A Preference Degree for Ranking Intuitionistic Fuzzy Numbers

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ABSTRACT
In spite numerous researches on ranking method based on intuitionistic fuzzy numbers, there has not been any study on preference degree based on intuitionistic fuzzy numbers. This paper extends a preference criterion for ranking intuitionistic fuzzy numbers inspire by a well known method of ranking fuzzy numbers. The main properties of the extended preference degree will be also studied into the space of intuitionistic fuzzy numbers. In addition, the feasibility and effectiveness of the proposed ranking method is examined via an applied example related to the multi-criteria group decision-making based on intuitionistic fuzzy linguistic terms. The proposed method also compared with some common methods of ranking intuitionistic fuzzy numbers. Through specific theoretical and numerical results, it is shown that the proposed preference criterion provide us with a useful and valuable way to handle intuitionistic fuzzy numbers in many practical applications of decision making such as multiple attributes group decision-making based on linguistic variables.

Keywords:
Preference degree; reciprocity; transitivity; deviation degree; aggregation operator; multi-criteria decision making.

1. Introduction

The problem of ordering fuzzy numbers plays very important roles in linguistic decision making and some other fuzzy application systems such as decision-making, data analysis, artificial intelligence, socioeconomic systems, statistical procedures, and etc. Because of the nature of uncertainties, many different strategies have been proposed for ranking uncertain quantities including fuzzy sets [6,8,14,29,30,33,36,37], Intuitionistic Fuzzy quantities [18,20,21,25,28,31, 34], Hesitant Fuzzy Sets [14], Hesitant Fuzzy Linguistic Term Sets [5, 18, 39]. These methods rely on coefficient of variation, distance measure, centroid point and original point, and weighted mean value, preference degree, and so on. Contrariwise, during the last decades, intuitionistic fuzzy numbers have been largely focused upon for their wide real applications in the real world. The (Atanassov's) intuitionistic fuzzy sets can present the degrees of membership and non-membership with a degree of indeterminacy, with the knowledge and semantic representation becoming more meaningful and applicable [2,3,4]. These generalizations have been extensively studied and applied in a variety of areas such as logic programming, decision making problems, medical diagnostics, etc (for instance, see [7,13,23,32,33,35,38]).

Essentially, there exist two approaches to construct a method for ranking uncertain data. The first approach is based on real-valued criteria and another is based on degrees of preference. Nayagam et al. [27] introduced a complete ordering of intuitionistic fuzzy numbers using upper lower dense sequence. Darehmiraki [11] suggested a parametric ranking method for intuitionistic fuzzy numbers based on the concept α-cuts and β-cuts of intuitionistic fuzzy numbers. Ali et al. [1] introduced a graphical ranking method based on the uncertainty index and entropy. Saikia [29] presented a method of ranking trapezoidal intuitionistic fuzzy numbers based on the concept of value and ambiguity at different levels of decision-making. Feng et al. [14] introduced a number of lexicographic orders of intuitionistic fuzzy numbers by means of several measures such as the membership, non-membership, score, accuracy and expectation score functions. Das and Guha [10] and Prakash et al. [28] proposed a ranking method of intuitionistic fuzzy numbers is developed by utilizing the concept of centroid point. Jeevaraj and P. Dhanasekaran [22] introduced a linear total ordering of trapezoidal intuitionistic fuzzy numbers using axiomatic set of eight different scores. Shakouri et al. [30] proposed a parametric method to rank generalized intuitionistic fuzzy numbers such that the decision maker's opinion about the decision level and hesitation degree parameters can affect the obtained ranking results. Faizi et al. [15] suggested a multi-criteria group decision making method by combining the e Characteristic object method for triangular intuitionistic fuzzy numbers. Huang et al. [21] introduced a complete ranking method for interval-valued intuitionistic fuzzy numbers by using a score function and three types of entropy functions. Chutia et al. [9] proposed a ranking criterion for ranking generalized triangular intuitionistic fuzzy numbers by adopting the value and ambiguity at level of decision-making. Garg et al. [17] proposed a ranking criterion for intuitionistic fuzzy sets by combining a possibility measure and some operational laws and aggregation operators. Hao et al. [19] suggested a trapezoidal intuitionistic fuzzy induced ordered weighted arithmetic averaging operator to solve multiple attribute decision-making problems with attribute values for trapezoidal intuitionistic fuzzy numbers. Velu et al. [34] developed a total ordering on the class of trapezoidal intuitionistic random variable using eight different score functions, namely, imprecise score, non-vague score, incomplete score, accuracy score, spread score, non-accuracy score, left area score, and right area score.

Unlike the previous methods, this study introduced a degree of preference for intuitionistic fuzzy numbers by extending a well-known degree of preference used to rank fuzzy numbers introduced by Yuan [40]. This criterion measures the degree of which one intuitionistic fuzzy number is greater than the other. Therefore, the proposed method suggested a novel criterion to rank intuitionistic fuzzy numbers. The main properties of the proposed preference degree including robustness, reciprocity and transitivity are then put into investigation. For practical reasons, we will illustrate the proposed ranking method using an applied example related to multi-criteria decision making.

This paper is classified as follows: Section 2 reviews some concepts about fuzzy numbers and intuitionistic fuzzy numbers. Section 3 presents a preference criterion for ranking intuitionistic fuzzy numbers. Some basic properties of the proposed ranking method are also studied. In Section 4 an applied example is employed to show the possible application of the proposed ranking method of intuitionistic fuzzy numbers. Concluding remarks are finally made in another section.

2 Intuitionistic fuzzy numbers

This section briefly reviews several concepts and terminology related to fuzzy numbers and intuitionistic fuzzy numbers used throughout the paper.

