Provide a model for calculating the economic capital of bank loan portfolio and compare it with regulatory capital

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ABSTRACT

The purpose of this article is to provide a model for calculating the economic capital of a bank loan portfolio and compare the obtained results with regulatory capital based on Basel II Models. The widely used asset value approach is used to model the default correlation. The method of estimating the parameters is based on the Method of moments and the method of maximum likelihood. Finally, the economic capital is calculated based on the distribution of losses obtained from the Monte Carlo simulation method. According to research hypothesis, calculated economic capital is compared with calculated regulatory capital based on Basel II Models. Regarding obtained results regulatory capital is more than economic capital. Considering the difference between economic capital and regulatory capital in the selected portfolio, it is not sufficient to rely on regulatory capital to assess banks' risk. In order to have an accurate assessment of banks' risk, economic capital must be calculated with proper modeling.

Keywords:

Credit Risk, The Probability of Default, Common Default, Credit Lose Distribution, Economic Capital and Value at Credit Risk.

1. Introduction

One of the important categories in risk management is calculating and maintaining an appropriate level of envelopment capital to deal with unexpected losses. Adequate economic capital in banks will be used as an envelopment against the risks that banks face. A bank can easily manage its expected losses; since this loss is expected and its occurrence can be easily quantified and the bank can manage it by receiving a profit. But what causes problems for financial institutions and banks is unexpected losses; since in this circumstances, the number of defaults is higher than expected items or the loan repayment rate is lower than the expected ones. According to the Basel Accords, one of the reasons for the escalation of the financial and economic crisis in 2007 was the high leveraging ratio at the top and bottom of the balance sheet items, which ultimately led to the gradual erosion of the amount and quality of basic capital. In the mentioned critical conditions, the banks did not also have enough liquidity buffer to absorb credit and commercial losses.

The Federal Reserve in the Journal of Bank Management Principles on the importance of capital pointed as follows "Capital is needed to stablish a company and keep it running. It also needs to finance the development of the company activities, to add new business lines and invest in technology." In other words, capital is important for establishment and development of bank further activities. In addition to the above mentioned, capital is envelopment, which protects the bank against unexpected risks and assures creditors that the targeted bank has the ability to repay their claims. Capital is important in businesses and companies other than the banks, but due to the high leverage ratio in banks, the importance of the capital is doubled. The main purpose of this research is to provide the model and to assess of economic capital for the sample credit portfolio. In this research according to financial models, the probability of default variables, *loss given default, exposure at default, calculated case and according it the expected loss all are estimated. Then by assessment of sample portfolio lose distribution, value at risk is also assessed. Ultimately* considering two factors of expected lose and unexpected lose the economic capital will be calculated. After modeling and assessment of needed economic capital, economic capital will be compared with regulatory capital required by the Basel II Standards and Iran's Central Bank Standards. According to this comparison, one can comment on the reassurance of regulatory capital to prevent the bankruptcy of banks.

2.Theoretical Background of Research

2.1. Economic Capital

Economic capital is the amount of capital that is expected to logically cover the bank's risks. Regulatory capital alone is not sufficient to cover bank's risks, since in regulatory capital, debt instruments are considered to cover the risk and this debt instrument is regarded as one of the bank's obligations. In this case, the risk is covered at the expense of those we are committed. Regarding to book capital which does not reflect the day value of assets and debts, it is an insufficient measure of shareholders' net worth (due to bank's risk capacity). Most institutions that are bankrupted have positive book capital. Capital based on market value was affected by general market variations and can only be calculated for stock banks, so it would not be an ideal measurement. To calculate nominal capital, we need to predict and apply hypothesis about the types of assets and debts that some of the assumptions and predications may be unreasonable. Also, due to this subject in the calculations related to capital the present value is considered, in case of significant change in market conditions, new calculations must be performed. (Resti & Sironi, 657: 2007)

Diagram (1) shows an overview of the loss distribution of a portfolio.In the loss distribution of diagram (1), there is a certain level of expected loss that can be observed with a high level of confidence. When we get away from the expected loss, the probability decreases. Economic capital is the difference between the Value at risk of loss at the specified confidence level minus the expected loss. Banks focus on both expected and unexpected losses. Expected loss is the mean of loss distribution and indicates the amount that the bank expects to lose in the credit portfolio on average. On the other hand, unexpected loss indicates the amount of fluctuation in credit losses and are usually considered as high percentages of loss distribution (such as 99.9%). (Bandyopadhyay, 277: 2016).

