Modeling Multiportfolio Selection Considering the Market Impact in Iran Stock Market

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ABSTRACT

In most of the existing research on investment portfolio optimization, it is assumed, usually implicitly, that investors' portfolios are managed individually and independently. However, in reality, portfolio managers typically manage the accounts (i.e., portfolios) of multiple client-investors simultaneously and decisions made for one client's portfolio may induce a market impact cost that impairs the performance of not only that client's account, but other clients' accounts as well. This suggests that there may be transaction-induced performance interdependencies across all portfolios. This implies that utility-maximization for all of an investment manager's clients (collectively) requires a multi-portfolio optimization model. That is the objective of this study. Specifically, this study models multi-portfolio optimization using data drawn from the Tehran Stock Exchange while considering market impact costs on all portfolios, and the fair allocation of such costs.

In other words, the main objective of the present study is to find the suitable model for market impacts and optimizing multiple portfolios with mutual behavioral effects on each other. For this purpose, ISTAR model is used to obtain market impacts and a model is introduced and implemented using data of selected stocks from Tehran Stock Exchange in 1398. Comparison of the results obtained from the model introduced in this paper and the classic optimization models indicate that the manager's performance and customers' utility, within the framework of the proposed model, are higher than they would be if the interdependence among the accounts is not taken into consideration. Thus, the proposed framework outperforms other models.

Keywords: Multiportfolio optimization, market impact, fair allocation

1. Introduction

To date, regarding the financial computations and stock selection for creating a portfolio, the existing the investments are taken into account in terms of risk degree and return rate in order that the investor can create his desirable portfolio considering his financial facilities and other policies.

In most of the studies of portfolio optimization, it is assumed that the investment manager manages a single account. However, in practice, an investment manager manages multiple accounts simultaneously and, practically, the optimization of each account independently and isolated from the others means disregarding the market dynamics and correlation of the decisions on one account with the results and performance of the other accounts. In other words, since investment managers are often in charge of providing services for multiple accounts at the same time and the customers' accounts have different risks and return profiles, the correlation and behavior of the management of different accounts have significant direct and indirect effects on the final performance (Iancu and Trichakis, 2014).

One of the most important advancements in the modern theory of portfolio optimization is the consideration of simultaneousness in the maximization of the interests of various accounts under a unique management. Joint portfolio optimization, which is known as a multiportfolio optimization model (MPO), considers the effectiveness and mutual dependency of the accounts and imports the optimal portfolio determination problem from an independent optimization space to a multi-component space. In traditional models, it was assumed that an investment manager executes an optimization model for each of the accounts under management independently. In the MPO models, the effectiveness and the dynamics of the decisions as well as the realities of the market place are taken into consideration.

Although MPO models attempt to capture the reality of mutual dependency, a crucial aspect of these models, which is still ignored, is the transaction costs resulting from the effect of a transaction on the market. A specific type of transaction cost, defined as market impact cost or market impact, which is the change in an asset's market price as a result of a transaction in that asset, will have an impact on the return of that asset for all portfolios under management that hold that asset as part of their portfolio. (For the balance of this study, the "assets" included in the portfolio will be stocks.) Market impact costs are among the most important transaction costs in that they result in reverse movement of stock prices. The market impact might result from an investor's demand for liquidity or from the informational content of the transaction. The liquidity demand cost is created when an investor buys or sells a stock when the market lacks good liquidity for that stock. In such situations, the investor, in order to complete the transaction, will absorb a cost in the form of paying a higher price when buying a stock and receive a lower price when selling a stock.

Based on the aforesaid, an investment manager, under real market conditions, is exposed to challenges that have been mainly ignored in classic studies. The first challenge is to consider transactions that affect the whole market. These transactions impact the possible benefits of portfolio rebalancing. The second challenge is to estimate the costs of these transactions and to model the market impact function. One of the major concerns for implementing an optimization model under real conditions is the lack of empirical studies and a realistic estimation of the market impact costs

Once the above-mentioned costs are estimated and the market impact costs are calculated, the third challenge is the allocation of these costs to different investor accounts. Finally, in addition to these challenges, the most important issue in modeling a problem and developing a classic and independent optimization model is to take into account the multiple portfolio optimization and the non-isolated conditions. Therefore, the Markowitz optimization model is exposed to some challenges including the mutual effect on the accounts that are under centralized management, the modeling of the function, the estimation of the market impact costs, and finally the allocation of the costs.

The main objective of the present study is to find the market impact functions and also model multiportfolio selection while taking account of the market impact function and the fair allocation of the costs in the Iran Stock Market.

By defining a four-step model, the present paper aims to model the above-mentioned challenges for an investment manager under real market conditions. In the first step, an independent optimization model is implemented under the assumptions of the classic framework. In the second step, the market impact costs for each account are estimated using ISTAR model. In this step, the utility impact for each client is considered equal to the difference between the returns to that client's portfolio and the market impact costs resulting from the behavior of that client's account. Nevertheless, the error resulting from the effect on portfolio returns of other accounts still exists and the dynamic existing in the optimization is not taken into account.

In the third step, we get closer to reality: market impact costs are estimated cumulatively, and the fair allocations among the under-management accounts are applied. Yet, the optimization of the portfolios is still performed independently. In the final step, which is the proposed model, the optimization is carried out simultaneously as a utility function by considering market impact costs and the allocated share to each account of the market impact costs.

These steps are executed using real data from Iran's capital market in 1398. The market impact function is estimated in order to determine the market impact costs of each stock. Finally, the performance of each step is calculated and then compared to each other. The proposed framework enables us to answer the study's question: How should multiportfolio selection be modelled considering the market impact costs and their fair distribution among the portfolio manager's accounts in the Iran Stock Market?

The rest of the present study is organized as follows. Section 2 includes a short review of previous studies in the field of market impact and portfolio optimization in both classic and multi-portfolio models. In Section 3, the market impact function is modeled, and the steps of the proposed multiple optimization model are presented. In this section, the assumptions and indices related to the determined model are introduced. The studied case, the range of the study, and the data used for the proposed model are introduced in Section 4. This section also provides the results obtained from the implementation of the model and the comparison of the results in different approaches. And finally, the conclusion is presented in the last section of the paper.

Literature Review

This section provides a review of some of the studies of portfolio optimization. This review starts with the basic Markowitz model and, then, focuses on the process of completion of the portfolio optimization models in both classic (independent optimization of a portfolio) and multiple modes. Additionally, subsequent to a review of studies on market impact, this section provides an evaluation and sum-up of these models, which will be followed by a clarification of the gap considered in the present paper.