A fuzzy set $\tilde{A}$ of $\mathbb{X}$ (the universal set) is defined by its membership function $\tilde{A}: \mathbb{X} \to [0,1]$. The set $\tilde{A}[\alpha] := \{x \in \mathbb{X}: \tilde{A}(x) \geq \alpha\}$ is called the α-cut of the fuzzy set $\tilde{A}$, for each $\alpha \in (0,1]$ [24]. The set $\tilde{A}[0]$ is also defined equal to the closure of the set $\{x \in \mathbb{X}: \tilde{A}(x) > 0\}$. A fuzzy set $\tilde{A}$ of $\mathbb{R}$ (the real line) is called a fuzzy number if it is normal, i.e. there exists a unique $x_{\tilde{A}}^* \in \mathbb{R}$ with $\tilde{A}(x_{\tilde{A}}^*) = 1$, and for every $\alpha \in [0,1]$, the set $\tilde{A}[\alpha]$ is a non-empty compact interval in $\mathbb{R}$. This interval will be denoted by $\tilde{A}[\alpha] = [\tilde{A}_\alpha^L, \tilde{A}_\alpha^U]$, where $\tilde{A}_\alpha^L = \inf\{x: x \in \tilde{A}[\alpha]\}$ and $\tilde{A}_\alpha^U = \sup\{x: x \in \tilde{A}[\alpha]\}$. A fuzzy number $\tilde{A}$ is called a LR-fuzzy number if there exist real numbers a, l_a and r_a with $l_a, r_a \geq 0$, and strictly decreasing and continuous functions $L, R: [0,1] \to [0,1]$ such that

$$\tilde{A}(x) = \begin{cases} L(\frac{a-x}{l_a}) & a - l_a \leq x \leq a, \\ R(\frac{x-a}{r_a}) & a < x \leq a + r_a, \\ 0 & x \in R - [a - l_a, a + r_a]. \end{cases}$$

In this case $\tilde{A}$ is denoted simply by $(a; r_a, r_a)_{LR}$. The most common used LR-fuzzy numbers in many real applications are the so-called triangular fuzzy numbers in which the shape functions L and R are given by $L(x) = R(x) = 1 - x$, for all $x \in [0,1]$. The membership function of triangular fuzzy number, denoted by $\tilde{A} = (a; r_a, r_a)_T$, is given by

$$\tilde{A}(x) = \begin{cases} \frac{x - a + l_a}{l_a} & a - l_a \leq x \leq a, \\ \frac{a + r_a - x}{r_a} & a \leq x \leq a + r_a, \\ 0 & x \in \mathbb{R} - [a - l_a, a + r_a]. \end{cases}$$

Remark 1 *For a given $\tilde{A} \in \mathcal{F}(\mathbb{R})$, assume $\tilde{A}_\alpha$ is defined for each $\alpha \in [0,1]$ by:*

$$\tilde{A}_\alpha = \begin{cases} \tilde{A}_{2\alpha}^L & \alpha \in [0,0.5], \\ \tilde{A}_{2(1-\alpha)}^U & \alpha \in (0.5,1]. \end{cases} \qquad (1)$$

Then, the α-cuts of a fuzzy number $\tilde{A} \in \mathcal{F}(\mathbb{R})$ is equivalent to $\tilde{A}[\alpha] = [\tilde{A}_{\alpha/2}, \tilde{A}_{1-\alpha/2}]$, $\alpha \in [0,1]$.

Example 1 For a given LR-fuzzy number $\tilde{A} = (a; l_a, r_a)_{LR}$, it is easily verified that

$$\tilde{A}_\alpha = \begin{cases} a - l_a L^{-1}(2\alpha - 1) & for \quad 0.0 \leq \alpha \leq 0.50, \\ a + r_a R^{-1}(2(1-\alpha)) & for \quad 0.50 \leq \alpha \leq 1.0. \end{cases}$$

Specially, if $\tilde{A} = (a; l, r)_T$ is a triangular fuzzy number, then

$$\tilde{A}_\alpha = \begin{cases} a + (2\alpha - 1) l_a & for \quad 0.0 \leq \alpha \leq 0.50, \\ a + (2\alpha - 1) r_a & for \quad 0.50 \leq \alpha \leq 1.0. \end{cases}$$

In the sequel, we shall review the basic definitions and terminology of the intuitionistic fuzzy sets (IFSs) (for further details, see [2]). An (Atanassov's) intuitionistic fuzzy set on the universal set $\mathbb{X}$ is defined by a set of ordered triples:

$A = \{\langle x, \tilde{A}^\mu(x), \tilde{A}^\nu(x): x \in \mathbb{X}\rangle\}$,

where, $\tilde{A}^\mu(x), \tilde{A}^\nu(x): \mathbb{X} \to [0,1]$ are the degrees of membership and nonmembership, respectively, and $0 \leq \tilde{A}^\mu(x) + \tilde{A}^\nu(x) \leq 1$ for all $x \in \mathbb{X}$. An intuitionistic fuzzy set $A = \{\langle x, \tilde{A}^\mu(x), \tilde{A}^\nu(x): x \in \mathbb{R}\rangle\}$ is called an intuitionistic fuzzy number (IFN) if and only if $\tilde{A}^\mu(x)$ and $1 - \tilde{A}^\nu(x)$ are the fuzzy numbers. We briefly denote a intuitionistic fuzzy number $\tilde{A}$ by $A = (\tilde{A}^\mu, \tilde{A}^{1-\nu})$ through the paper. The set of all intuitionistic fuzzy number of $\mathbb{R}$ is denoted by $\mathcal{IF}(\mathbb{R})$. The addition of two **IFN**s A and B is denoted by $A \oplus B = (\tilde{A}^\mu \oplus \tilde{B}^\mu, \tilde{A}^{1-\nu} \oplus \tilde{B}^{1-\nu})$. Moreover, the multiplication of a scaler $\lambda \in \mathbb{R} - \{0\}$ and a **IFN** A is given by $\lambda \otimes A = (\lambda \otimes \tilde{A}^\mu, \lambda \otimes \tilde{A}^{1-\nu})$.