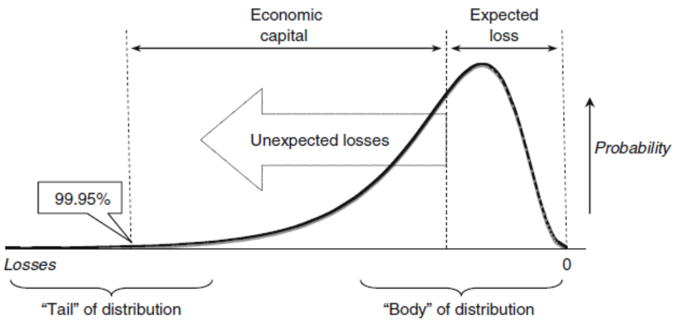


Diagram (1) - Expected and unexpected losses in the distribution of losses of a portfolio
Source: (Bandyopadhyay, 277:2016)

2.2. Explaining of Merton model to predict the probability of default

Merton's asset value model was first proposed in the article by Merton and Black Schultz. Imagine a firm with an asset value of A ≥ 0 that is financed with shares of E ≥ 0 and the rest is financed through bonds at a nominal price of F whose maturity is T. At time t we have the following equation: (1)

$$A_t = E_t + D_t \quad (1)$$

From time 0 to T, the firm struggles with opportunities and threats that can affect the value of the firm's assets. Asset A in These conditions of uncertainty with the *Stochastic Brownian motion* will be as follows:

$$dA_t = \mu A_t d_t + \sigma_A d_z \quad (2)$$

Which σ_A is the fluctuation of the asset and d_z the Winery Process that is $d_z = \sqrt{t}\varepsilon$. In period t, the market value of the asset will be in accordance with equation (3).

$$\log A_t = \log A_0 + \left(\mu - \frac{\sigma_A^2}{2}\right)T + \sigma_A \sqrt{T}\,\varepsilon$$

(3)

In equation (3), default occurs when the value of an asset is less than the nominal value of the debt. The PD or the probability that the firm will not be able to pay the debt at nominal value of F at maturity T, it will be according to equation (4):

$$p\left[A_T < F\right] = \Phi_N(-d_2)$$
$$= \Phi_N\left(-\frac{\ln(A_0/F)}{\sigma_A\sqrt{T}} + \frac{\sigma_A\sqrt{T}}{2}\right) \quad (4)$$

(Wangstel &Business, 168:2009)

2.3. Explaining the asset value model for modeling default correlation

Correlation models can be divided into two groups of single-factor models and intensity models.(Mals 266:2011)

In this research, the method used to estimate the default correlation is the asset value approach. This method models the default correlation with respect to linking the default to a continuous variable namely the asset value A. Borrower i will default if the value of asset (A) falls below the threshold value d_I

$$Default \Leftrightarrow A_i \le d_i$$
$$NoDefault \Leftrightarrow A_i \ge d_i \quad (5)$$

If the value of the asset has a normal distribution, we will have:

$$d_i = \Phi^{-1}(PD_i) \quad (6)$$

Which Φ is the standard normal cumulative distribution. The correlation in this model is calculated through factor models. If the asset has a systemic factor z and an exclusive factor ε.

$$A_i = \omega_i Z + \sqrt{1-\omega_i^2}\varepsilon_i , \mathrm{cov}(\varepsilon_i , \varepsilon_j) = 0 \qquad (7)$$

The correlation of i and j assets is defined as follows:

$$\rho_{ij}^{asset} = \frac{\mathrm{cov}(A_i , A_j)}{\sigma(A_i)\sigma(A_j)}$$

$$= \frac{\mathrm{cov}(\omega_i Z + \sqrt{1-\omega_i^2}\varepsilon_i , \omega_j Z + \sqrt{1-\omega_j^2}\varepsilon_j)}{1\times 1}$$

$$= \mathrm{cov}(\omega_i Z , \omega_j Z) = \omega_i \omega_j \ \mathrm{var}(Z)$$

$$= \omega_i \omega_j \qquad (8)$$

Default probability and common default probability are also defined as follows:

$$\mathrm{Prob}\left(A_i \le d_i\right) = p_i = \phi\left(d_i\right)$$

$$\mathrm{Prob}\left(A_i \le d_i , A_j \le d_j\right) = p_{ij} = \phi_2\left(d_i , d_j , \rho^{asset}_{ij}\right) \qquad (9)$$

