where μij's are the fuzzy membership functions. By using Model 1, the decision makers are able to find the feasible solutions accordance with their preemptive priority structure among the fuzzy goals as well. In the decision theory, the characters of decision makers can be divided into three categories of risk-averse, risk-seeking and risk-neutral as shown in Fig. 1. Here, the membership function μ(x) denotes the satisfaction level related to the goals of decision makers. From Fig. 1, it can be inferred that if decision makers achieve their goals at least (at most) to a certain level, a unit less than (more than) that level will cause a lower degree of satisfaction for a risk-seeker than that for a risk-averse.

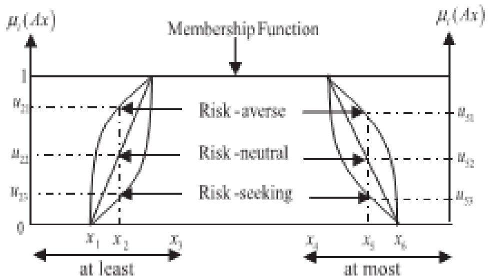


Fig. 1. Different risk groups and membership functions

In the general fuzzy goal programming structure, the fuzzy constraints can be defined as follows:

$$G_{i,j} : g_{i,j}(x_1, x_2, \ldots, x_n) \equiv (Ax)_{i,j} \simeq B_{i,j} \quad \text{(around)} \qquad (4)$$

$$G_{i,j} : g_{i,j}(x_1, x_2, \ldots, x_n) \equiv (Ax)_{i,j} \lesssim B_{i,j} \quad \text{(at most)} \qquad (5)$$

$$G_{i,j} : g_{i,j}(x_1, x_2, \ldots, x_n) \equiv (Ax)_{i,j} \gtrsim B_{i,j} \quad \text{(at least)} \qquad (6)$$

$$x \geqslant 0; \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, p$$

Here, the symbols "$\lesssim$", "$\gtrsim$" and "$\simeq$" denote the fuzzified aspiration levels with respect to the linguistic terms of ''at most'', ''at least'' and ''around'' defined in Eqs. 1–3, respectively. If desired, the different kinds of membership functions can be used in accordance with the strategies of decision makers as $(\mu_{i,j}(Ax))^{c_{i,j}}$. Values of $c_{i,j}$ between 0 and 1 correspond to risk averseness while values larger than 1 reflects risk seeking behavior. In the risk neutral cases, $c_{i,j}$ is 1.

For instances; if $c_{i,j}$ = 1, then $(\mu_{i,j}(Ax))^{c_{i,j}}$ is a monotonically piecewise linear membership function; otherwise it is a nonlinear membership function for ci,j = 2 of contraction or $c_{i,j}$=0.5 of dilation. As a result of contraction and dilation, the fuzzy linear goal programming transforms into the nonlinear programming. According to properties of dilation (c = 0.5), contraction (c = 2) and motionless (c = 1) of membership functions, Model 2 can be solved by different characters of decision makers: risk-averse, risk-seeking and risk-neutral; respectively. Let's use the exponents of the membership functions to take into account the different types of decision maker strategies, Model 1 can be rearranged as follows:

Model 2

$$Max \sum_{i=1}^{m} \sum_{j=1}^{p} \mu_{i,j}^{c_{i,j}}$$

By means of Model 2, the decision makers not only use the preemptive priority for the fuzzy goals, but also solve this model with respect to the different kinds of their strategies. Besides, it is possible to use the three types of constraints defined in Eqs. 4–6 as well. That is, the decision makers can set different kinds of membership functions depending on the constraint types, and then solve the problem accordance with their strategies.

3. Methodology

Determining the goals

After constructing the fuzzy membership functions for risk, return and beta; the goals corresponding to these quantities can be defined under restrictions

$$\sum_{i=1}^{n} x_i = M_0$$

and

$$0 \leqslant x_i \leqslant M_0 (i = 1, 2, \ldots, n)$$

as follows:

Model 4

$$G_1: \quad \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \lesssim Z^* \qquad (20)$$

$$G_2: \quad \sum_{i=1}^{n} r_i x_i \gtrsim R^* \qquad (21)$$

$$G_3: \quad \sum_{i=1}^{n} \beta_i x_i \simeq \beta^* \text{ or } \sum_{i=1}^{n} \beta_i x_i \gtrsim \beta^* \text{ or } \sum_{i=1}^{n} \beta_i x_i \lesssim \beta^* \qquad (22)$$

where G_j's (j = 1,2,3) are the fuzzy goals for risk, return and beta, and Z^* ، R^* and β^* are the target values of them, respectively. Here, the decision makers can prefer any one of three constraints for G_3 defined in Eq. (22) with respect to the market trend. According to Chen and Tsai's (2001) fuzzy goal programming approach, the decision makers is able to define the different priorities among the fuzzy goals. Similarly, the different priorities can be determined for the fuzzy goals defined in Model 4 accordance with market movements.