The membership function of a LR-intuitionistic fuzzy number (**LRIFN**) is denoted by $A = (a; l_a, r_a; l'_a, r'_a)_{LR}$ where

$$\tilde{A}^{\mu}(x) = \begin{cases} L(\frac{a-x}{l_a}) & a - l_a \le x < a, \\ 1 & x = a, \\ R(\frac{x-a}{r_a}) & a < x < a + r_a, \\ 0 & x \in \mathbb{R} - [a - l_a, a + r_a], \end{cases}$$

and

$$\tilde{A}^{\nu}(x) = \begin{cases} 1 - L'(\frac{a-x}{l'_a}) & a - l'_a \le x < a, \\ 0 & x = a, \\ 1 - R'(\frac{x-a}{r'_a}) & a < x < a + r'_a, \\ 1 & x \in \mathbb{R} - [a - l'_a, a + r'_a], \end{cases}$$

in which $(l'_a > l_a > 0, r'_a > r_a > 0)$ and $L(L')$ and $R(R')$ are continuous and strictly decreasing functions with these properties that $L(0) = R(0) = 1 (L'(0) = R'(0) = 1)$ and $L(1) = R(1) = 0 (L'(1) = R'(1) = 0)$. $\tilde{A}$ is called a triangular intuitionistic fuzzy number (**TIFN**) if and only if $L(x) = R(x) = 1 - x$, for all $0 \le x \le 1$ which is denoted by $A = (a; l_a, r_a; l'_a, r'_a)_T$.

Here, we introduce a distance for **IFN**s. We will apply this distance in multi-criteria decision making based on **IFN**s in next Section.

Definition 1 *Let $A, B \in \mathcal{IF}(\mathbb{R})$. The distance between two A and B is defined as follows:*

$$d^*(A,B) = \int_0^1 \int_0^1 |A_\alpha^\beta - B_\alpha^\beta| d\beta d\alpha,$$

where

$$A_\alpha^\beta = \begin{cases} (1-\beta)\tilde{A}_\alpha^{\mu} + \beta \tilde{A}_\alpha^{1-\nu} & 0.0 \le \alpha \le 0.50, \\ (1-\beta)\tilde{A}_\alpha^{1-\nu} + \beta \tilde{A}_\alpha^{\mu} & 0.50 \le \alpha \le 1.0. \end{cases}$$

It is easy to verify that $d^*: \mathcal{IF}(\mathbb{R}) \times \mathcal{IF}(\mathbb{R}) \to [0, \infty)$ has the following properties.

Lemma 1 *For three **IFN**s A, B and C, d^* has the following properties:*

- $d^*(A,B) = 0$ if and only if $A = B$ (i.e. $\tilde{A}^{\mu} = \tilde{B}^{\mu}$ and $\tilde{A}^{\nu} = \tilde{B}^{\nu}$),
- $d^*(A,B) = d^*(B,A)$,
- $d^*(A,C) \le d^*(A,B) + d^*(B,C)$.

Proof. The assertion 2) is immediately followed. To prove 1), first it is readily seen that $d^*(A,A) = 0$. In reverse, assume that $d_p^*(A,B) = 0$. It concludes that $A_\alpha^\beta = B_\alpha^\beta$ for any $\alpha, \beta \in [0,1]$ that is $\tilde{A}^{\mu} = \tilde{B}^{\mu}$ and $\tilde{A}^{\nu} = \tilde{B}^{\nu}$ or $A = B$. To prove the assertion 3), first note that $d^*(A,B) = E||\mathfrak{A} - \mathfrak{B}||_p$, where $E||\mathfrak{A}|| = (\int_0^1 \int_0^1 |A_\alpha^\beta| d\beta d\alpha)^{1/p}$, $p \ge 1$. Therefore, we get $d^*(A,C) = E||(\mathfrak{A} - \mathfrak{B}) - (\mathfrak{B} - \mathfrak{C})|| \le E||\mathfrak{A} - \mathfrak{B}|| + E||\mathfrak{B} - \mathfrak{C}|| = d^*(A,B) + d^*(B,C)$ by triangular inequality which completes the proof.

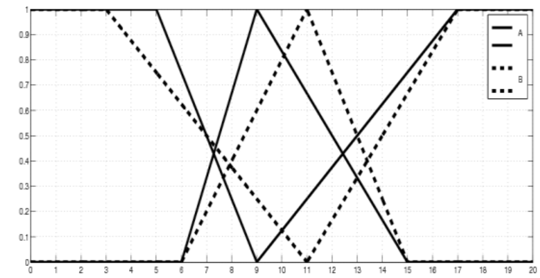

Figure 1: The membership functions of IFNs *A* and *B* in Example 2.

Example 2 *Consider the following two **TIFN**s $A = (9; 3,6; 4,8)_T$ and $B = (11; 5,4; 8,6)_T$ (as drawn in Fig. 1). For any $\beta \in [0,1]$, first note that:*

$$A_\alpha^\beta = \begin{cases} (1-\beta)(9 + 3(2\alpha - 1)) + \beta(9 + 4(2\alpha - 1)) & 0.0 \le \alpha \le 0.50, \\ (1-\beta)(9 + 6(2\alpha - 1)) + \beta(9 + 8(2\alpha - 1)) & 0.50 \le \alpha \le 1.0, \end{cases}$$

$$= \begin{cases} 6\alpha - \beta + 2\alpha\beta + 6 & 0.0 \le \alpha \le 0.50, \\ 12\alpha - 2\beta + 4\alpha\beta + 3 & 0.50 \le \alpha \le 1.0, \end{cases}$$

and

$$B_\alpha^\beta = \begin{cases} (1-\beta)(11 + 5(2\alpha - 1)) + \beta(11 + 8(2\alpha - 1)) & 0.0 \le \alpha \le 0.50, \\ (1-\beta)(11 + 4(2\alpha - 1)) + \beta(11 + 6(2\alpha - 1)) & 0.50 \le \alpha \le 1.0, \end{cases}$$

$$= \begin{cases} 10\alpha - 3\beta + 6\alpha\beta + 6 & 0.0 \le \alpha \le 0.50, \\ 8\alpha - 2\beta - 4\alpha\beta + 7 & 0.50 \le \alpha \le 1.0. \end{cases}$$

From Eq. (2), the distance between A and B can be then evaluated as follows:

$$D(A,B) = \int_0^1 \int_0^1 \left| A_\alpha^\beta - B_\alpha^\beta \right| d\beta d\alpha =$$

$$\int_0^{0.5} \int_0^1 |(6\alpha - \beta + 2\alpha\beta + 6) - (10\alpha - 3\beta + 6\alpha\beta + 6)| d\beta d\alpha +$$

$$\int_{0.5}^1 \int_0^1 |(12\alpha - 2\beta + 4\alpha\beta + 3) - (8\alpha - 2\beta - 4\alpha\beta + 7)| d\beta d\alpha =$$

$\int_0^{0.5}\int_0^1 |-4\alpha+2\beta-4\alpha\beta|d\beta d\alpha+\int_{0.5}^1\int_0^1 |4\alpha+4\beta+8\alpha\beta-4|d\beta d\alpha=0.9432.$

The above double integration can be evaluated using Mathematical software.