(Loffler &Posch131:2011)

2.3.1. Explaining Method of Moments to estimate the parameters

The estimation of average default rate is generally as follows:

$$\hat{p} = \frac{1}{T}\sum_{t=1}^{T}\frac{D_t}{N_t} \qquad (10)$$

If the available information is not sufficient, we assume that the default rate is the same for all recipient facilities; so

$$p_i = p_j = p$$

The default threshold with the above mentioned assumption will be in accordance with equation (11).

$$d_i = d_j = d = \Phi^{-1}(p) \qquad (11)$$

In equation (10), we divide the number of observed defaults by the total possible defaults. We do this for common default. If we have a D number of default, the number of common default that we can have will be according to equation (12).

$$\binom{D_t}{2} = \frac{D_t(D_t - 1)}{2} \qquad (12)$$

The total number of common defaults is also obtained from equation (13).

$$\binom{N_t}{2} = \frac{N_t(N_t - 1)}{2} \qquad (13)$$

So the number of common default in t year will be in accordance with equation (14).

$$p_{2t} = \frac{\frac{D_t(D_t - 1)}{2}}{\frac{N_t(N_t - 1)}{2}} \qquad (14)$$

After a period of T years, the estimation of common default probability will be obtained according to equation (15).

$$p_2 = \frac{1}{T}\sum_{t=1}^{T} p_{2t} = \frac{1}{T}\sum_{t=1}^{T}\frac{D_t(D_t - 1)}{N_t(N_t - 1)} \qquad (15)$$

Common the probability of default is also shown in equation (16).

$$prob(A_i \le d_i , A_j \le d_j) = p_{ij} = \Phi_2(d_i , d_j , \rho_{ij}^{asset}) \qquad (16)$$

In (16) relation ɸ is the function of cumulative normal distribution and ρ is the correlation coefficient.

$$p_{ij} = \Phi_2(d_i , d_j , \rho_{ij}^{asset}) \qquad (17)$$

From equation (10) we can calculate d_i and d_j from equation (15) we can calculate p_{ij}. Therefore, equation (17) will change to an unknown equation with ρ. We use numerical methods to solve the above equation. (Loffler &Bosch; 133: 2011).

2.3.2. Explaining the maximum likelihood approach for estimating parameters

The logical way for the maximum likelihood method in the asset value approach is that we determine the probability of default and the sensitivity of the factor in such a way as to maximize the probability of observing default historical data. We first need to describe the default behavior through the proper distribution function. The probability of default subject to factor Z can be written as follows:

$$p_i(Z) = prob(A_i \le \Phi^{-1}(p_i) \mid Z) \qquad (18)$$

By writing the factor model, we will have for the value of the asset as follows:

$$p_i(Z) = prob(w_i Z + \sqrt{1-w_i^2}\,\varepsilon_i \le \Phi^{-1}(p_i)) = \Phi\left[\frac{\Phi^{-1}(p_i) - w_i Z}{\sqrt{1-w_i^2}}\right] \qquad (19)$$

Subject to factor Z, the defaults are independent of each other. Each default variable y_i is a random variable of 0-1, which the probability of its success is $p_i(Z)$. By applying a binomial distribution function with the probability of its success $p(Z)$ in one year and for a k part, the likelihood function will be according to equation (20).

$$L_{kt} = \int_{\infty}^{\infty} \binom{N_{kt}}{D_{kt}} p_k(Z)^{D_{kt}} (1-(p_k(Z))^{N_{kt}-D_{kt}} \, d\Phi(Z)$$

(20)

And if we suppose that each part is affected by a systematic factor, the likelihood function will be in accordance with equation (21).

$$L = \prod_{t=1}^{T} \int_{\infty}^{\infty} \prod_{k=1}^{K} \binom{N_{kt}}{D_{kt}} p_k(Z)^{D_{kt}} (1-(p_k(Z))^{N_{kt}-D_{kt}} \, d\Phi(Z)$$

(21)

We use the Gauss -Hermite process to maximize the likelihood, which approximates the integral as a total weight. In other words, the integral is based on a number of discrete points (horizontal length) and these values are weighted based on special functions. Gauss – Hermite approximates the integral of a function as follows:

$$\int_{-\infty}^{\infty} f(x)dx \approx \sum_{i=1}^{n} w(x_i)\exp(x_i^2) f(x_i) \qquad (22)$$

In (22) equation x_i is the horizontal length and $w(x_i)$ is the related weight.