Let's put the membership functions of risk, return and beta into the Model 2, the following portfolio selection model can be constituted:

Model 5

$$Max\ \mu_Z^c + \mu_R^c + \mu_B^c \tag{23}$$

s.t.

$$\mu_i^c \geqslant \mu_j^c \quad (i \neq j)i,\ j = Z,\ R,\ B; c = 0.5,\ 1,\ 2 \tag{24}$$

$$\sum_{i=1}^{n} x_i = M_0,\quad 0 \leqslant x_i \leqslant M_0 (i = 1, 2, \ldots, n) \tag{25}$$

where Z, R and B are the abbreviations of risk, return and beta respectively. Here, the decision makers desire the larger objective function values to increase the satisfaction levels of preemptive fuzzy goals under the system constraints. In this model, it is possible to define different importance and priorities among fuzzy goals using inequality system in Eq. (24). Besides, if desired, this model can be solved for different kinds of strategies corresponding to risk-averse, risk-seeking and risk-neutral using the properties of dilation (c = 0.5), contraction (c = 2) and motionless (c = 1) of membership functions, respectively. Thus, the decision makers can use different strategies with respect to market trends. For instance; if the priority among fuzzy goals is determined as Z ≥ R ≥ B, thus Model 5 can be written as follows:

Model 6

$$Max\ O = \mu_Z^c + \mu_R^c + \mu_B^c \tag{26}$$

s.t.

$$\mu_Z^c \geqslant \mu_R^c \tag{27}$$

$$\mu_R^c \geqslant \mu_B^c\ c = 0.5,\ 1,\ 2 \tag{28}$$

$$\sum_{i=1}^{n} x_i = M_0,\quad 0 \leqslant x_i \leqslant M_0 (i = 1, 2, \ldots, n) \tag{29}$$

Let's use the membership functions of risk, return and beta into Model 6, this model can be given explicitly as follows:

$$\text{Max } O = \left[1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} - Z_0}{Z_1 - Z_0}\right]^c + \left[1 - \frac{E(R_{\max}) - \sum_{i=1}^{n} r_i x_i}{(E(R_{\max}) - E(R_{\min}))}\right]^c + [\mu_\beta(x)]^c \tag{30}$$

s.t.

$$\left[1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} - Z_0}{Z_1 - Z_0}\right]^c \geqslant \left[1 - \frac{E(R_{\max}) - \sum_{i=1}^{n} r_i x_i}{(E(R_{\max}) - E(R_{\min}))}\right]^c \tag{31}$$

$$\left[1 - \frac{E(R_{\max}) - \sum_{i=1}^{n} r_i x_i}{(E(R_{\max}) - E(R_{\min}))}\right]^c \geqslant [\mu_\beta(x)]^c \tag{32}$$

$$\sum_{i=1}^{n} x_i = M_0,\quad 0 \leqslant x_i \leqslant M_0,\quad c = 0.5,\ 1,\ 2 \quad (i = 1, 2, \ldots, n) \tag{33}$$

Here, it is worth noting that Model 7 includes the notation of beta membership function $\mu_\beta^{(x)}$ instead of its explicit form because the decision makers can prefer any one of three types of membership functions depending on market trends.

4. Results

In this section, to examine how investors should behave in accordance with the market moving trends, three investment terms in the top 50 companies index are handled separately. The sample data used in these implementations include the daily closed prices of stocks traded in the top 50 companies. In the first implementation, top 50 companies index recorded in Farvardin 1396 that has an upward (bullish) moving trend is considered. In this interval, the daily closed prices of the stocks traded in Khordad 1396 are used to construct the portfolio selection models. To compare daily returns of estimated portfolios, daily closed prices of the stocks traded in Farvardin 1396 are utilized. In the second implementation, the top 50 companies index recorded in Tir 1396 that has a downward (bearish) moving trend is considered. In this interval, the daily closed prices of the stocks traded in Tir 1396 are used to construct the portfolio selection models. To compare daily returns of estimated portfolios, daily closed prices of the stocks traded in Tir 1396 are utilized. In the third implementation, an investor profile who asks for chasing the top 50 companies index to make the investments is examined. Similarly to previous implementations, while the daily closed prices in Mordad 1396 are used to construct the portfolio selection models.

In the all analyses, the classical portfolio selection models are used as well as the proposed fuzzy models in terms of comparing their return performance over test periods. Specifically, Markowitz's (1952) model minimizes the covariance between all the stock S_i and stock S_j whereas Konno and Yamazaki's (1991) model minimizes the mean absolute deviations based on the differences between the expected and period returns of all the stock S_j. Young's (1998) model maximizes the minimum portfolio returns considered in all the periods; in other words, this approach is similar to Maximin criterion in the Game Theory where the players aim to maximize their expected minimum returns. Although Minimax criterion is commonly used in the portfolio selection literature, Maximin is more comprehensible in terms of constructing the related model. Therefore, the second notation is preferred in this study. In order to construct the proposed fuzzy models, the required statistics and beta coefficients of stocks were calculated using Matlab2011. While the nonlinear portfolio selection problems were solved by Generalized Reduced Gradient Algorithm, the linear ones were solved by the Simplex Method using the Solver Toolbox is available in Excel 2013. In the all implementations, the total fund M_0 is taken as 100 currency unit due to its mathematical simplicity. According to Stocks traded in the top 50 companies; min, max and average levels of the expected returns of all the stocks in the analysis periods are given in Table 1. The min and max beta values of all the stocks for analysis periods are given in the Table 2. The priority ordering to risk classes are given in the Table 3.