Extending a preference degree for ranking IFNs

In this section, we extend an criterion for comparing **IFN**s inspired by Yuan [40]. It should be noted that, for two fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the Yuan preference degree is defined as follows:

$$P_D(\tilde{A},\tilde{B})=\frac{\Delta_{\tilde{A}\tilde{B}}}{\Delta_{\tilde{A}\tilde{B}}+\Delta_{\tilde{B}\tilde{A}}},$$

where

$$\Delta_{\tilde{A}\tilde{B}}=\int_0^1\int_{\{\beta:\tilde{A}_\alpha>\tilde{B}_{1-\alpha}\}}\left(\tilde{A}_\alpha-\tilde{B}_{1-\alpha}\right)d\alpha,$$

and

$$\Delta_{\tilde{B}\tilde{A}}=\int_0^1\int_{\{\beta:\tilde{B}_\alpha>\tilde{A}_{1-\alpha}\}}\left(\tilde{B}_\alpha-\tilde{A}_{1-\alpha}\right)d\alpha.$$

Here, we extend such an idea for comparing two **IFN**s. Then, we will study its useful properties of proposed method on the space of **IFN**s.

Definition 2 *For two **IFN**s A and B, the preference index $P_D:\mathcal{IF}(\mathbb{R})\times\mathcal{IF}(\mathbb{R})\to[0,1]$ is defined by*

$$P_D(A,B)=\frac{\Delta_{AB}}{\Delta_{AB}+\Delta_{BA}},$$

where

$$\Delta_{AB}=\int_0^1\int_{\left\{\beta:A_\beta^\alpha>B_{1-\beta}^{1-\alpha}\right\}}\left(A_\beta^\alpha-B_{1-\beta}^{1-\alpha}\right)d\beta d\alpha,$$

and

$$\Delta_{BA}=\int_0^1\int_{\left\{\beta:B_\beta^\alpha>A_{1-\beta}^{1-\alpha}\right\}}\left(B_\beta^\alpha-A_{1-\beta}^{1-\alpha}\right)d\beta d\alpha.$$

Example 3 *Here, we present an example to demonstrate the preference degree between two **IFN**s introduced in Example 2. For any **IFN**s **TIFN**s $A=(a,l_a,r_a;l'_a,r'_a)_T$ and $B=(b,l_b,r_b;l'_b,r'_b)_T$, first note that:*

$$A_\alpha^\beta=\begin{cases}(1-\beta)(a+l'_a(2\alpha-1))+\beta(a+l_a(2\alpha-1)) & 0.0\le\alpha\le0.50,\\ (1-\beta)(a+r_a(2\alpha-1))+\beta(a+r'_a(2\alpha-1)) & 0.50\le\alpha\le1.0,\end{cases}$$

and

$$B_{1-\alpha}^{1-\beta}=\begin{cases}(1-\beta)(b+(1-2\alpha)r'_b)+\beta(b+(1-2\alpha)r_b) & 0.0\le\alpha\le0.50,\\ (1-\beta)(b+(1-2\alpha)l'_b)+\beta(b+(1-2\alpha)l_b) & 0.50\le\alpha\le1.0.\end{cases}$$

Now, considering $A=(9;3,6;4,8)_T$ *and* $B=(11;5,4;8,6)_T$, *we get*

$$A_\alpha^\beta=\begin{cases}6\alpha-\beta+2\alpha\beta+6 & 0.0\le\alpha\le0.50,\\ 12\alpha-2\beta+4\alpha\beta+3 & 0.50\le\alpha\le1.0,\end{cases}$$

and

$$B_{1-\alpha}^{1-\beta}=\begin{cases}-12\alpha-2\beta-4\alpha\beta+13 & 0.0\le\alpha\le0.50,\\ -16\alpha-3\beta+6\alpha\beta+19 & 0.50\le\alpha\le1.0.\end{cases}$$

Next, we need to evaluate both Δ_{AB} and Δ_{BA} introduced in Definition 2. Calculations show that

$$\Delta_{AB}=\int_0^1\int_{\left\{\beta:A_\beta^\alpha>B_{1-\beta}^{1-\alpha}\right\}}\left(A_\beta^\alpha-B_{1-\beta}^{1-\alpha}\right)d\beta d\alpha=$$

$$=\int_0^{0.5}\int_{\{\beta:(6\alpha-\beta+2\alpha\beta+6)>(-12\alpha-2\beta-4\alpha\beta+13)\}}\big((6\alpha-\beta+2\alpha\beta+6)-(-12\alpha-2\beta-4\alpha+13)\big)d\beta d\alpha$$

$$+\int_{0.5}^1\int_{\{\beta:(12\alpha-2\beta+4\alpha\beta+3)>(-16\alpha-3\beta+6\alpha\beta+19)\}}\big((12\alpha-2\beta+4\alpha\beta+3)-(-16\alpha-3\beta+6\alpha\beta+19)\big)d\beta d\alpha$$

$$=\int_0^{0.5}\int_{\{\beta:\beta>(7-18\alpha/_{1+6\alpha})\}}(18\alpha+\beta+6\alpha\beta-7)d\beta d\alpha+ \tag{3}$$