Studies on the Classic Portfolio Optimization Model

Harry Markowitz (1952) proposed a basic portfolio optimization model, which has become the foundation of modern portfolio theory. He introduced the concept of efficient portfolios. An efficient portfolio is a portfolio of assets such that the risk level (defined as variance of return) is minimized relative to a specified expected rate of return (defined as the mean rate of return). Accordingly, investors can specify an efficient portfolio by selecting an expected return rate and then solving for the portfolio that achieves that expected rate of return at the lowest possible risk. While the basic Markowitz model is recognized as the starting point of portfolio optimization modeling, it lacks in some respects. For example, it does not address all portfolio considerations, including, but not limited to, constraints that might be imposed on the portfolio manager[1], how an investor's optimal portfolio might vary as a function of the length of the investor's investment horizon, and transactions costs (including market impact cost) However, over time, this model has been exposed to some changes in different aspects. Over the years, advances in portfolio optimization have led to new models and adaptations of the original model to account for financial market realities. The introduction of new models and the adaptations of Markowitz's original model can be described and explained in terms of the methods of problem-solving, risk modeling, and return estimation.

Ehrgott (2004) introduced a model for portfolio optimization based on the development of Markowitz's mean-variance model. He applied five specific objectives associated with risk and return and imported the considerations of the individuals' preferences into the model using the decision-maker's utility function. Subbu (2005) introduced an approach for multi-objective hybrid optimization, which involved a combination of the evolutionary algorithms with linear

[1] For example, the portfolio manager might be limited as to the percentage of the portfolio that can be invested in any one stock, or any one sector

programming aimed at portfolio optimization. In this regard, numerous evolutionary algorithms, including the Artificial Bee Colony (ABC) algorithm, the Firefly algorithm, local search, simulated annealing (SA), Tabu search (TS), and Genetic algorithm (GA), have been used independently by Ehrgott (2004), Subbu (2005), Hochreiter (2007), Yang (2011), and Tuba and Bacanin (2014) in their respective efforts to improve portfolio optimization.

Gulpinar et al. (2007) expanded the mean-variance optimization framework for designing the robust worst case with risk and return scenarios. Their approach requires a min-max algorithm as well as a multi-period mean-variance optimization framework for the random aspects of the scenario tree. Since the investment decision is made based on a min-max strategy, the robustness is guaranteed by the lack of a low min-max. The optimal portfolio is created simultaneous with the worst case in order that all of the rival scenarios can be used. The portfolio is revised at each period considering the measurable (non-fixed) transaction cost. Additionally, its relative performance is assessed in terms of the returns and deviation of the returns. Moon and Yao (2011), using a robust optimization approach, solved the problem with the mean absolute deviation (MAD) risk measure, while Huang (2010) and Guastaroba (2011) solved it with a conditional value at risk (CVaR) measure. Huang also modeled the creation of a robust portfolio by considering experts’ opinions (prior distribution) through solving the sequence of second-order conic and linear programming problems.

Many researchers, including Xidonas (2009), Hadavandi (2010), Yunusoglu (2013), Kamley (2015), Dymova (2016), and Amin Naseri (2019), have employed rule-based expert systems for optimal portfolio selection. These systems attempt to select portfolios by considering the degree of risk tolerance of the investors and employing both fundamental and technical indices. Anagnostopoulos and Mamanis (2011) focused on the investment portfolio selection problem in a three-objective optimization mode and attempted to find a balance among risk, return, and the number of assets in the portfolio. They incorporated the constraints of class and value into the model in order to limit the fraction of the invested portfolio in assets with common features and prevent inclusion of small assets. The obtained result was a mixed-integer multi-objective optimization model.

Bermudez (2012) proposed a genetic algorithm to solve the portfolio optimization problem that was constrained by a maximum number of stocks included in the portfolio. He modeled the uncertainty of the return rates as a trapezoidal fuzzy number and applied low risks for the decision-maker's risk aversion criterion. Chen (2015), by considering the return rates of the problem as fuzzy, solved the problem using the ABC algorithm and then compared the obtained results with those of the GA and simulated annealing. Rostami (2015) applied the entropy criterion, which is not dependent on the assets' return distribution symmetry, in contrast to the variance, as the risk measure to optimize the fuzzy portfolio. The proposed model is aimed at solving the problem based on mean-entropy-skewness. Also, due to the linearity of the proposed model, it has been solved using linear programming.

Sun (2015) and Liu (2016) have addressed the problem from a different perspective. They addressed the problem as a multi-period one. They aimed to find the ratio of the assets at each period and attempted to simplify the portfolio optimization problem. Furthermore, Mehlawat (2016) has solved the problem as a multi-period multi-objective fuzzy problem.

Kaucic et al (2019) introduced a novel strategy for portfolio selection. They used semi-variance, conditional value-at-risk, and a combination of both as the risk criteria for loss-averse investors. Moreover, they proposed a new version of a genetic algorithm and used five publicly available datasets to address small- to large-sized portfolio optimization problems. They assessed the capabilities of their procedures in terms of four performance metrics and applied various statistical tests in an effort to assess the robustness of their findings. They concluded that the proposed algorithm outperformed others with respect to all criteria.

Yeh and Liu (2020) considered the challenges of a weight-scoring approach in stock selection models. Their study employed a mixture of experimental designs to collect the weights of stock-picking concepts and portfolio performance data to predict portfolio performance. They also used a sample of stocks listed on the Taiwan Stock Exchange in 1997 for modelling and in 2015 for testing. Based on the data from the training period, their results indicated that mixture experimental designs and multivariable polynomial regression can be used to construct performance prediction models. Furthermore, the methodology can discover interactions between the weights of stock-picking concepts. They also indicated that selected stock portfolios can meet various investor preferences. Such portfolios are obtained through the optimal combination of weights of factors, determined by proposed optimization techniques. They concluded that the proposed method could overcome the challenges of the classical weighted-scoring approaches.

Rahiminezhad et al (2020) developed a method for applying multiple criteria to evaluate and select portfolios. The FANP approach was used to rank portfolios in consideration of uncertain conditions and decision-makers' judgments. Although most studies consider return and risk as the only decision-making criteria, this study finds that profitability, growth, market, and risk could be efficient indicators in portfolio selection models. The main contribution of the study is that it applied a new approach, FANP, to assess and select portfolios.

As noted above, the Markowitz model was constructed to identify the portfolio with minimum risk for a specified level of expected return (or, equivalently, identify the portfolio with the maximum expected return for a specified level of risk). This model has served as the basis of studies on portfolio optimization models in terms of the optimization methods and algorithms, the application of the uncertainties in the return rate, the use of various criteria for risk estimation, and the application of different constraints. All the models reviewed above assume, usually implicitly, independence in the optimization of each account while the mutual effects of simultaneous and joint maximization are ignored.