(Loffler&Posch, 136:2011)

2.4. Creating the distribution of credit portfolio

To create the loss distribution of credit portfolio, the following four processes are required:

A. Determining the probability of default of each credit portfolio firms (PD).
B. Determining the default losses for each credit portfolio firms (LGD).
C. Determining default correlations and if possible to determine the correlation between LGDs.
D. We obtain the distribution of portfolio value according to steps 1- to 3.

2.4.1. Adjustment of distribution by the importance of sampling method

If the number of companies in a portfolio are high and the exposures are more widely distributed among companies, due to diversification, the effect importance of the former will be greater than the latter one; this is why some companies will have favorable conditions ($\varepsilon_i > 0$) and others will have unfavorable conditions) ($\varepsilon_i < 0$) as a result they will neutralize each other's effects on the portfolio. In high-loss scenarios, we can create Z based on the normal distribution with a mean less than zero. In impotence sampling model, the probability of each iteration is equal to the 1 / M multiplied by the likelihood ratio $\frac{\phi(Z_j)}{\phi(Z_J - \mu)}$. In this ratio, ϕ the standard normal distribution and Z_j the amount of factor in iteration is j. The likelihood ratio can be written as follows:

$$\frac{\phi(Z_j)}{\phi(Z_j-\mu)}=\frac{(2\pi)^{-1/2}\exp(-Z_J^2/2)}{(2\pi)^{-1/2}\exp(-(Z_J-\mu)^2/2)} \quad (99)$$

$$=\exp(-\mu Z_j+\mu^2/2)$$

As a result, the probability of loss in j repetition will be equal to:

$$\Pr ob_j=\exp(-(Z_J-\mu)^2/2)/M \quad (100)$$

In this case we have two vectors of simulated losses and likelihood ratios. Therefore, we first arrange the two vectors based on the amount of loss, and then by starting from the maximum loss we add the j probabilities, and finally by determining the desired percentile α, we obtain the maximum loss that has a cumulative probability higher than $1-\alpha$. The optimal amount of transfer depends on the percentiles we want, the closer the percentiles get to the maximum, the closer the optimal amount of transfer gets to the maximum. (Loffler & Posch; 156: 2011, 20)

A simple way to do this is to transfer the mean to a value that is less than the maximum loss distribution percentile we want to find. If we consider the mean -1.5, in the normal case the standard -1.5 is more than the probability of 93.3%. This amount is less than the percentages above 95% desired by risk managers. Therefore, in this research, we consider the mean value to be -1.5. (Glassman & Li, 2005).

2.4.2. Adjustment of distribution by Monte Carlo quasi-method

Due to the randomness of the data in the simulations performed, the properties of the simulated numbers may deviate from the distribution from which they were extracted. If the number of iteration in the simulation decreases, this deviation increases. One way to eliminate this problem is to use Monte Carlo-quasi numbers. By using these numbers, simulated numbers can be created much closer to the desired distribution. For example, Halton sequence numbers with base 2 lead to pseudo-random numbers that have a coordinated distribution at unit distances as follows:

$$\frac{1}{2},\frac{1}{4},\frac{3}{4},\frac{1}{8},\frac{5}{8},\frac{3}{8},\frac{7}{8},\ldots$$

Therefore, according to the mentioned cases, we first create random numbers with the Halton sequence in base 2 and then we create the Z factor based on the Halton random numbers. (Loffler &Posch, 2011)

2.5. Calculation of capital based on IRIB Basel II model

To determine how the required capital should change with the risk of a loan, the Basel Committee uses a single-factor model for credit portfolio risk. In this model, default begins with a continuous latent variable that is often interpreted as the value of the borrower's assets. Borrower's asset value is dependent on Z systematic risk and ε_i firm-specific factor:

$$A_i=\omega_i Z+\sqrt{1-\omega_i^2}\,\varepsilon_i\ ,\quad \operatorname{cov}(\varepsilon_i,\varepsilon_j)=0, i\neq j \quad (23)$$

Which Z and ε_i are standard normal variables. In this model the probability of default is equal to:

$$PD_i=\Pr ob(A_i\leq\Phi^{-1}(PD_i))=PD_i \quad (24)$$

Factor sensitivity is the determinative of asset correlation and consequently default correlation.