$=\int_{0.5}^1\int_{\{\beta:\beta>(16-28\alpha/_{1-2\alpha})\}}(28\alpha+\beta-2\alpha\beta-16)d\beta d\alpha=2.5402.$

Similarly , we have

$$B_\alpha^\beta=\begin{cases}10\alpha-3\beta+6\alpha\beta+6 & 0.0\le\alpha\le0.50,\\ 8\alpha-2\beta-4\alpha\beta+7 & 0.50\le\alpha\le1.0,\end{cases}$$

and

$$A_{1-\alpha}^{1-\beta}=\begin{cases}-16\alpha-2\beta+4\alpha\beta+21 & 0.0\le\alpha\le0.50,\\ -8\alpha-\beta+2\alpha\beta+11 & 0.50\le\alpha\le1.0.\end{cases}$$

This concludes that

$$\Delta_{BA}=\int_0^1\int_{\left\{\beta:B_\beta^\alpha>A_{1-\beta}^{1-\alpha}\right\}}\left(B_\beta^\alpha-A_{1-\beta}^{1-\alpha}\right)d\beta d\alpha=$$

$$\int_0^{0.5} \int_{\{\beta:\beta>({15-2\alpha}/_{1-2\alpha})\}} (26\alpha-\beta+2\alpha\beta-15)d\beta d\alpha$$
$$+\int_{0.5}^{1} \int_{\{\beta:\beta<({4-16\alpha}/_{1+6\alpha})\}} (16\alpha-\beta-6\alpha\beta-4)d\beta d\alpha$$
$$= 3.002.$$

We employ the Mathematica software to compute the above double integrals. Therefore, $\Delta_{AB}=2.5402$ and $\Delta_{BA}=3.002$. From Eq. (3), the preference degree that "A is preferred to B" can be then evaluated as $P_D(A,B)=\frac{2.4502}{2.4502+3.2002}=0.4336$.

Definition 3 *For two **IFN**s A and B, A is said to be preferred to B, denoted by $A \succ_{P_D} B$, if $P_D(A,B) \geq 0.5$.*

Lemma 2 *Let A, B, C, and D be four **IFN**s. Then*

- P_D is reciprocal, i.e. $P_D(A,B)=1-P_D(B,A)$.
- P_D is transitive, i.e. $A \succ_{P_D} B$ and $B \succ_{P_D} C$ imply $A \succ_{P_D} C$.
- $P_D(A \succ B)=1$ if and only if $\tilde{B}_1^{1-\nu} \leq \tilde{A}_0^{1-\nu}$.
- $A \succ_{P_D} B$ if and only if $A \oplus C \succ_{P_D} B \oplus C$, for any **IFN** C.
- If $A \succ_{P_D} B$ and $C \succ_{P_D} D$ then $A \oplus C \succ_{P_D} B \oplus D$.

Proof. Let A, B, C and D be four **IFN**s. From Eq. (3), it is easily seen that $1-P_D(B,A)=1-\frac{\Delta_{BA}}{\Delta_{BA}+\Delta_{AB}}=P_D(A,B)$. Therefore the assertion 1) is immediately verified. To proof the assertion 2), first note that,

$$\int_0^1 \int_{\{\beta:A_\beta^\alpha>B_{1-\beta}^{1-\alpha}\}} (A_\beta^\alpha - B_{1-\beta}^{1-\alpha})d\beta d\alpha = \int_0^1 \int_{\{\beta:A_{1-\beta}^\alpha>B_\beta^{1-\alpha}\}} (A_{1-\beta}^\alpha - B_\beta^{1-\alpha})d\beta d\alpha,$$

where

$$\int_0^1 \int_{\{\beta:A_\beta^\alpha>B_{1-\beta}^{1-\alpha}\}} (A_\beta^\alpha - B_{1-\beta}^{1-\alpha})d\beta d\alpha = \int_0^1 \int_{\{\beta:A_{1-\beta}^{1-\alpha}>B_\beta^\alpha\}} (A_{1-\beta}^{1-\alpha} - B_\beta^\alpha)d\beta d\alpha,$$

and

$$\int_0^1 \int_{\{\beta:A_{1-\beta}^\alpha>B_\beta^{1-\alpha}\}} (A_{1-\beta}^\alpha - B_\beta^{1-\alpha})d\beta d\alpha = \int_0^1 \int_{\{\beta:A_\beta^{1-\alpha}>$$

Therefore, we get $\Delta_{AB}=\frac{\delta_{AB}^1+\delta_{AB}^2}{4}$ in which

$$\delta_{AB}^1 = \frac{\int_0^1 \int_{\{\beta:A_\beta^\alpha>B_{1-\beta}^{1-\alpha}\}} (A_\beta^\alpha - B_{1-\beta}^{1-\alpha})d\beta d\alpha + \int_0^1 \int_{\{\beta:A_\beta^{1-\alpha}>B_{1-\beta}^\alpha\}} (A_\beta^{1-\alpha} - B_{1-\beta}^\alpha)d\beta d\alpha}{2},$$

and

$$\delta_{AB}^2 = \frac{\int_0^1 \int_{\{\beta:A_{1-\beta}^\alpha>B_\beta^{1-\alpha}\}} (A_{1-\beta}^\alpha - B_\beta^{1-\alpha})d\beta d\alpha + \int_0^1 \int_{\{\beta:A_{1-\beta}^{1-\alpha}>B_\beta^\alpha\}} (A_{1-\beta}^{1-\alpha} - B_\beta^\alpha)d\beta d\alpha}{2}.$$