A Review of the Selected Papers on Market Impact Models

There are a number of studies focusing on transaction costs in the market microstructure literature. Some include market impact cost models. For example, the primary theoretic models introduced by Kyle (1985), Glosten and Milgrom (1985), and Hasbrouck (1991) focused on microstructure models that describe the market impact of asymmetric information. These models take a traditional microstructure approach to the financial markets and mainly focus on the effect of informed traders' behavior versus that of uninformed traders. Although these models are unable to describe the effects of market impact, they are considered a starting point for further work in the field.

The Kyle model (1985) makes the simple assumption is that market impact cost is linearly related to trading volume and has a permanent impact over time. Also, this model justifies the assumption that the market maker clears the orders of the informed and uninformed traders. Hasbrouck (1991) considered the form of the market impact function with regard to the size of the orders. In a study of the market impact curve conducted by Lillo, Farmer, and Mantegna (2003), the shape of the curve was found to be concave.

Market impact is commonly classified into two types: temporary impacts and permanent impacts. Temporary impacts reflect liquidity demand costs while permanent impacts, i.e., long-term impacts, reflect information arrival in the market. Clearly, if the market identifies a big buyer or seller, it will be seen as a strong signal and will influence the asset's price. A single large buy or sell order might have a greater temporary impact than a set of small orders. However, the permanent impacts of small orders are similar to the result of the signal generated by a large order (Johnson 2010). It is quite difficult to decompose the market impact into temporary and permanent components. Some of price estimation models, assuming that the transaction wouldn't cause a considerable general impact, consider the permanent impact to be zero. However, the permanent impact can be estimated to some extent using the estimation model proposed by Kissell et al. (2004). Stoll (1997) states that the execution of large orders will result in temporary and permanent market impacts. This hypothesis has been tested by Biais (1995) and it has not been rejected.

In some other studies of market impact, the trades' market impact has been assumed for a single stock. These studies have treated the impact as a concave function of the trades' volume. Lillo et al. (2003) showed that the market impact of single stock trades is an exponential function of the trades' volume with exponents ranging from 0.2 to 0.5.

Instead of investigating the market impact from a selling or buying trade, some studies have focused on the market impact associated with a set of selling and buying trades over a specified time interval. These studies sought to determine the cost incurred by all of the trades made during the given interval. Patzelt and Bouchaud (2017) investigated whether the basic market impact functions can explain the concavity and nonlinearity of the market impact. For this purpose, in these studies, the transactions (trades) were classified into two categories: first, transactions that cause changes in prices and, second, transactions with no impact on prices. It was shown that price changes depend on the flow of irregular orders.

Other studies have investigated the market impact of big organization orders, which are known as hidden orders. Bouchaud’s study is of great importance in this regard (Bouchaud, 2009). Generally, the empirical literature on the market impact of hidden orders is limited due to the difficulty of getting access to relevant information. One study of note on the impact of hidden orders was conducted by Almgren (2003). The market impact model (AC) was used in the study. This model estimates the total cost of the order based on the sequence of the transactions. This is considered a bottom-to-top approach. Within the framework of this model, the total cost of the order is determined based on the real order size and the sequence of the transactions. Kissell et al. (2004) introduced the ***I Star*** model, an approach which is a top-to-bottom allocation of the costs. In this model, the total cost of the order is first estimated and, then, the estimated cost is allocated to the transaction periods based on the transaction schedule. The I-Star function includes liquidity, fluctuations, imbalances, and in-day transactions. Huberman & Stanzl (2001) and Farmer et al. (2004) showed that the specific feature of the reaction or linear impact of Kyle's model is that it does not allow manipulation of the price. Different market impact modeling approaches can also be found in the theoretical literature including Wagner (1991), Kissell and Glantz (2003), Chan and Lakonishok (1997), Bertismas and Lu (1998), Lillo et al. (2003), and Gatheral (2010).

A notable point in this regard is the adoption of an appropriate approach to obtain a “functional” description of the market impact model. The existing studies only identify and determine the main variables taken into account for estimating a market impact function. The main issue is the possibility of estimating the impact function based on the available data in financial markets and evaluating its effect on portfolios.

A Review of the Selected Papers on Multiportfolio Selection Models

The multiportfolio optimization problem was first proposed by O’Cinneide et al. (2006), who noticed some suspicious interactions among investment accounts and identified problems relevant to fairness and potential profit from simultaneous rebalancing of accounts. In the model proposed in their study, social welfare maximization was assumed as the objective function. As such, a single optimization model could lead to multiple portfolio optimizations (multiple accounts) as it can represent all possible states and all transactions for all accounts under management. They argued that work on multiple optimizations could resolve the joint transactions' problems. Additionally, fairness was achieved since the outcome demonstrated a competitive balance for liquidity among the accounts participating in the market. O’Cinneide et al. (2006) believed that the multiple optimizations would yield the same decision for the customers as those they would make if they wanted to compete in the market for liquidity. This claim cannot be validated within the framework of the proposed model; thus, the major reason to reject this claim is the level of access to the information for decision-making. The individual investor knows nothing about the behavior and decisions of other competitors and actors. On the contrary, the investment managers are the asset managers and investment consultants that manage multiple accounts simultaneously with different levels of information, so that, normally, such difference in levels of information can affect their decisions.

O’Cinneide et al. also emphasized that making transactions decisions must be associated with fairness. Therefore, they formulated their optimization problem in such a way that it could optimize the portfolios of all customers and, consequently, towards social welfare. This modeling has been performed as the summation of the objective functions of the personal accounts, and the transaction costs have been imported into the model nonlinearly. The framework of their proposed model was built based on the assumption that the total transaction cost is a nonlinear function of the total volume of the transactions. The authors have claimed that the proposed method ensured fairness and they have called this process the multi-account optimization. Liquidity allocation in multiple optimization is a set of Pareto solutions, thus liquidity cannot be made better for one customer without being decreased for another customer. As a result, all customers will achieve optimal portfolios (O'Cinneide et al., 2006).

Stubbs and Vandenbussche (2009), Savelsbergh (2010), and Yang (2013) conducted comprehensive investigations of the issues surrounding the multiportfolio optimization techniques. They discussed the advantages and disadvantages of the Cournot-Nash equilibrium economic approach and the collusive solution and, thereby, presented an integrated framework that could solve the problem using both methods. In the collusive solution method, the total welfare is maximized, meaning that the sum of the objective functions of all accounts is maximized. In the Cournot-Nash equilibrium method, besides maximizing the total welfare, each target account optimizes itself while assuming that the transaction decisions of all the accounts participating in this joint transaction are specified and fixed. Savelsbergh (2010) compared the results of the two methods and concluded that both methods have their own advantages and disadvantages so that neither of them can be preferred over the other. However, later, Iancu and Trichakis (2014) proved that the Cournot-Nash equilibrium method not only isn't suitable for the establishment of fairness but, also, it doesn't necessarily yield the optimal solution, because the accounts participate in a fake game in which the Securities and Exchange Commission's rules are violated and, thus, the obtained results cannot be reliable.