Table 1
Return levels in the periods (%).

Periods	Return levels (%)		
	Min	Ave	Max
farvardin	−0.94	−0.02	1.13
ordibeheshht	−0.85	0.30	1.15
shahrivar	−1.73	−0.75	0.56

Table 2
Beta's in the periods.

Periods	Beta coefficient	
	Min	Max
farvardin	0.26	2.06
ordibehsht	0.14	1.49
shahrivar	0.15	1.79

Table 3
Priority ordering to risk classes.

Risk classes	Risk neutral	Risk seeking	Risk aversion
Priority ordering	$Z \geqslant R \geqslant \beta$	$Z \geqslant R \geqslant \beta$	$Z \geqslant R \geqslant \beta$
	$Z \geqslant \beta \geqslant R$	$Z \geqslant \beta \geqslant R$	$Z \geqslant \beta \geqslant R$
	$R \geqslant Z \geqslant \beta$	$R \geqslant Z \geqslant \beta$	$R \geqslant Z \geqslant \beta$
	$R \geqslant \beta \geqslant Z$	$R \geqslant \beta \geqslant Z$	$R \geqslant \beta \geqslant Z$
	$\beta \geqslant Z \geqslant R$	$\beta \geqslant Z \geqslant R$	$\beta \geqslant Z \geqslant R$
	$\beta \geqslant R \geqslant Z$	$\beta \geqslant R \geqslant Z$	$\beta \geqslant R \geqslant Z$

4.1. Analysis of upward moving trend (Bullish) in the top 50 companies index

In order to analyze the investor behaviors when the market has upward moving trend, the stock returns realized in Farvardin 1396(illustrated in Fig. 7)was handled. After the portfolio selection models were determined using the returns realized in Farvardin 1396 of stocks traded in the top 50 companies, their return performances were evaluated over the stock returns realized in Ordibehesh1396 as a test period(illustrated in Fig. 8) .

As mentioned before, to construct the proposed portfolio selection model, the first step is to determine the appropriate membership functions accordance with market trend and investor's strategy.

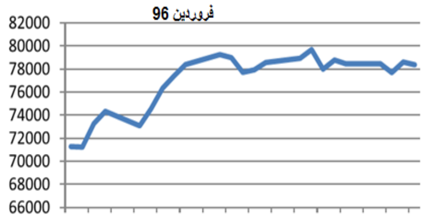


Fig. 7

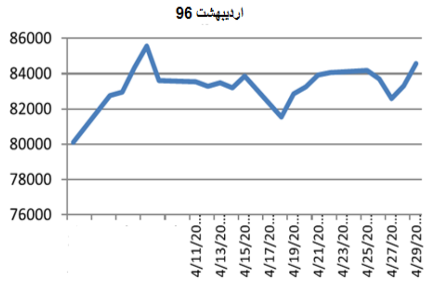


Fig. 8

In order to constitute the membership function of risk defined in Eq. (14), firstly Markowitz's model was solved at the min and max return levels (illustrated in Table 1) realized in Farvardin 1396 separately (using the total fund with 100 currency unit), and then risk borders Z0 and Z1 were evaluated as 0.52 and 19.51. By using these risk borders, the membership function of risk was constructed as defined in Eq. (14). To arrange the membership function of return defined in Eq. (15), min and max return rates in Table 1 were used. Because of upward moving trend of 50 companies, the beta membership function μ_{β}^{C} was constituted as a monotonically increasing function as given in Eq. (17) and Fig. 4 in accordance\ with risk aversion, risk neutral and risk seeker.

After all the membership functions were determined, the proposed portfolio selection model was solved by using different priorities among the fuzzy goals and the risk classes defined in Table 3. In order to examine the performances of Markowitz's, Konno–Yamazaki's and Maximin models over the test period, there solved at min, max and mean levels of average returns of all stocks in Ordibehesht 1396. Lastly, return performances of all the portfolios were evaluated over their returns realized in test period, and then these results were given in Table 4.

In Table 4, the only feasible solutions are summarized with respect to the risk classes and the priorities among the fuzzy goals: Risk (Z), Return (R) and Beta (B). These results show the daily returns of portfolios realized on selling days. The expected returns of portfolios evaluated over the test period are given in the last row of Table 4. In addition, daily returns and expected returns over test period are given in Figs. 9with respect to the different risk.

From Figs. 9, it can be seen that if the investors prefer the priority B≥ R≥ Z for all the risk classes (risk neutral, risk aversion and risk seeker), they can get better positive returns than ones of the other hierarchies because beta goal allows them to make the portfolios having the stocks with larger positive beta coefficients. Besides, the priority B≥ R≥ Z is the second best strategy in terms of expected return over test period for different risk classes. Although the beta goal has the second priority in the hierarchy for B≥ R≥ Z, this configuration ensures existing the stocks with larger positive beta coefficients in the portfolios as well.

From Figs. 10, 12 and 14, it can be seen that if Beta goal has the first or the second importance in the defined hierarchies, then risk neutral and risk seeker investors can get much greater positive returns than the risk aversion.