Wasisk (2002) and Gordy (2003) developed Merton's (1974) single asset model into a portfolio model, focusing on an infinity small portfolio in which each borrower is independent of the other. According to mentioned issues, the IRB formula of Basel II is written in the form of (25) equation:

$$C\ \ Loss_{h,(1-\alpha)}=$$

$$\sum_{s=1}^{S}\sum_{i=1}^{n}EAD_{i,s,t}LGD_{i,s,t}\Phi\left(\frac{\Phi^{-1}(PD_{i,s,t})-\omega_{i,s}\Phi^{-1}(1-\alpha)}{\sqrt{1-\omega_{i,s}^2}}\right) \quad (25)$$

In the above formula, n is the number of loans in s portfolio and S is the total number of portfolios. Unexpected loss is obtained from the difference between $C\ \ Loss_{h,(1-\alpha)}$ and expected loss EL:

$$UL_{h,(1-\alpha)} = \sum_{s=1}^{S}\sum_{i=1}^{n} EAD_{i,s,t} LGD_{i,s,t} \Phi(\frac{\Phi^{-1}(PD_{i,s,t}) - \omega_{i,s}\Phi^{-1}(1-\alpha)}{\sqrt{1-\omega_{i,s}^{2}}} - PD_{i,s,t}) \quad (26)$$

Risk-weighted assets are obtained by inverse multiplication of 8% in equation (26) and overall adjustment of 1.06:

$$RWA = 12.5 \times 1.06 \sum_{i=1}^{n} EAD_{i,s,t} LGD_{i,s,t} \Phi(\frac{\Phi^{-1}(PD_{i,s,t}) - \omega_{i,s}\Phi^{-1}(0.999)}{\sqrt{1-\omega_{i,s}^{2}}} - PD_{i,s,t}) adj(M) \quad (27)$$

In relation (27) at the end of the formula, the adjustment coefficient related to maturity has also been added. The percentile of 0.999 corresponds to $(1-\alpha)$ is in equation (26). Regarding IRIB, there are two approaches, which consists the basic approach and the advanced approach. In the basic approach, the bank calculates the PDs internally and assumes the rest of the parameters as given. While in the advanced approach, calculations related to PD, LGD and EAD are performed with the bank's internal models. According to the guidelines of the Basel Committee on Banking Supervision (2005), banks should have a meaningful distribution of exposure in each rating, as they do not focus too much on customer rating scales and facility rating scales. To achieve this goal, banks must have at least 7 ranks for non-defaulted customers and one rank for defaulted customers (Bellini, 24: 2019).

2.6. The Background of conducted researches

2.6.1. The Background of foreign researches

In the studies of Nagpal and Bahar (2001) about various sectors in the United States from 1981 to 1999, they concluded that default is correlated due to economic or industrial factors. Cervigny and Renault (2002) based on Standard and Poor's (S&P) data obtained evidence regarding default correlation. They concluded that the default correlation is higher for companies with low credit quality in compare to companies with high credit quality.Bajaj (2010) according to the study about Indian companies, concluded that the probability of default and correlation estimate change with variables of time, credit ranking and economic activities of borrower and also correlation between companies with equal credit ranking and the companies with same industry is higher due to special factors of borrower and the industry.

Accornero et al. (2017) used a multi-factor structural model in their study which is developed by *Duellman and Puzanova* (2006). In multifactorial models, default correlations are created by Y latent risk factors that affect the $X_{i,s}$ return on assets of i firm, which depends on s segment. In these models, the probability of common default is modeled based on the dependence of economic sectors. So that bank loans are grouped in the form of a portfolio of economic sectors and the distribution of loss potential for each economic sector is estimated separately. In another study, Arindam calculated the economic capital of public sector banks in northern India through simulations. In this study, distributions (normal, logarithmic normal and beta) are used to calculate the value at risk and the distribution of losses. (Bandiopadia, 280: 2016). Other studies such as Gordy and Howells (2006) and Repullo et al. (2009) show that the required capital in Basel II are relatively low when the economy is performing well, and the required capital increases when the economy is in recession.

Credit strategists in JP Morgan (2004) conducted a study on default correlation in which they publicly disseminated the concept of basic correlation. *Hashimoto has* concluded the research on defaults data of Japanese banks. In the mentioned research it is concluded that the correlation of assets can vary based on industry, credit rating, region and size. Also, according to the study, asset correlation is more for large industries and less for small industries (*Eluerkhaoui*, 13: 2017).