In Iancu and Trichakis (2014), the authors presented a review of the literature and a comprehensive discussion on existing practices in the financial services industry. They identified three main challenges with which financial service providers must deal. First, if the problematic mutual effects among the transactional activities of multiple accounts are ignored, the advantages of rebalancing might be reduced significantly. Second, there is a considerable potential profit in the joint optimization structure and coordination in rebalancing the individual portfolios. And finally, the last challenge is to know which information should be published at what time in order to achieve a fair distribution of benefits among the portfolios. Accordingly, they proposed a novel and appropriate approach that includes a model in which the market impact is considered along with the three above-mentioned challenges. In this model, the market impact cost, unlike that in previous studies, is not exogenous and not in the form of weight-division among the accounts. Instead, it is treated as a stochastic variable. On this basis, it can be said that the major feature of their study is the endogenous consideration of the market impact cost of all the individual accounts in contrast to the common assumption of the exogenous estimation.

Jing Fu (2017) proposed an information pooling game for multi-portfolio optimization which differs from the classical ones in several aspects, with a key distinction of allowing the clients to decide whether and to what extent their private trading information is shared with others, which directly affects the market impact cost split ratio. The empirical results suggest that within this framework, information pooling has non-negative impact on all participants' perceived fairness, although it may hurt some account's realized benefit compared to null information pool.

Ji et al. (2018) proposed a class of stochastic risk budgeting multi-portfolio optimization models that impose portfolio as well as marginal risk constraints. The models permit the simultaneous and integrated optimization of multiple sub-portfolios in which the marginal risk contribution of each individual security is accounted for. A risk budget defined with a downside risk measure is allocated to each security.

Zhang et al. (2019) considered market impact cost in multiportfolio optimization model. The main contribution of their study was using Conditional Value-at-Risk (CVaR) for risk measurement and modeling market impact cost in joint optimization framework. The study proposed a model while market impact costs accounted as the unique feature of the model. Results show joint optimization model incurs less market impact cost than the independent decision.

Yu et al. (2020) developed a target-oriented framework that optimizes the rebalancing trades and the market impact costs incurred by trading jointly with consideration of target and distributional uncertainty. To evaluate multiple portfolios' uncertain payoffs in achieving their targets, they first proposed a type of performance measure, called the fairness-aware multi participant satisficing (FMS). In MPO, they focused on the FMS criterion with the underlying risk measure being conditional value-at-risk.

Lampariello et al. (2021) analyzed a Nash equilibrium problem arising when trades from different accounts are pooled for execution. They introduced a multi-portfolio model and state conditions for the monotonicity of the underlying Nash equilibrium problem. Monotonicity makes it possible to treat the problem numerically and, for the case of nonunique equilibria, to solve hierarchical problems of equilibrium selection. They also gave sufficient conditions for the Nash equilibrium problem formulation to be a potential game.

Sum-up

This concluding section represents a review of studies conducted on the portfolio optimization problem that are directly relevant to the targeted challenges of the present study. Undoubtedly, Markowitz's work is of great importance and serves as the starting point for all portfolio optimization studies. Subsequent to that seminal work, further developments can be classified as follows:

- Diversity in solution methods of the optimization problem.
- Development of models based on multiportfolio optimization (MPO).
- Development of models that take the market microstructure into consideration.
- Diversity in risk and return estimation indices and new modeling constraints or other indices that can be determined proportionate to the behavior of investors and markets.

The present study is mainly focused on multiportfolio optimization with regard to market microstructure. As indicated by this review, prior studies were not aimed at endogenous consideration of market microstructure and modelling using real data. They were focused on the development of the theoretical fundamentals. This is why an estimation of a market impact function based on real data and the application of a multiportfolio optimization model based on it will pave the ground for an integration of the work of the previous studies.

3. Modeling

3.1. Market Impact Model

The market impact function used in this study has been adopted from the I* model proposed by Kissell and Glantz in 2003. The I* model is a cost allocation approach in which the market activists incur some costs based on the size of their orders. Also, it follows the economic supply-demand equilibrium. In fact, the cost incurred by the investor is created in the case that the total order in the market is made at once. Moreover, it is assumed as the total payment required to attract excess customers and sellers to the market. In economy, I* is the incremental cost incurred by the demanders due to the supply-and-demand disequilibrium.

The model used in this study is as follows:

$$I^* = \theta\left(\frac{Q}{ADV}\right)^y \quad (1)$$

where Q is the market imbalance, which represents the trade volume in the I* model.

The imbalance in the given period is obtained from the instantaneous data related to the volume of the trades made during market hours. Based on the modified tick rule, transactions are divided into two categories of trades, including those performed by the buyer and those performed by the seller. In this rule, the price of the trades is compared with the mean price gap (last buy and sell offer before the trade). Accordingly, the trades with a price higher than the mean price gap are assumed as the buyer-made trades and those with a price lower than the mean price gap are assumed as the seller-made trades. But in the case of equal prices, the classification of the transaction will be made based on the price of the previous trade; so that, if the price of the previous trade is lower, it will be assumed as the buyer-made trade, and in the case of a higher price, the trade will be assumed as the seller-made trade. Further, in the case of the same prices, the price of the trade made before the previous one will be taken into account. This procedure is continued until the price is changed.

After classifying the transactions into two buyer-made and seller-made groups, in this study, only the buyer-made trades have been considered in order that only the positive market impact, which increases the price, is taken into account.

In this case, the imbalance is obtained from the difference between the total offered volume of the three-level buy and the total offered volume of the three-level sale.

In this equation, ADV is the average daily volume of the trades in T days (during transactional hours). Also, $v_i(day)$ is the total volume of the transactions made on the i^{th} day.

$$ADV = \frac{1}{T}\sum_{I=1}^{T} v_i(day) \quad (2)$$

The value of the market impact is calculated for modelling from the difference between the trade price and the last buy offer price before the trade. θ and y are the model's parameters.

3.2. Multiportfolio Optimization Model

In this section, a multiportfolio optimization model with 4-step optimization schemes, based on Zhang et al. (2019), will be introduced. The presented research model is a multiple optimization framework.