Now, note that $P_D(A,B) \geq 0.5$ if and only if $\Delta_{AB}-\Delta_{BA} \geq 0$ which is equivalent to $\int_0^1 \int_0^1 A_\alpha^\beta d\beta d\alpha \geq \int_0^1 \int_0^1 B_\alpha^\beta d\beta d\alpha$. Therefore, if $A \succ_{P_D} B$ and $B \succ_{P_D} C$, then we get $\int_0^1 \int_0^1 A_\alpha^\beta d\beta d\alpha \geq \int_0^1 \int_0^1 B_\alpha^\beta d\beta d\alpha \geq \int_0^1 \int_0^1 C_\alpha^\beta d\beta d\alpha$ which means that $\int_0^1 \int_0^1 A_\alpha^\beta d\beta d\alpha \geq \int_0^1 \int_0^1 C_\alpha^\beta d\beta d\alpha$ or $A \succ_{P_D} C$. Therefore, the assertion 2) is verified. Now, note that $P_D(A \succ B)=1$ if and only if $\Delta_{BA}=0$ or $A_\alpha^\beta \geq B_{1-\alpha}^{1-\beta}$ for any $\alpha \in [0,1]$ and $\beta \in [0,1]$. which is equivalent to $\tilde{A}_0^{1-\nu} \geq \tilde{B}_1^{1-\nu}$. Therefore, the assertion 3) is verified. To proof the assertion 4), first note that for any **FN**s $\tilde{A}$ and $\tilde{B}$ we have $(\tilde{A} \oplus \tilde{B})_\alpha = \tilde{A}_\alpha + \tilde{B}_\alpha$ for all $\alpha \in [0,1]$ which consequences that $(A \oplus B)_\beta^\alpha = A_\beta^\alpha + B_\beta^\alpha$ for any $\alpha,\beta \in [0,1]$. Therefore, if $A \oplus C \succ_{P_D} B \oplus C$ we get $\int_0^1 \int_0^1 (A \oplus C)_\beta^\alpha d\beta d\alpha \geq \int_0^1 \int_0^1 (B \oplus C)_\beta^\alpha d\beta d\alpha$ or $\int_0^1 \int_0^1 (A_\beta^\alpha + C_\beta^\alpha)d\beta d\alpha \geq \int_0^1 \int_0^1 (B_\beta^\alpha + C_\beta^\alpha)d\beta d\alpha$ if and only if $\int_0^1 \int_0^1 A_\alpha^\beta d\beta d\alpha \geq \int_0^1 \int_0^1 B_\alpha^\beta d\beta d\alpha$ or $A \succ_{P_D} B$. Moreover, if $A \succ_{P_D} B$ and $C \succ_{P_D} D$, then we have

$$\int_0^1 \int_0^1 (A \oplus C)_\beta^\alpha d\beta d\alpha = \int_0^1 \int_0^1 (A_\beta^\alpha + C_\beta^\alpha)d\beta d\alpha$$

$$\geq \int_0^1 \int_0^1 (B_\beta^\alpha + D_\beta^\alpha)d\beta d\alpha = \int_0^1 \int_0^1 (B \oplus D)_\beta^\alpha d\beta d\alpha,$$

which means that $A \oplus C \succ_{P_D} B \oplus D$.

According to Example 3, since $P_D(B,A)=1-P_D(A,B)=1-0.4336=0.5664$, it can be concluded that $B \succ_{P_D} A$.

Remark 2. A set of **IFN**s $\{A_1,A_2,\ldots,A_n\}$ are then ranked by the following algorithm from the above lemma:

• **Step 1.** Construct a preference matrix $P_D = [P_D(A_i, A_j)]_{ij}$ for $i = 1, \dots, n$, $j = 1, \dots, n$. From Lemma 2, note that we only need to calculate $n \times (n-1)/2$ preference values.

• **Step 2.** Sort $\{A_1, A_2, \dots, A_n\}$ into $\{A_{k_1}, A_{k_2}, \dots, A_{k_n}\}$ so that for any $i < j$, $A_i \succ_{P_D} A_j$. It is point out that the feasibility of the sorting is guaranteed by Lemma 2. Based on the sorting, therefore, A_{k_1} is the most preferred choice, A_{k_2} is the second, etc.

Table 1: Linguistic terms and their corresponding TIFNs in application example.

Linguistic terms	**TIFN**s
Very low (VL)	$(0.00; 0.00, 0.05; 0.00, 0.10)_T$
Low (L)	$(0.10; 0.05, 0.10; 0.03, 0.20)_T$
Medium low (ML)	$(0.30; 0.10, 0.20; 0.20, 0.20)_T$
Medium (M)	$(0.5; 0.10, 0.10; 0.20, 0.20)_T$
Medium high (MH)	$(0.70; 0.10, 0.10; 0.20, 0.20)_T$
High (H)	$(0.90; 0.10, 0.05; 0.20, 0.10)_T$
Very high (VH)	$(1.00; 0.05, 0.00; 0.10, 0.00)_T$

Table 2: Evaluation TIFNs results provided by decision makers in application example.

Attribute	**Resources**	**Politics and Policy**	**Economy**	**Infrastructure**
A_1	A_{11} =VH	A_{12} =M	A_{13} =H	A_{14} =MH
A_2	A_{21} =H	A_{22} =VH	A_{23} =ML	A_{24} =M
A_3	A_{31} =H	A_{32} =H	A_{33} =MH	A_{34} =H
A_4	A_{41} =VH	A_{42} =H	A_{43} =H	A_{44} =M
A_5	A_{51} =VH	A_{52} =H	A_{53} =H	A_{54} =MH
A_6	A_{61} =H	A_{62} =M	A_{63} =H	A_{64} =VH
A_7	A_{71} =VH	A_{72} =VH	A_{73} =ML	A_{74} =MH