However, the portfolio returns obtained from Markowitz, Konno–Yamazaki and Maximin models (solved at the different return levels) are fewer than those obtained from the proposed models, because they are not able to take into accounts the market trends and the different kinds of investors simultaneously.

Among the classical models, Konno–Yamazaki's model solved at average return level gives much better return than Markowitz and Maximin because it is based on minimizing the absolute deviations from average returns of stocks. For this reason, this feature provides better returns in the upward moving trend [of market] cases.

Table 4
Selling prices of portfolios (over 100 currency unit) in test period for April 2011.

Sale days	Proposed models												Markowitz		Konno–Yamazaki		Max–min	
	Risk neutral				Risk aversion				Risk seeker				Ave. ret.	Max ret.	Ave. ret.	Max ret.	Ave. ret.	Max Ret.
	Z, R, B	*R, Z, B*	*R, B, Z*	*B, R, Z*	*Z, R, B*	*R, Z, B*	*R, B, Z*	*B, R, Z*	*Z, R, B*	*R, Z, B*	*R, B, Z*	*B, R, Z*						
4	101.8	103.4	103.7	104.9	100.9	101.8	103.1	104.9	101.3	103.3	103.9	104.9	100.5	101.2	101.1	100.5	100.9	100.5
5	101.5	103.7	103.9	105.4	100.5	101.5	102.6	105.4	101.1	103.4	104.2	105.4	100.5	100.9	100.8	100.5	100.6	100.5
6	102.3	105.5	106.2	108.8	100.3	102.3	104.1	108.8	101.7	105.4	106.8	108.8	100.5	101.7	100.8	100.5	101.7	100.5
7	103.6	107.0	108.2	111.6	101.1	103.5	105.8	111.6	102.8	106.9	109.0	111.6	100.5	102.6	102.4	100.5	102.6	100.5
8	100.9	104.6	105.4	108.0	99.7	100.9	103.2	108.0	100.2	104.2	105.8	108.0	100.0	100.6	100.0	100.0	100.5	100.0
11	101.0	104.8	105.8	108.0	100.2	101.0	103.6	108.0	100.1	104.4	106.2	108.0	100.5	100.8	100.4	100.5	100.3	100.5
12	100.6	104.7	106.1	108.5	100.5	100.6	103.3	108.5	99.7	104.5	106.5	108.5	100.5	100.3	99.8	100.5	99.7	100.5
13	100.9	105.1	106.4	108.0	101.4	100.9	103.7	108.0	99.8	104.8	106.5	108.0	100.5	100.5	100.1	100.5	99.9	100.5
14	100.8	105.0	106.8	108.0	101.4	100.8	104.3	108.0	99.8	104.5	106.6	108.0	100.5	100.4	100.8	100.5	99.7	100.5
15	101.9	105.6	107.0	108.0	101.7	101.9	104.8	108.0	100.9	105.1	106.9	108.0	100.5	101.6	101.1	100.5	101.1	100.5
18	99.3	102.6	103.4	104.1	100.3	99.3	101.3	104.1	98.3	101.9	103.1	104.1	100.0	99.4	99.0	100.0	99.0	100.0
19	101.0	104.3	105.1	105.4	102.0	101.0	103.2	105.4	100.1	103.8	104.8	105.4	100.5	101.1	100.4	100.5	100.5	100.5
20	101.4	104.5	105.0	104.9	103.7	101.3	103.7	104.9	100.6	103.9	104.5	104.9	100.5	101.7	100.8	100.5	101.1	100.5
21	102.0	105.1	105.3	104.7	104.3	102.0	104.1	104.7	101.3	104.2	104.7	104.7	100.5	102.6	101.1	100.5	102.0	100.5
22	102.8	105.0	105.1	104.1	105.2	102.8	104.3	104.1	95.9	104.1	104.3	104.1	100.5	103.4	102.1	100.5	102.7	100.5
25	103.9	105.4	105.4	104.7	104.6	103.9	104.5	104.7	102.7	104.5	104.7	104.7	99.5	104.0	103.5	99.5	103.2	99.5
26	104.8	105.2	105.1	103.9	102.4	104.8	104.0	103.9	103.4	104.2	104.2	103.9	100.0	104.5	105.3	100.0	103.8	100.0
27	105.1	103.2	102.6	100.8	102.2	105.1	102.5	100.8	103.6	102.3	101.5	100.8	100.5	104.5	106.0	100.5	103.6	100.5
28	107.1	103.9	102.9	100.8	104.1	107.2	103.0	100.8	105.1	102.9	101.7	100.8	101.4	106.3	108.9	101.4	104.8	101.4
29	107.1	105.2	104.0	102.1	106.5	107.2	103.9	102.1	105.1	104.1	102.8	102.1	100.5	101.2	108.6	101.4	104.8	101.4
E(R)	102.5	104.7	105.2	105.7	102.1	102.5	103.6	105.7	101.3	104.1	105.0	105.7	100.4	101.9	102.1	100.4	101.6	100.4

Priority levels: (*Z* = Risk; *R* = Return; *B* = Beta) E(R) = Expected return.