Before introducing the main model, first, the method used to determine the market impact costs, the distribution of costs among different accounts, and the utility function as the objective function of the optimization model are introduced and described. Also, the modeling assumptions and the used symbols are presented. To construct the model, the following assumptions are made:

- The problem is considered and executed in a single-period framework.
- The portfolio selection problem, considered as a single-portfolio one, aims to maximize the utility (net profit), which is equal to the return of the portfolio minus the market impact cost. In the multiportfolio optimization, the net return is optimized in joint with a multiple-objective optimization problem.
- Short selling is not possible.

The symbols used in this model include:
k: index of portfolio or user account, k=1,2, …, m
i.j: index of stocks
n: number of stocks
x_{ki}: a volume of the i^th stock, which is selected for the k^th account
X_{ki}: vector of the selected percentage of the stocks
C_k: capital of the k^th user account
$\bar{r}_i$: expected return of the i^th stock
$\bar{r}_{p_k}$: expected return of the k^th individual's portfolio
R: vector of the expected return
p_i: price of the i^th stock
σ_k: the lowest risk level for k^th account that its value is the result of solving the first step
k_k: risk-aversion coefficient of the k^th account (individual)
θ_i: market impact coefficient
y_i: market impact parameter
ADV_i: average daily volume of the i^th stock
u_k: utility of the k^th account
$f(U_1, U_2, \ldots, U_m)$: welfare function

A. Market Impact Cost

The market impact cost of the whole volume of the i^th stock, which has been purchased by all portfolios, is equal to:

$$t_i\left(\sum_{k=1}^{m} x_{ki}\right) = \theta_i\left(\frac{\sum_{k=1}^{m} x_{ki}}{ADV_i}\right)^{y_j} \quad (3)$$

The total market impact is:

$$t_T = \sum_{i=1}^{n} \theta_i\left(\frac{\sum_{k=1}^{m} x_{ki}}{ADV_i}\right)^{y_j} \quad (4)$$

B. Method of Cost Division Among the Accounts

In the proposed model, the total market impact cost is divided among all the portfolios using the pro-rata method. Thus, the total market impact cost imposed on the k^th portfolio is:

$$t_k = \sum_{i=1}^{n} \frac{x_{ki}}{\sum_{a=1}^{m} x_{ai}}\theta_i\left(\frac{\sum_{a=1}^{m} x_{ai}}{ADV_i}\right)^{y_j} \quad (5)$$

C. Utility Functions

The most common description for the quantification of utility is to consider the return. Thus, utility of the k^th account is:

$$u_k = R^T X_k \quad (6)$$

The net expected utility U_k for the k^th account is equal to the total expected return (U_k) for the k^th account so that a fraction of this account is subtracted as the costs affecting the market.

$$U_k = u_k - t_k \quad (7)$$

D. Modeling

The model used in this study has been designed in four steps so that, at the end, the results of each step are compared. In this model, the variance is used as the risk measure. Estimating the market impact cost and dividing the costs have been performed using the I* model and the pro-rata method, respectively.

Step-1: The portfolio optimization problem is solved for each account independently, and the objective function is the variance of the account that is minimized.

$$minZ_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{x_{ki}p_i}{C_k}\frac{x_{kj}p_j}{C_k}cov(\bar{r}_i,\bar{r}_j) \quad (8)$$

$s.t$:

$$\sum_{i=1}^{n} x_{ki}p_i\bar{r}_i \geq \bar{r}_{p_k}C_k \qquad \forall k$$

$$\sum_{i=1}^{n} x_{ki}p_i \leq C_k \qquad \forall k$$

$$x_{ki} \geq 0 \qquad \forall k, \mathrm{i}$$

The first step is the same as the classic Markowitz model, based on which the minimization of the account's variance considering three constraints, including the minimum expected return, the accessible resources of the account's owner, and the minimum value of the variable. At this step, in addition to the mutual effects of the optimization, the market impact costs are also ignored.

Step-2: At this step, again, the accounts are optimized independently, and the utility of each account is maximized individually. The model is executed based on two constraints, including the maximum portfolio risk and minimum value of the variable. At this step, the market impact cost is estimated, but it is assumed that the transactions of different accounts are independent of each other, and the market impact cost of each account is considered in the utility of that account.

$$\max Z_2 = \sum_{i=1}^{n}\bar{r}_i\frac{x_{ki}p_i}{C_k} - \sum_{i=1}^{n}\theta_i\left(\frac{x_{ki}}{ADV_i}\right)^{y_j} \qquad \forall k \quad (9)$$

s.t:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{x_{ki}p_i}{C_k}\frac{x_{kj}p_j}{C_k}cov(\bar{r}_i,\bar{r}_j) \leq k_k\sigma_k \qquad \forall k$$

$$x_{ki} \geq 0 \qquad \forall k$$

The output of the execution of the model at this step is the optimal vector x_i, which is represented by X_{ki}^{IND} due to the independence of the accounts. Ignoring the mutual effects of the accounts causes a significant reduction in the actual costs of the market impact as a result of the transactional activity of each account. Due to this unreal cost, the output deviates from the optimal output. To make the costs more real, all the transactions and the resulting costs should be estimated. Accordingly, the total transactions of each stock made by the manager is:

$$\sum_{k=1}^{m} X_{ki}^{IND} \quad (10)$$

The market impact cost of the i[th] stock is:

$$t_i\left(\sum_{k=1}^{m} X_{ki}^{IND}\right) = \theta_i\left(\frac{\sum_{k=1}^{\mathrm{m}} X_{ki}^{IND}}{ADV_i}\right)^{y_i} \quad (11)$$

Consequently, the market impact cost of all the stocks is calculated as follows:

$$t_T = \sum_{i=1}^{n}\theta_i\left(\frac{\sum_{k=1}^{\mathrm{m}} X_{ki}^{IND}}{ADV_i}\right)^{y_i} \quad (12)$$

Step-3. At this step, the market impact cost estimation for each account is modified. For this purpose, the total market impact cost on all accounts is determined, finally, the distribution of these costs is performed fairly using the pro-rata method. At this step, despite the independent optimization of the accounts, the effect of the transactions of different accounts on each other is taken into account. To do this, the total market impact is distributed among all the accounts using the pro-rata method and the net utility calculated at the previous step becomes more real due to the correction of the costs. Since the effects of the accounts' transactions on each other are taken into account, the results of this step are closer to the reality. The net utility for the i[th] account is calculated using Eq. (10):

$$\frac{\sum_{a=1}^{m} X_{ai}^{IND}}{ADV_i})^{y_i} \qquad \forall k \qquad (13)$$

Step-4. At this step, the multiportfolio optimization is carried out through integrating the market impact costs that have been modified at the third step. This step is aimed at the simultaneous optimization of the accounts (multiportfolio optimization), and the max-min function represents describes the trade-off between welfare (total utility) and fairness (fair allocation of the utilities). According to the third step, the market impact costs are allocated to the accounts through fair division among the accounts.