2.6.2. Background of internal research

In articles, teases and internal researches on credit risk and credit test stress, studies have been conducted. However, no study has been done on the distribution of credit losses and economic capital of a loan portfolio according to the model of this research. Shakeri and Dadashi (2011) in their research have obtained the distribution of credit portfolio losses using auxiliary predictor variables and the concept of frailty. They introduced 4 default models using different combinations of predictor variables in their research and by using the available data the parameters of estimates of all 4 default models with and without frailty variables were compared with function inside the sample. Finally, it obtained the loss distribution for 4 models and investigated the effect of frailty on the loss distribution.

Moshiri and Abdolshah in an article using quarterly information of macroeconomic variables and the banking industry during the period 2004 to the second quarter of 2016, estimated the distribution of losses due to credit risk of banks using stress test and they identified the minimum required capital by banks for resilience them against stress scenarios. In the above article, in the first step, the probability of default estimation is conducted. Then, using Monte-Carlo simulation, the probabilities of default in the one-year time horizon are simulated under the baseline and stress scenarios, and then the portfolio loss distribution is calculated using the values at default and losses due to default.

3. Research Methodology

Statistical society and spatial territory of the present research are 30 firms of bank corporate banking companies. These companies are classified into three industries, including the chemical industry, the automotive industry, and the food industry. Time domain and research data to calculate the probability of default include 262 data that are daily from August1, 2018 to August 28, 2019.

The selected companies for automotive industry and accessories include (Electric Khodro Shargh , Iran Khodro, Ashtad Iran, IRCA Part Sanat, Bahman, Saipa Azin, Iran Casting Industries, Mehvar Khodro (VAMCO), Mehvarsazan and Niroo Moharrekeh) . The selected companies for chemical industry include (Pars Petro Chemical, Parsan, Tolypers, Rangin, Zagros Petrochemical, Herbicide Production, Kaf, Marun, Iran Salt Mining and Nirou Chlor). The selected companies for food industry include (Pak Dairy, Pakdis, East Azerbaijan Pegah, Tabarok, Behshahr Industries Development, Shadab, and Dasht-e- Morghab, Mahram, Minioo Shargh and Noosh Pouneh Mashhad). In this research we use widely applied method of asset value approach to model default correlation. The parameters estimate method are based on torques method and maximum likelihood method. Mont Carlo simulation method is used to create portfolio loss distribution.

4. Research findings

4.1. The results of probability of default prediction

The results of the probability of default based on Merton Model for 30 companies is as below table:

Table (1) - the results of the probability of default prediction based on Merton Module for 30 companies in research

Automotive Industry	The probability of Default (%)	Chemical Industry	The probability of Default (%)	Food Industry	The probability of Default (%)
Electric Khodro Shargh	2.41	Pars Petrochemical	0	Pak Dairy	1.48
Iran Khodro	5.94	Parsan	0.2	Pakdis	0.0023
Ashtad Iran	0.048	Tolypers	28.7	East Azerbaijan Pegah	0.0018
IRCA Part Sanat	0.0003	Rangin	0	Tabarok	0.0001
Bahman	0.00004	Zagros Petrochemical	0	Behshahr Industries Development	0.000001
Saipa Azin	3.03	Herbicide production	0.07	Shadab	0.004
Iran Casting Industries	7.3	Kaf	24.53	Dasht-e- Morghab	0.0052
Mehvar khodro (VAMCO)	0.0035	Marun	0.02	Mahram	2.71

Mehvarsazan	0.0006	Iran Salt Mining	0.0077	Minioo Shargh	0.008
Niroo Moharrekeh	0.0017	Nirou Chlor	0	Noosh Pouneh Mashhad	0.28

Source: Research findings

4-2-Calculate of default correlation for three automotive, food and chemical industries

With the maximum likelihood method, we can conduct default correlation or factor sensitivity for different industries. A summary of the performed modeling results for three industries is as below table.

Zero factor sensitivity in the food industry indicates that the assets of companies in this group do not have a common factor, but the assets can all be specified in the form of a company-specific factor. This result is not far from expectation, as companies in the food industry have a very wide range, and each has its own financial and risk special conditions.

Table (2) - The results of default correlation for three automotive, food and chemical industries.