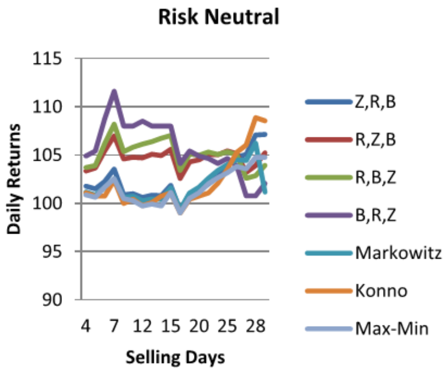
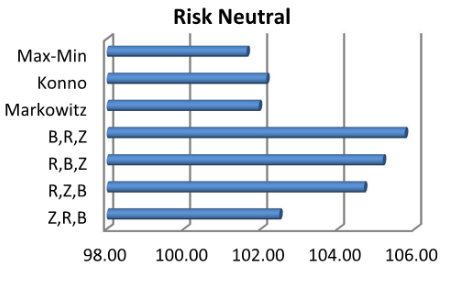


Fig. 9. Returns of different priorities for risk neutral.



	Z,R,B	R,Z,B	R,B,Z	B,R,Z	Markowitz	Konno	Max-Min
E(R)	102.48	104.68	105.17	105.74	101.95	102.14	101.63

Fig. 10. Expected returns for risk neutral.

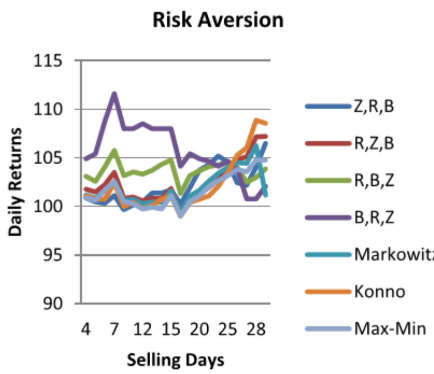
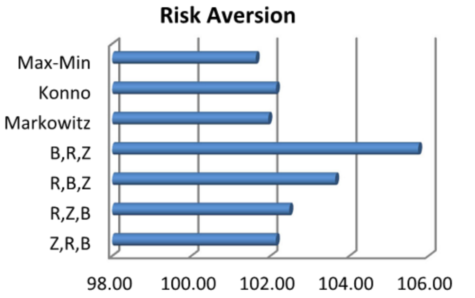


Fig. 11. Returns of different priorities for risk aversion.



	Z,R,B	R,Z,B	R,B,Z	B,R,Z	Markowitz	Konno	Max-Min
E(R)	102.14	102.48	103.64	105.74	101.95	102.14	101.63

Fig. 12. Expected returns for risk aversion.

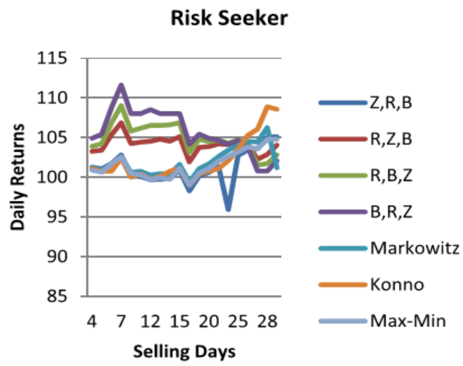
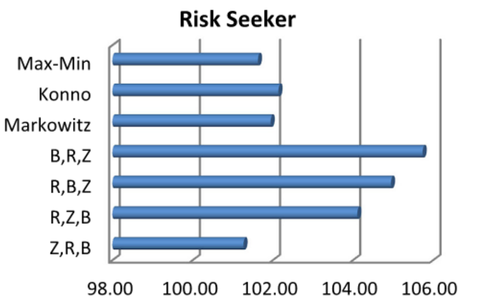


Fig. 13. Returns of different priorities for risk seeker.



	Z,R,B	R,Z,B	R,B,Z	B,R,Z	Markowitz	Konno	Max-Min
E(R)	101.27	104.11	104.95	105.74	101.95	102.14	101.63

Fig. 14. Expected returns for risk seeker.

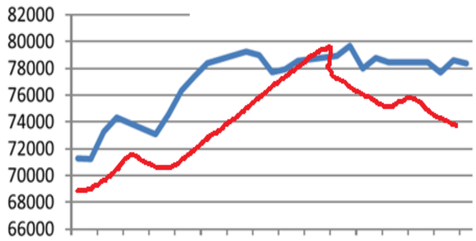


Fig. 15- Ordibehesht & Farvardin

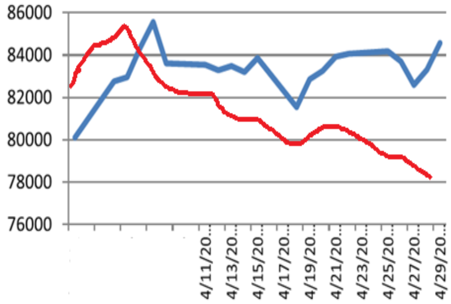


Fig. 16- Ordibehesht & Shahrivar

4.2. Analysis of downward moving trend (Bearish) in top 50 companies index

In order to analyze the investor behaviors when the market has downward moving trend, the stock returns realized in Farvardin 1396 was handled. After the portfolio selection models were determined using the returns realized in Farvardin 1396 of stocks traded in top 50 companies, their return performances were evaluated over the stock returns realized in Ordibehesht 1396 as a test period .