At this step, the investment manager optimizes all accounts in joint, and the allocation of the market impact costs will be determined endogenously in the objective function and simultaneous with solving the problem. The important point is the different approach of the fourth step in executing the optimization, in which the mutual effects and dependencies among the accounts are considered and, unlike the previous steps, the output is not based on the assumption of the independence of each account.

The objective function, $f(U_1, U_2, \dots, U_n)$, is a welfare function that is made as follows:

$$f(U_1, U_2, \dots, U_m) = \min\left\{\frac{U_i - U_i^{IND}}{U_k^{IND}}\right\} \qquad (14)$$

U_k^{IND} is the net utility of the k[th] account, which has been derived from the independent framework while the net utility is obtained from the framework of joint optimization with U_k.

The risk measure for the proposed multiportfolio optimization model is the variance, and the output of the model at this step is how to allocate the capital of each account holder to different types of the existing assets and the construction of the optimal portfolio by an investment manager for various accounts. The advantage of the output is that the impact of the transactions of each account on the performance and return of other accounts is taken into account, as the market impact costs, simultaneous with the optimization.

$$\max Z_3 = \min\left\{\begin{array}{c}\frac{(\sum_{i=1}^{n}\bar{r}_i\frac{x_{1i}p_i}{C_1} - \sum_{i=1}^{n}\frac{x_{1i}}{\sum_{a=1}^{m}x_{ai}}.\theta_i(}{U_1^{IND}} \\ \vdots \\ \frac{(\sum_{i=1}^{n}\bar{r}_i\frac{x_{mi}p_i}{C_m} - \sum_{i=1}^{n}\frac{x_{mi}}{\sum_{a=1}^{m}x_{ai}}.\theta}{U_m^{IND}}\end{array}\right. \qquad (15)$$

s.t:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{x_{ki}p_i}{C_k}\frac{x_{kj}p_j}{C_k}cov(\bar{r}_i,\bar{r}_j) \le k_k\sigma_k \qquad \forall k$$

$$\sum_{i=1}^{n}\bar{r}_i\frac{x_{ki}p_i}{C_k} - \sum_{i=1}^{n}\frac{x_{ki}}{\sum_{a=1}^{m}x_{ai}}.\theta_i(\frac{\sum_{a=1}^{m}x_{ai}}{ADV_i})^{y_i} \ge U_k^{IND} \qquad \forall k$$

$$x_{ki} \ge 0 \qquad \forall k, i$$

Since the min-max function has been used for the optimization, the objective function maximizes the least increase in the net utility relative to the utility obtained in the previous step. The model's constraints include the risk level of each account regarding the minimum risk level and the increase in the utility compared to the utility in its independent mode.

4. Implementation and Results

In this section, first, characteristics of the sample used to implement the model as well as the resources used to get access to the research data are introduced. The second step includes the estimation of the market impact functions considered in the proposed optimization model. Then, using the above-mentioned functions and real data of the Tehran Stock Exchange, the model introduced in the third section is implemented and the relevant results are extracted under different assumptions. Afterwards, the results obtained from different models are analyzed and compared.

4.1. Data Used for Model Implementation

The present work was conducted using the data of the transactions of Tehran Stock Exchange in 1398. On this basis, initially, 50 stocks with the highest liquidity during the research period were extracted. The list of the stocks with the highest liquidity was prepared based on two indicators, namely the volume and the total number of transactions. Subsequently, from among these stocks, ten stocks with the highest market value were assumed as the chosen stocks in the sample. It should be noted that the above-mentioned data were obtained through Rahavard 365 software and filtering on the website of Tehran Securities Exchange Technology Management Company. Characteristics of the used sample along with the data of price, average return, and standard deviation are presented in Table 1. Also, Table 2 represents the data of the variance-covariance matrix[2] of the selected stocks.

[2] To make the table easier to read, all data is multiplied by 1000.

Table1 Characteristics of the research sample

No.	Company Name	Stock Symbol	Standard Deviation	Average Return	Price
1	Saipa Automotive Group	Khesapa	0.442836358	0.004167274	1371
2	Melat Bank	Vabemelat	0.35805007	0.004820278	1746
3	Isfahan Mobarakeh Steel	Foolad	0.376654044	0.005135377	5184
4	Pars Khodro	Khepars	0.39716239	0.00176162	987
5	National Iran Cooper Industries	Femeli	0.507297023	0.003383883	3890
6	Isfahan Oil Refinery	Shepna	0.432466584	0.004372896	7148
7	Tamin Petroleum & Petrochemical Investment	Tapiko	0.361901142	0.001699412	1904
8	Zamyad	Khezamiya	0.43624912	0.001897637	964
9	Iran Khodro	Khodro	0.359222547	0.000940988	3042
10	Kharazmi Investment	Vekharazm	0.355608557	0.001986713	1082

Ref: Tehran securities exchange technology management co.

Table 2 Variance-covariance matrix of the research sample

Var-Cov	Shepna	Tapiko	Khezamiya	Khepars	Khesapa	Khodro	Femeli	Foolad	Vabemelat	Vekharazm
Shepna	0.79	0.39	0.17	0.18	0.19	0.16	0.29	0.37	-0.18	0.12
Tapiko	0.39	0.56	0.13	0.09	0.12	0.04	0.36	0.42	-0.04	0.16
Khezamiya	0.17	0.13	0.81	0.64	0.64	0.51	-0.02	0.09	0.04	0.18
Khepars	0.18	0.09	0.64	0.67	0.62	0.51	-0.04	0.07	0.03	0.17
Khesapa	0.19	0.12	0.64	0.62	0.83	0.48	0.00	0.08	0.04	0.18
Khodro	0.16	0.04	0.51	0.51	0.48	0.55	-0.02	0.03	0.00	0.14
Femeli	0.29	0.36	-0.02	-0.04	0.00	-0.02	1.09	0.48	-0.09	0.20
Foolad	0.37	0.42	0.09	0.07	0.08	0.03	0.48	0.60	-0.06	0.15
Vabemelat	-0.18	-0.04	0.04	0.03	0.04	0.00	-0.09	-0.06	0.54	0.04
Vekharazm	0.12	0.16	0.18	0.17	0.18	0.14	0.20	0.15	0.04	0.54

Ref: Tehran securities exchange technology management co.