Industry	Probability of Default	Factor Sensitivity	Default Correlation	Likelihood Value
Automotive	13.47%	36.41%	13.26%	-127.2619
Food	22.45%	0.00%	0.00%	
Chemical	17.46%	2.08%	0.04%	

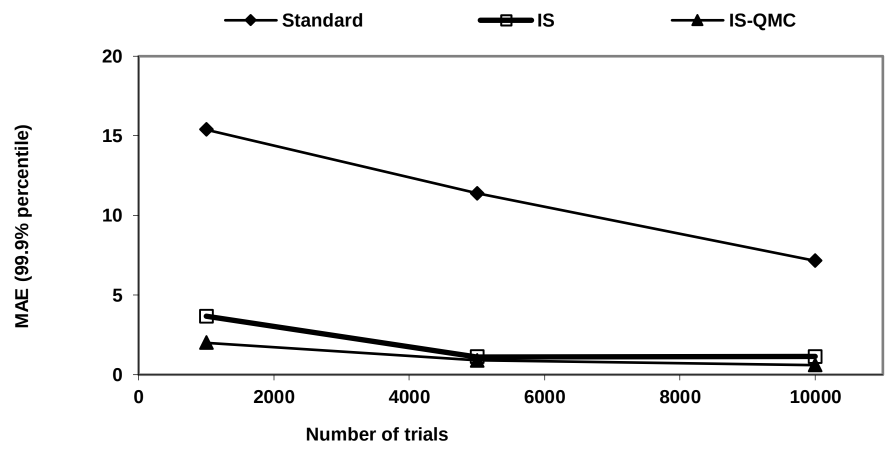
Source: Research findings

4.3. Evaluation of simulated loss distribution

To evaluate which simulation method (simulation without adjustment, simulation with importance sampling adjustment and simulation with importance sampling adjustment and quasi Monte Carlo) has less error in low iterations, we perform the following process:

1) We do simulation with a lot of iterations (1million iterations).
2) We do simulation for each of the above methods with less iteration (1000, 5000, 10000). We calculate the difference of this stage in comparison with the first stage.
3) We repeat the second stage sufficiently to obtain an accurate estimate of mean difference.

Finally, we calculate the mean absolute error37 (MAE) between the above modes. In diagram 2 the absolute mean of the error is shown at 99.9% confidence level and 95% confidence level for loss distribution in the mentioned methods.



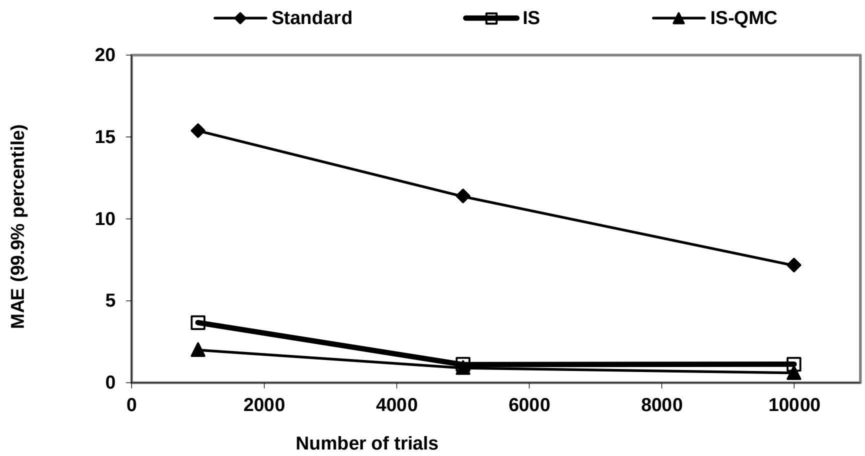


Diagram (2)- Evaluation of loss distribution error based on three simulation models at 99.9% confidence level and 95%(standard simulation, simulation with importance sampling adjustment and simulation with importance sampling adjustment and quasi Monte-Carlo)

Source: Research findings

According to both diagrams, it is clear that the accuracy of the simulated model with the importance sampling adjustment method (IS) is more than the standard model (Standard) and has a lower error level in all three iterations (1000, 5000 and 10000). Also, according to the above diagram, the presented model with quasi Monte Carlo adjustment method and importance sampling (IS-QMC) has more accuracy and lower error level than the importance sampling adjustment method.