In order to set the membership function of risk, Markowitz's model was solved for min and max return levels (illustrated in Table 1) realized in Farvardin 1396 separately, and then risk borders Z0 and Z1 were evaluated as 0.68 and 29.16 respectively. By using these risk borders, the membership function of risk was constructed as defined in Eq. (15). Because of downward moving trend of 50 companies, the beta membership function μ_β^C was constituted as a monotonically decreasing function as given in Eq. (18) and Fig. 5 in accordance with risk aversion, risk neutral and risk seeker. To arrange the membership function of return defined in Eq. (15), min and max return rates in Table 1 were used.

After all the membership functions were determined, the proposed portfolio selection model was solved by using different priorities among the fuzzy goals and the risk classes defined in Table 3. In order to examine the performances of Markowitz's, Konno–Yamazaki's and Maximin models over the test period, there solved at min, max and mean levels of average returns of all stocks in Farvardin 96. Lastly, return performances of all the portfolios were evaluated over their returns realized in test period, and then these results were given in Table 5. In Table 5, only feasible solutions are summarized with respect to the risk classes and the priorities among the fuzzy goals: Risk (Z), Return (R) and Beta (B). In addition, daily returns and expected returns realized over test period are illustrated in Figs. 17–22 with respect to different risk classes and priorities.

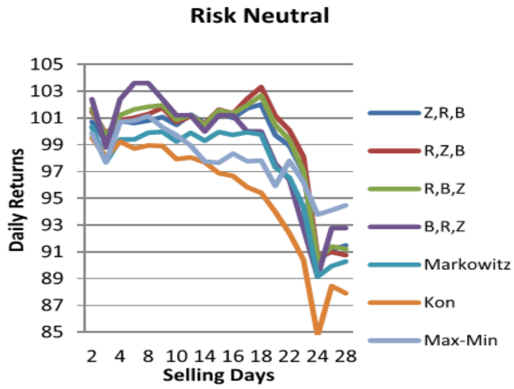


Fig. 17. Returns of different priorities for risk neutral.

According to these charts, if the risk neutral investors prefer the priority R≥Z≥B, they can get greater expected return than the other strategies as seen in Fig. 18. Besides, the priorities B≥R≥Z and R≥B≥Z bring much more positive returns from February 2 to 18 as seen in Fig. 17 and Table 5 because beta goals allow investors to make the portfolios having the stocks with negative beta coefficients in the case of downward market trend. For risk aversion, the priority R≥B≥Z ensures greater expected return than the other strategies as seen in Fig. 20. Besides, the priority B≥R≥Z brings much more positive returns from February 2 to 18 as seen Fig. 19 and Table 5. For risk seeker, R≥Z≥B brings greater expected return than the other strategies. Besides, B≥R≥Z and R≥B≥Z ensure much more positive returns from February 2 to 18 as seen Fig. 21 and Table 5. (

However, the portfolio returns obtained from Markowitz, Konno–Yamazaki and Maximin models (solved at the different return levels) are fewer than those obtained from the proposed models, because they are not able to take into accounts the market trends and the different kinds of investors simultaneously.

Table 5
Selling prices of portfolios (over 100 currency unit) in test period for February 2011.