Application example

Here, the possible application of the proposed ranking method is examined in a multi-criteria decision making situation based on **IFN**s. We employ the data set introduced by Hu et al. [20] for **IFN**s linguistic terms instead of interval type-II fuzzy number ones. The overseas investment department (in China) has decided to find a selected pool of alternatives from seven foreign countries A_1, A_2, A_3, A_4, A_5, A_6 and A_7 all over the world based on preliminary surveys. During the assessment, four factors including “Resources”, “Politics and Policy”, “Economy”, and “Infrastructure” are considered with respect to the previous investment experiences of the department. The experts and executive managers use linguistic terms to evaluate the criteria values showed in Table 1. Based on the some criteria such as surveys on the countries, knowledge, and experience, the experts and managers made a final decision information based on the given linguistic terms as shown in Table 2. Assume the experts were not certain about the weights, so they gave the ranges of weights under each criterion instead of exact weight coefficients, and the ranges of weights were provided as $w = (w_1, w_2, w_3, w_4)$ where $0.25 \le w_1 \le 0.4$, $0.3 \le w_2 \le 0.4$, $0.2 \le w_3 \le 0.3$ and $0.2 \le w_4 \le 0.35$ in which $\sum_{j=1}^{4} w_j = 1$. Let $S(w) = \sum_{j=1}^{4}\sum_{i=1}^{7}\sum_{k=1}^{7} w_j D(A_{ij}, A_{kj})$ be the sum of deviation degree of all alternatives where $D(A, B)$ denotes the distance between A and B introduced in Definition 2. Then, the optimal weights are obtained based on the maximizing deviation method, that is:

$$S(w) = 3w_1 + 10w_2 + 13w_3 + 9w_4,$$

Subject to:

$$\begin{cases} 0.25 \le w_1 \le 0.4, \\ 0.3 \le w_2 \le 0.4, \\ 0.2 \le w_3 \le 0.3, \\ 0.2 \le w_4 \le 0.35. \end{cases}$$

Solving the above linear optimization problem leads to $\hat{w} = (0.3, 0.3, 0.2, 0.2)$. For this purpose, the ``Maximize" command prompt was employed to optimize the target functions through the use of Mathematica Software. This software is capable of providing a fast algorithm for such non-linear optimization problems.

Table 3: Preference degrees between IFNs $\boldsymbol{TIFN-WAA(A_1)-TIFN-WAA(A_7)}$.

Preference degrees	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
$P_D(TIFN-WAA(A_1), TIFN-WAA(A_j))$	0.50	0.68	0.18	0.19	0.08	0.38	0.35
$P_D(TIFN-WAA(A_2), TIFN-WAA(A_j))$	0.32	0.50	0.08	0.09	0.03	0.23	0.19
$P_D(TIFN-WAA(A_3), TIFN-WAA(A_j))$	0.82	0.92	0.50	0.54	0.36	0.73	0.74
$P_D(TIFN-WAA(A_4), TIFN-WAA(A_j))$	0.81	0.91	0.46	0.50	0.73	0.82	0.83
$P_D(TIFN-WAA(A_5), TIFN-WAA(A_j))$	0.92	0.97	0.64	0.27	0.50	0.86	0.87
$P_D(TIFN-WAA(A_6), TIFN-WAA(A_j))$	0.62	0.77	0.17	0.18	0.14	0.500	0.48
$P_D(TIFN-WAA(A_7), TIFN-WAA(A_j))$	0.65	0.81	0.26	0.17	0.13	0.52	0.50

In this example, we try to find the four best alternatives. For this purpose, similar to that of Hu et al.'s method, the following multi-criteria decision making procedure can be applied as listed below:

• **Step 1.** Calculate the deviation degree under all criteria based on Equation (4) respectively. In this regard, the concrete weight coefficients were obtained as $\widehat{w}_1 = 0.3$, $\widehat{w_2} = 0.3$, $\widehat{w}_3 = 0.2$, $\widehat{w}_4 = 0.2$ (by solving the linear optimization given in Eq. (9)).

• **Step 2.** Aggregate all the criteria value for alternatives based on an aggregation operator (**TIFN-WAA)** given by the following **IFN**s:

$$TIFN-WAA(A_i) = \left(\oplus_{k=1}^{4}\left(\widehat{w}_k \otimes \tilde{A}_{ik}^{\mu}\right), \oplus_{k=1}^{4}\left(\widehat{w}_k \otimes \tilde{A}_{ik}^{1-\nu}\right)\right), i = 1, \ldots, 7$$

where $\oplus$, $\otimes$ are the addition and multiply operations of fuzzy numbers and $\widehat{w} = (\widehat{w}_1, \widehat{w}_2, \widehat{w}_3, \widehat{w}_4)$ was determined in **Step 1**. Therefore, based on the concrete weights of $\widehat{w} = (0.3, 0.3, 0.2, 0.2)$ and linguistic terms in Table 2, the weight coefficients of **TIFN-WAA**s can be calculated as follows:

$TIFN-WAA(A_1)$
$= (0.770; 0.085, 0.060; 0.170, 0.120)_T$,
$TIFN-WAA(A_2)$
$= (0.730; 0.085, 0.075; 0.170, 0.110)_T$,
$TIFN-WAA(A_3)$
$= (0.860; 0.100, 0.060; 0.200, 0.120)_T$,
$TIFN-WAA(A_4)$
$= (0.850; 0.085, 0.045; 0.170, 0.090)_T$,
$TIFN-WAA(A_5)$
$= (0.890; 0.085, 0.045; 0.170, 0.090)_T$,
$TIFN-WAA(A_6)$
$= (0.800; 0.090, 0.055; 0.180, 0.110)_T$,
$TIFN-WAA(A_7)$
$= (0.800; 0.070, 0.060; 0.140, 0.080)_T$.
For instance,

$$TIFN-WAA(A_1) = \left((TIFN-\widetilde{W}AA(A_1))^{\mu}, (TIFN-\widetilde{W}AA(A_1))^{1-\nu}\right),$$

where

$$(TIFN-\widetilde{W}AA(A_1))^{\mu} = \oplus_{k=1}^{4}\left(w_k \otimes \tilde{A}_{1k}^{\mu}\right) = \left(\widehat{w}_1 \otimes \tilde{A}_{11}^{\mu}\right) \oplus \left(\widehat{w}_2 \otimes \tilde{A}_{12}^{\mu}\right) \oplus \left(\widehat{w}_3 \otimes \tilde{A}_{13}^{\mu}\right) \oplus \left(\widehat{w}_4 \otimes \tilde{A}_{14}^{\mu}\right) =$$
$$(0.3 \otimes (1.00; 0.05, 0.00)_T \oplus (0.3 \otimes (0.30; 0.10, 0.20)_T) + (0.2 \otimes (0.90; 0.10, 0.05)_T \oplus (0.2 \otimes (0.70; 0.10, 0.10)_T) = (0.770; 0.085, 0.060)_T,$$