4.4. Calculation of Economic Capital

To obtain economic capital, according to the explanations provided in the previous sections, we choose the Monte Carlo simulation model with importance of sampling adjustment and quasi Monte Carlo. The portfolio loss distribution is obtained with 10,000 iterations according to diagram (3).

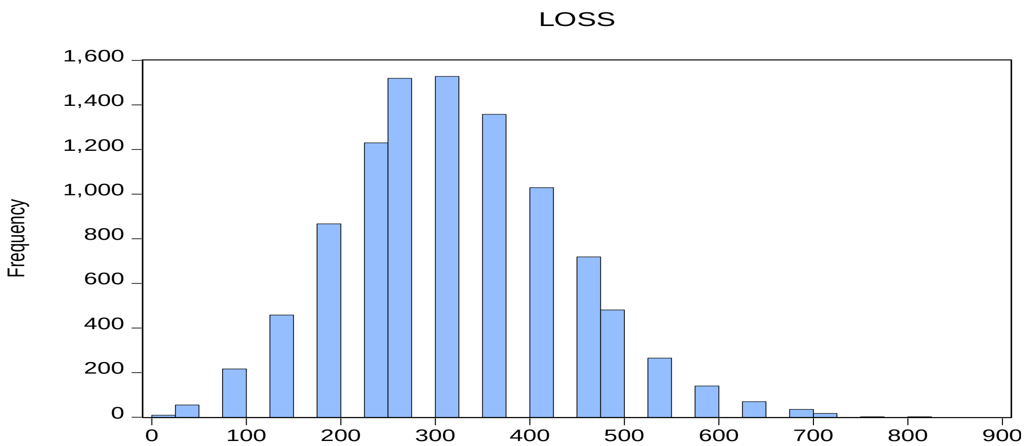


Diagram (3) - portfolio loss distribution of 30 companies with Monte Carlo simulation method and importance of sampling adjustment and quasi Monte Carlo

Source: Research findings

The mean of above distribution is equal 318. The maximum amount of simulated losses with mentioned coordinates is equal 810. In table (3) the amount of loss has shown at the specified confidence level.

Table (3) - The amount of losses in confidence level 90%, 95%, 99% and 99.9%,

losses	Confidence Level
405	90%
405	95%
495	99%
675	99.99%

Source: Research findings

To obtain economic capital in this case, we subtract the amount of loss at a confidence level of 99.99% (675 units) from the mean loss distribution (318 units), which economic capital will be equal to 360 units.

4.5. The calculation of capital based on IRIB Basel II model

According to the PDs calculated for 30 companies in the portfolio and considering LGD equal to 45% and EAD for each loan equal to 100 units, the required regulatory capital is calculated with the Basel II formula and the results are shown in the table below. Considering that the economic capital has been calculated for a period of one year, in these calculations, the desired period is also considered for one year.

Table (4) - Capital calculation based on IRIB Basel II

Required Capital	Required Capital (%)	Probability of Default
15.91	15.91%	13.17%
15.91	15.91%	13.74%
…	…	…
18.29	18.29%	22.45%
515.14 **Total of required capital**		

Source: Research findings

Based on the calculations shown in Table (4), the required regulatory capital is equal to 515. Regarding to the formula presented in Basel II, this capital is the difference between the loss in stress and the confidence level of 99.99% and the expected loss.

5.Results and suggestions

In this research the widely applied method of asset value approach was used to model default correlation and the estimate of parameters was conducted based on method of Moments and maximum likelihood method. The Monte-Carlo simulation method is also used to create portfolio loss distribution. Factor sensitivity was 36.41% in the automotive industry, zero in the food industry and 2.08% in the chemical industry. Zero sensitivity in the food industry indicates that the assets of the companies in this group do not have a common factor. Among the studied industries, the food industry had the most probability of default and the automotive industry had the least probability of default.

The required regulatory capital has also calculated based on presented formula in Basel II. Comparing economic capital with regulatory capital, it can be observed that in the selected sample portfolio, regulatory capital is more than economic capital. In this circumstances regulatory capital will not be accurate guidance for the amount of bank risk and capital required to cover the risk. If the pricing of loans is done properly, the required capital for covering portfolio risk will be less than the specified amount by Basel standard model. Given the difference between economic capital with regulatory capital in the selected portfolio, it is not sufficient to rely on regulatory capital to evaluate bank risk. And in order to have correct assessment of the bank's risk, economic capital must be calculated with proper modeling.

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