Sale days	Proposed models												Markowitz		Konno–Yamazaki		Max–min	
	Risk neutral				Risk aversion				Risk seeker				Ave. ret.	Max ret.	Ave. ret.	Max ret.	Ave. ret.	Max Ret.
	Z, R, B	*R, Z, B*	*R, B, Z*	*B, R, Z*	*Z, R, B*	*R, Z, B*	*R, B, Z*	*B, R, Z*	*Z, R, B*	*R, Z, B*	*R, B, Z*	*B, R, Z*						
2	100.7	101.5	101.7	102.4	100.7	100.7	101.7	102.4	100.7	101.5	101.7	102.4	100.4	98.1	99.5	98.1	99.9	98.1
3	99.3	99.8	99.6	98.8	99.3	99.3	99.6	98.8	99.3	99.9	99.6	98.8	97.7	95.8	98.1	95.8	97.7	95.8
4	100.8	100.9	101.2	102.4	100.7	100.7	101.2	102.4	100.7	100.8	101.2	102.4	99.4	97.0	99.2	97.0	100.7	97.0
7	100.6	101.0	101.7	103.6	100.5	100.5	101.7	103.6	100.5	101.0	101.7	103.6	99.4	97.0	98.7	97.0	100.8	97.0
8	100.8	101.3	101.8	103.6	100.7	100.7	101.8	103.6	100.7	101.2	101.8	103.6	99.9	96.6	99.0	96.6	101.2	96.6
9	101.1	101.8	102.0	102.4	101.0	101.0	102.0	102.4	101.0	101.8	102.0	102.4	100.0	98.5	98.9	98.5	100.3	98.5
10	100.5	100.7	100.9	101.2	100.3	100.3	100.9	101.2	100.4	100.7	100.9	101.2	99.3	97.4	97.9	97.3	99.7	97.3
11	101.3	101.1	101.2	101.2	101.1	101.1	101.2	101.2	101.1	101.2	101.2	101.2	99.9	99.6	98.1	99.6	98.9	99.6
14	100.6	100.6	100.5	100.0	100.5	100.5	100.5	100.0	100.5	100.7	100.5	100.0	99.3	97.4	97.7	97.3	97.8	97.3
15	101.5	101.6	101.6	101.2	101.3	101.3	101.6	101.2	101.3	101.7	101.6	101.2	100.0	95.8	96.9	95.8	97.7	95.8
16	101.0	101.3	101.4	101.2	100.8	100.8	101.4	101.2	100.9	101.4	101.4	101.2	99.7	96.2	96.7	96.2	98.3	96.2
17	101.7	102.4	102.0	100.0	101.5	101.5	102.0	100.0	101.6	102.6	102.0	100.0	99.9	95.8	95.8	95.8	97.8	95.8
18	102.0	103.3	102.7	100.0	101.8	101.8	102.7	100.0	101.9	103.6	102.7	100.0	99.8	95.8	95.4	95.8	97.8	95.8
21	99.7	101.2	100.5	97.6	99.6	99.6	100.5	97.6	99.6	101.5	100.5	97.6	97.3	92.1	94.0	92.0	95.9	92.0
22	98.9	100.1	99.4	96.4	98.7	98.7	99.4	96.4	98.8	100.4	99.4	96.4	96.6	89.0	92.4	89.0	97.8	89.0
23	97.0	98.1	97.1	92.8	96.8	96.8	97.1	92.8	96.8	98.5	97.1	92.8	94.4	84.1	90.4	84.1	96.2	84.1
24	90.3	90.6	90.4	89.2	90.1	90.1	90.4	89.2	90.1	90.8	90.4	89.2	89.2	82.0	84.7	82.0	93.8	82.0
25	91.2	91.0	91.4	92.8	91.0	91.0	91.4	92.8	91.0	91.0	91.4	92.8	89.9	87.3	88.4	87.3	94.1	87.3
28	91.5	90.8	91.2	92.8	91.3	91.3	91.2	92.8	91.3	90.7	91.2	92.8	90.3	84.9	87.9	84.8	94.5	84.8
E(R)	99.0	99.4	99.4	98.9	98.8	98.8	99.4	98.9	98.8	99.5	99.4	98.9	97.5	93.7	95.3	93.7	97.9	93.7

Priority levels: (Z = Risk; R = Return; B = Beta) $E(R)$ = Expected return.

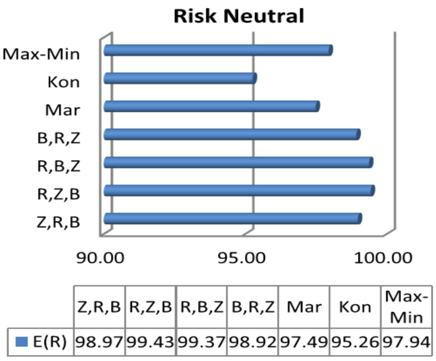


Fig. 18. Expected returns for risk neutral.

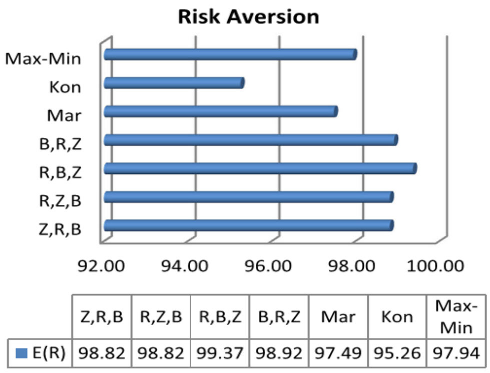


Fig. 20. Expected returns for risk aversion.

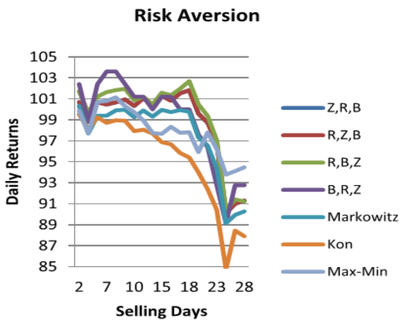


Fig. 19. Returns of different priorities for risk aversion.

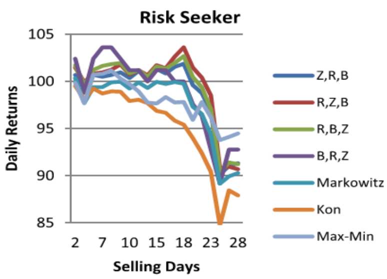
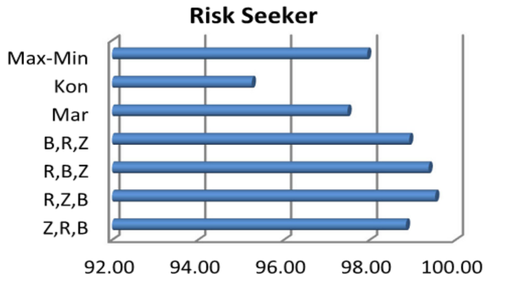


Fig. 21. Returns of different priorities for risk seeker.