and

$$(TIFN-\widetilde{W}AA(A_1))^{1-\nu} = \oplus_{k=1}^{4}\left(\widehat{w}_k \otimes \tilde{A}_{1k}^{1-\nu}\right) = \left(\widehat{w}_1 \otimes \tilde{A}_{11}^{1-\nu}\right) \oplus \left(\widehat{w}_2 \otimes \tilde{A}_{12}^{1-\nu}\right) \oplus \left(\widehat{w}_3 \otimes \tilde{A}_{13}^{1-\nu}\right) \oplus \left(\widehat{w}_4 \otimes \tilde{A}_{14}^{1-\nu}\right) =$$
$$= (0.3 \otimes (1.00; 0.10, 0.00)_T \oplus (0.3 \otimes (0.5; 0.20, 0.20)_T) + (0.2 \otimes (0.70; 0.20, 0.20)_T \oplus (0.2 \otimes (0.70; 0.20, 0.20)_T) = (0.770; 0.170, 0.120)_T.$$

This concludes that $TIFN-WAA(A_1) = (0.770; 0.085, 0.060; 0.170, 0.120)_T$. The other values of $TIFN-WAA(A_2) - TIFN-WAA(A_7)$ can be calculated in a similar manner.

• **Step 3.** Calculate the preference degree of every two **TIFN-WAA**s using Equation (3). For this purpose, there is need to evaluate:

$$P_D\left(TIFN-WAA(A_{\mathrm{i}}), TIFN-WAA(A_{\mathrm{j}})\right) = \frac{\Delta_{TIFN-WAA(A_{\mathrm{i}})TIFN-WAA(A_{\mathrm{j}})}}{\Delta_{TIFN-WAA(A_{\mathrm{i}})TIFN-WAA(A_{\mathrm{j}})} + \Delta_{TIFN-WAA(A_{\mathrm{j}})TIFN-WAA(A_{\mathrm{i}})}},$$

for all $1 \leq i < j \leq 7$. Applying the calculation procedure illustrated in Example 3, one can produce a matrix of preference degrees (P_D) as shown in Table 3.

• **Step 4.** Rank all **TIFN-WAA**s in a descending order using Remark 2 for preference degrees of matrix P_D and then select the best one(s). In this regard, note that

$P_D\left(TIFN-WAA(A_4),TIFN-WAA(A_{\mathrm{j}})\right) \geq 0.5$, for all $j = 1,2,\ldots,7$,
$P_D\left(TIFN-WAA(A_5),TIFN-WAA(A_{\mathrm{j}})\right) \geq 0.5$, for all $j = 1,2,3,5,6,7$,
$P_D\left(TIFN-WAA(A_3),TIFN-WAA(A_{\mathrm{j}})\right) \geq 0.5$, for all $j = 1,2,6,7$,
$P_D\left(TIFN-WAA(A_7),TIFN-WAA(A_{\mathrm{j}})\right) \geq 0.5$, for all $j = 1,2,6$,
$P_D\left(TIFN-WAA(A_6),TIFN-WAA(A_{\mathrm{j}})\right) \geq 0.5$, for all $j = 1,2$,
$P_D\left(TIFN-WAA(A_1),TIFN-WAA(A_2)\right) \geq 0.5$.

Acoording to the ordering principal in Remark 2, all the alternatives can be then ranked as $A_4 \succ_{P_D} A_5 \succ_{P_D} A_3 \succ_{P_D} A_7 \succ_{P_D} A_6 \succ_{P_D} A_1 \succ_{P_D} A_2$. Thus, A_4, A_5, A_3, and A_7 are the four ordered alternatives which can be considered in the selection pool for subsequent further evaluations. In order to investigate the effectiveness of the proposed method compared to other existing ranking methods, we focus on a well-established ranking criteria used for (unimodal) intuitionistic fuzzy numbers. In this regard, the focuse is on some relevant methods introduced by Ali et al. [1], Darehmiraki [11], Feng et al. [16], Prakash et al. [28], Shakouri et al. [30]. The results of such methods are listed in Table 4. Accordingly, our method gives similar results to that the other methods. However, unlike the other methods, our method provides additional information in ranking procedure by assigning a preference degree to each order as given in Table 3.

Table 4: Ordering results of $TIFN-WAA(A_1)$-$TIFN-WAA(A_7)$ based on some common methods in application example.

Method	Ranking
Ali et al.	$A_4 > A_5 > A_3 > A_7 > A_6 > A_1 > A_2$
Prakash et al.	$A_4 \geq A_5 \geq A_3 \geq A_7 \geq A_6 \geq A_1 \geq A_2$
Feng et al.	$A_4 > A_5 > A_3 > A_7 > A_6 > A_1 > A_2$
Shakouri et al.	$A_4 \geq A_5 \geq A_3 \geq A_7 \sim A_6 \geq A_1 \geq A_2$
Darehmiraki	$A_4 > A_5 \geq A_3 > A_7 \geq A_6 \geq A_1 \geq A_2$

Conclusion

This paper describes an approach for comparing intuitionistic fuzzy numbers based on an extended preference degree and presents an algorithm to rank a set of such data. Some basic properties of the proposed ranking criterion, including transitivity, and reciprocity are investigated. The effectiveness and advantages of the proposed ranking method are described through an applied example revelent to multiple-attributes group decision-making. Therefore, the suggested preference relationships provide us with a useful and valuable way to handle intuitionistic fuzzy numbers in many practical applications of decision-making. The numerical evaluations revealed that the proposed method lead to same ordering results to that of some well-established techniques. However, the main advantage of our method is that it provides a preference degree which allows a decision maker to measure the degree to which an intuitionistic fuzzy number is greater than another one. It should be noted that the proposed ranking method can only be applied for normal intuitionistic fuzzy numbers with normal fuzzy numbers. Extending the proposed ranking criterion between trapezoidal or generalized intutionistic fuzzy numbers is a potential subject for future study.

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