Due to the large volume of the data of the market microstructure (tick-by-tick transactions), the estimation of the market impact function and the calculation of the parameters utilized in the market impact model and multiportfolio optimization were carried out by using merely the data of the first six months of the given year. Also, the data related to before 9:00:00 (pre-opening time) and after 12:00:00 were not taken into account.

The price used in this study was the closing price of the last trading day, and the logarithmic average of the closing prices was assumed as the average return. The number of working days in the first 6 months of the year was T = 116 and that of the whole year was 240. Given these values, the average daily volume of each stock will be as given in Table 3.

Table3 Average daily value of each stock

Stock Symbol	ADV	Stock Symbol	ADV
Foolad	65422172.5	Khesapa	74844547
Khepars	33068266.8	Vabemelat	59840242
Vekharazm	26371710.8	Khodro	11541061.2
Khezamiya	36039549.2	Femeli	46801703.3
Tapiko	31547762.8	Shepna	21719012.8

Ref: Research findings

In the present work, to achieve results with higher reliability, the data of the final step were monitored and the case data events were excluded from the study. For this purpose, the filters proposed in Kissell's study (2013) were used:

$$\text{Daily volume} \leq 3*\text{ADV} \quad (16)$$

$$\frac{-4\sigma}{\sqrt{240}} \leq \log price\ change(close - to - close) \leq \frac{4\sigma}{\sqrt{240}} \quad (17)$$

4.2. Market Impact Function Estimation

Once the average daily volume of each stock was obtained, the data of disequilibrium (Q) and market impact (I*) of different stocks were prepared regarding the explanations given in Section 3. Finally, the market impact function was estimated using the ordinary least square (OLS) method in EVIEWS software (for this estimation, the zero values of ISTAR as well as the negative and zero values of Q were not taken into account). Also, to avoid false regression error in the implementation of the model, the Augmented Dickey-Fuller (ADF) test was used to evaluate the stationary hypothesis of the time series data. Accordingly, results of the estimated market impact function's parameters are presented in Table 4.

Table4 Market impact function estimation coefficients

Dependent Variable = logistar	***Variable***		***Std.Error***	***T***	***Prob.***	***R^2***	***Observations***
Khesapa	C	3.8507	0.04161	92.544	0.000	0.245678	3157
	Logsize	0.2713	0.0084	32.055	0.000		
Vabemelat	C	4.5115	0.0590	76.392	0.000	0.205920	2345
	logsize	0.2639	0.0107	24.649	0.000		
Foolad	C	7.1234	0.1261	56.454	0.000	0.360434	2799
	logsize	0.6033	0.0189	31.823	0.000		
Khepars	C	4.2815	0.0491	87.044	0.000	0.305598	4400
	logsize	0.4554	0.0103	43.994	0.000		
Femeli	C	6.0844	0.620	98.026	0.000	0.364181	3530
	logsize	0.4448	0.0098	44.952	0.000		
Shepna	C	6.8445	0.0784	87.19	0.000	0.427873	2573
	logsize	0.5796	0.0132	43.849	0.000		
Tapiko	C	6.3146	0.0707	89.262	0.000	0.413225	2851
	logsize	0.6200	0.0138	44.792	0.000		
Khezamiya	C	3.8292	0.0356	107.31	0.000	0.331452	3608
	logsize	0.3695	0.0087	42.282	0.000		
Khodro	C	5.7442	0.0818	70.165	0.000	0.276934	2876
	logsize	0.4135	0.0154	26.790	0.000		
Vekharazm	C	5.2932	0.0509	103.86	0.000	0.353578	3101
	Logsize	0.3988	0.0096	41.171	0.000		

Ref: Research findings

The outputs of the market impact function's regression are given in Table 4. As can be seen, all coefficients of the model are significant at the 95%level (large values of t and probabilities close to zero). The values of R^2 also due to the high number of observations (more than 2000 data), which indicates the proper explanatory strength of the estimated function.

The market impact represents the changes in the stock price due to the transactions and orderings. In accordance to this function, different accounts incur a cost known as the transaction cost or market impact cost, proportionate to the volume of the orderings. In other words, each order results in a kind of imbalance in the market that imposes a cost on the account owner and other investors, proportionate to the stock's liquidity and market conditions, to complete and execute the order. Despite the same volume of the orderings in the two types of the stocks, the differences resulting from the inherent and microstructural features and different market conditions would lead to different market impact of the stocks. Thus, the order volume is not assumed as the only factor and variable affecting the market impact cost and, thereby, it is necessary to estimate a specific function for each stock.

According to studies in this field, the graph of the market impact function in the stock markets of different countries is concave for different volumes. As indicated by the estimations carried out with the data of the selected sample, the market impact function in Iran Stock Market is also concave. Figure 1 shows the comparison of the behavioral pattern of the market impact costs of the selected stocks.

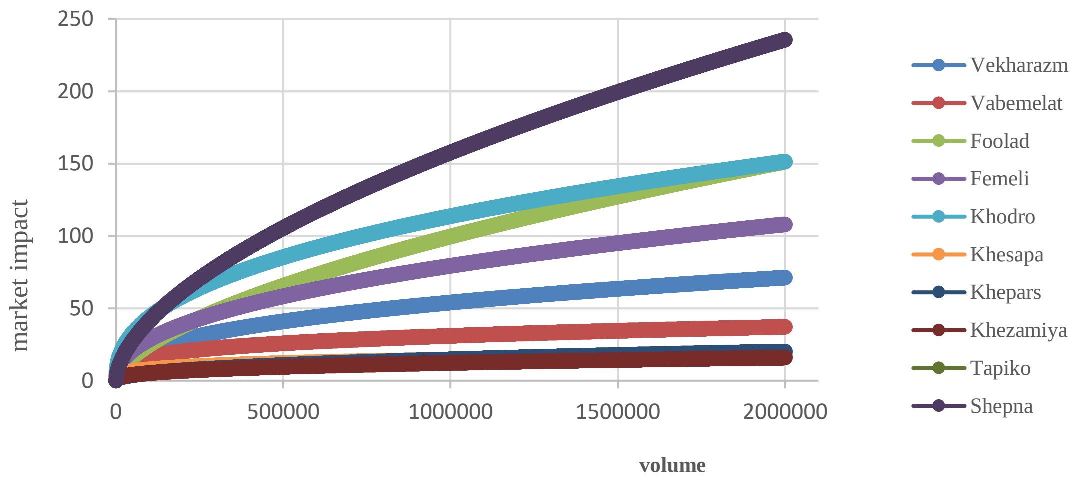

Figure 1. Comparison of the graphs of the market impact functions.