	Z,R,B	R,Z,B	R,B,Z	B,R,Z	Mar	Kon	Max-Min
E(R)	98.85	99.53	99.37	98.92	97.49	95.26	97.94

Fig. 22. Expected returns for risk seeker.

for this reason, these models bring the negative returns over the test period as seen in Table 5 and Figs. 17–22.

From these results, it can be concluded that the risk-neutral and risk-seeker investors gain much more profit than the risk-aversion.Especially, if they carry away the beta goal into first priority, then it is possible to get much more positive returns relatively than the other priorities from Ordibehesht 2 to 18 as seen Table 5. In analysis, even if the classical models were constructed at the different return levels, they failed to get positive returns because of their conservative nature and not considering the current market trend.

5. Discussion and Conclusions

In this article, a novel portfolio selection model is developed by means of fuzzy goal programming techniques which allow the researchers to reconcile their objectives at certain importance and priority. This portfolio selection model can be constructed in accordance with different types of investor behaviors against the market moving trends as well as considering risk-return tradeoff. In the different moving trend cases, modeling the different types of investor behaviors contributes a novelty to the proposed portfolio selection model which is constructed by considering certain importance and priority over objectives.

As well known, if the return rate desired over an asset or portfolio is greater than expected one, the risk level related to this return becomes high proportionally. In the excessive risk cases, the satisfaction of investors decrease in terms of undertaken risk, since they might exhibit more sensitive behaviors to risk.

In context of expert and intelligent systems, all the concepts encountered in the investment process are handled by fuzzy modeling theory. For this reason, the specific fuzzy membership functions are constituted for risk, return and CAPM beta coefficient with respect to different types of investor behaviors. By means of fuzzy goal programming approaches, the fuzzy goals are assigned to risk, return and beta coefficient defined as the objectives in the investment process. Lastly, considering different importance and priority among these fuzzy goals, a novel portfolio selection model is developed in accordance with the investor behaviors and market moving trends.

In the application section, three investment terms with different market moving trends are examined separately in the top 50 companies index. In these analyses, to enhance the intelligibility and simplicity of implementations, the daily closed prices of stocks traded in the top 50 companies are preferred. If desired, the researchers can work on the different time scales as minute, hourly, session, weekly, etc. rather than daily data in the certain investment term as well. Here, it can be inferred that the proposed model is able to be modified for the investment problems with different time scales. In addition, the top 50 companies index composes of 50 national stocks. If the researchers are interested in the much bigger benchmark indexes, they can use the proposed portfolio selection model easily to make the investment analysis.

In the first implementation, the top 50 companies index recorded in Mordad 1396 has an upward moving trend. From the analysis result, it can be seen that if the decision makers constitute the diversified portfolios including enough stocks having positive beta coefficients greater than 1 in the upward moving trend case, then they can get much more positive returns than the other configurations in the selling days. In the other words, if the proposed portfolio selection model can be solved at the hierarchal orders where beta coefficient has higher importance and priority with respect to risk seeker or risk neutral strategies, then it is possible to estimate more reasonable portfolios.

In the second implementation, the top 50 companies index recorded in Farvardin 1396 has a downward moving trend. From the analysis result, it can be seen that if the decision makers constitute the diversified portfolios including enough stocks having negative beta coefficients or smaller than 1 in the downward moving trend case, then they can get much more positive returns in the selling days. In these cases, the proposed portfolio selection model can be solved at the hierarchal orders where beta coefficient has higher importance and priority with respect to risk seeker or risk neutral strategies.

In the third implementation, an investor profile who asks for chasing the top 50 companies index to make an investment decision is examined. If the investors are expecting a plausible increasing at the top 50 companies in the next time periods, they mostly tend to invest a reasonable number of stocks which move together with this index according to their historical returns. In this analysis, the portfolio selection models are determined according to the daily closed prices in Shahrivar 1396. From analysis results, it can be seen that if the investors estimate the portfolios having the reasonable number of stocks that move together with the top 50 companies index, they can get better returns than other configurations.

However, according to analysis results, the classical models proposed in Markowitz (1952), Konno and Yamazaki (1991) and Young (1998) models could not able to take into accounts the different types of investor behaviors and market moving trends simultaneously in the three implementations too. Therefore, the portfolios obtained from these classical approaches could not give reasonable returns in the related investment periods. Actually, this situation is common shortcoming of the conventional portfolio selection models and their derivatives. As a result, the proposed model is able to fulfill this shortcoming because it handles all the objectives simultaneously in accordance with the conditions of investment periods and investor strategies.

Despite of superiority of the proposed model to conventional models, it requires an expert knowledge and interpretation to constitute the fuzzy membership functions in accordance with investor behaviors and market moving trends. Besides, in this article, only daily returns of stocks are used to enhance the intelligibility and integrity of analysis. In the future works, to simplify constructing and designing the portfolio selection models, user-friendly software of this model can be written, and then various trend cases in the different time scales for another benchmark index can be examined. In addition, the other factors such as liquidity, transaction cost etc. encountered in the investment process can be handled as objectives of an investor in the fuzzy nature, and then the suitable fuzzy membership function can be constituted. Thus, it is possible to extend the proposed portfolio selection model much more comprehensive model too. Another future direction is to evaluate all the objectives in the stochastic and fuzzy natures simultaneously. Actually, this attempt will provide much more realistic approaches in the financial investment process.

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