4.3. Model Implementation Results

The model presented in Section 3 was implemented assuming 3 independent accounts (m) and 10 stocks (n). The total capital (C_k) and risk-aversion coefficient[3] (K_k) of each customer were assumed as the model's inputs. The input parameters are presented in Table 5. The input values for the risk-aversion coefficient and the initial capital of the account holders are selected by the customer or the investment manager within a certain range. In the present work, these values were determined randomly.

Table-5: Model's inputs

Initial capital (1000 Rials)		Risk tolerance	
C_1	250000	K_1	5
C_2	100000	K_2	16
C_3	140000	K_3	8

In this study, the model was executed using GAMS software, which is an exact solution method, the obtained results of which are presented and analyzed below. Tables 6 and 7 show the variations of risk, return, market impact, and utility from Step 1 to Step 4.

Table-6: Variance of different accounts in modelling steps

Account	Variance		
	Step1	Step2	Step3
1	9.297514×10^{-7}	5.578508×10^{-6}	5.578508×10^{-6}
2	3.047395×10^{-6}	3.656874×10^{-5}	3.656874×10^{-5}
3	1.950333×10^{-6}	1.755299×10^{-5}	1.755299×10^{-5}

Ref: research findings

[3]The term $k_k\sigma_k$ on the right side of the risk limit is an upper bound for the customer's risk tolerance. Since $\sigma_k$is the minimum risk that is expected to be imposed on a portfolio, k_k is always greater than or equal to 1. But to calculate the upper bound of k_k, the set of solutions that results in maximum efficiency in the second step without considering risk constraint is obtained and used to calculate the maximum risk and upper bound of k_k.

Table-7: Outputs of the model in different steps of the model execution

account	Return			Market Impact			Utility			Improve (%)
	Step 2	Step 3	Step 4	Step2	Step3	Step4	Step 2	Step 3	Step 4	
1	13.104	13.104	11.955	0.661	3.737	1.659	12.443	9.367	10.296	9%
2	31.142	31.142	34.110	0.683	3.840	1.210	30.359	27.302	32.9	20%
3	21.546	21.546	22.542	0.519	2.827	0.974	21.047	18.719	21.569	15%

Ref: Research findings

4.4. Analysis of Results

The variance values for each account have been calculated separately at different steps, which are given in Table 6. Also, the values of return, market impact, and utility are presented in Table 7. As mentioned earlier, at the first step, for calculating the variance and return, the accounts were considered independent and the market impact was ignored. The computational variance at this step is, indeed, the minimum variance in the independent optimization of each account. This can be also inferred form the smaller values in the first step of Table 6.

At the second step, the market impact has been also considered in the model and the net utility of the expected portfolio for independent accounts has been maximized. At this step, it was assumed that the transactions of different accounts do not affect each other, the accounts were optimized independently, and the respective market's impact and return were obtained. Moreover, the utility was calculated as the difference between these two values. Since the mutual effects of different accounts were not considered, the results seem to be different from what happens in reality, and the obtained utility is unreal and deviated due to the underestimation of the costs and ignoring the correlation of accounts. At this step, since the market impact has been taken into account, the risk is increased compared to the previous step; on the other hand, since the effects of the transactions of different accounts on each other have been ignored, the market impact cost is underestimated and the return and, consequently, the utility are overestimated in comparison the reality. These results are represented in Tables 6 and 7. Also, importantly, what is extracted from the model for the second step is not reliable for programming due to the weakness of the above-mentioned assumptions in relation with the logical relationship between the accounts.

The third step addresses a case in which, despite the independent optimization of the accounts, the effects of transactions of different accounts on each other are taken into account. In such case, the total market impact is divided among all the accounts using the pro-rata method, and then the obtained value is subtracted from the return of the independent transactions (calculated in the second step), and utility is calculated. As shown in Table 7, as the market impact cost increases and the return remain unchanged, the utility decreases compared to the second step. Since the effects of the transactions of the accounts on each other are taken into account, the results of this step are closer to the reality.

At the fourth step, the problem of the simultaneous optimization of the accounts (multiportfolio optimization) has been discussed, and the proposed max-min function represents the trade-off between welfare (sum of utilities) and fairness (fair allocation of utilities). The market impact has been distributed among the accounts proportionately using the pro-rata method. As indicated by the obtained results, the final utility of all accounts in this method has been increased by nearly 9.5% compared to the utility of the independent mode (third step). The increased utility confirms the better performance of the multiportfolio optimization method compared to the independent optimization of the accounts. The main reason for such a change is the reduced market impact for the case of aggregated transactions. On the other hand, the equal increase in the utility of all accounts represents the observation of fairness in this method. Thus, determining the transactions of all accounts managed by the investment manager using the proposed method can yield almost the same profit for all customers.

5. Conclusion

5.1 Results and Discussion

The studies on the investment portfolio optimization are commonly based on the assumption that the accounts are managed independently. But, in real conditions, the management of multiple accounts at the same time is performed by one investment manager. On this basis, the present study is principally aimed to model the multiportfolio selection for Iran Stock Market considering the market impact costs.

- A major challenge in modeling the existing problem and answering the given question in the present work is to model the market impact costs resulting from the transactions.
- The fair allocation of these costs to different accounts should be also regarded as a notable problem in the optimization process.

To answer the above-mentioned questions and challenges, a 4-step framework, including the optimization of the investment accounts under different assumptions, was implemented using GAMS software, the results of which were then compared. Besides, since most of the previous studies had been conducted using sample data and due to their inconsistency with the real conditions and realities of financial markets, the present work was conducted using the data of selected stocks from Tehran Stock Exchange.

The implementation of the designed models showed that the simultaneous optimization of multiple portfolios using the IStar market impact model and the allocation of the estimated costs using the pro-rata method can be a quite suitable approach for the investment managers for managing different accounts. The proposed model, besides being executable in the real world using market data, appropriately reveals the market impact costs and improves the utility of the accounts. As indicated by the results of the implementation of the model, in the case of independent estimation of the accounts and market impact costs, although a higher utility might be obtained (model of the second step), the obtained utility and output cannot be reliable due to the unreal calculation and underestimation of the market impact costs.

5.2 Suggestions for Future Work

Despite studies in this field, many gaps are still observed. For example, it is still possible to develop and expand the results based on meta-heuristic methods that can increase the number of the accounts and overcome computational complexities. Also, the use of other risk modeling indices can increase the accuracy and flexibility of the model. In addition to the above-mentioned points, using different approaches for modelling the implicit and non-implicit transaction cost simultaneously is one of the interesting and attractive subjects for developing the present study. Finally, in order for using the proposed model in real conditions, it is suggested to adopt approaches that can model the uncertainty conditions in the financial markets.

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