Comparative Comparison of the Efficiency of Hybrid Model of an Agent-based & Recursive Neural Network in Automating Algorithmic Trading Strategies in Global Financial Markets

Mohammad Hossein Poostforoush
Ph.D. Student in Information Technology Management, Department of Management, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran.
Email: m.poustfroush@gmail.com

Amirhassan Monajemi
Senior Lecturer, School of Computing, National University of Singapore, 117417, Singapore.
(Corresponding Author)
Email: amir@comp.nus.edu.sg

Saeed Daei-Karimzadeh
Associate Professor of Economics, Department of Economics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran. Email: saeedkarimzade@yahoo.com

Saeed Samadi
Associate Professor of Economics, Department of Economics, Faculty of Administrative Science and Economics, University of Isfahan, Iran.
Email: s.samadi@ase.ui.ac.ir

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ABSTRACT

The purpose of this study is to investigate and compare the efficiency of using a hybrid model of an agent-based and recursive neural network to automate algorithmic trading strategies in global financial markets and the Tehran Stock Exchange. The model consists of two groups of agents, including traditional agents and intelligent agents. The group of traditional agents is divided into three categories: liquidity providers, liquidity consumers, and noise traders. Historical data was used to predict stock prices in the intelligent agent group, and model simulations were used to generate trading signals and update the limited order book.

To extract the historical data, information from the financial markets of New York, Frankfurt, and Tokyo from 2013 to 2020 AD and the Tehran Stock Exchange from 1392 to 1399 Persian calendar was extracted from the official websites of these markets.

To compare the efficiency of the model, autocorrelation and Hurst exponent tests were performed on the time series of the model price and the time series of the closing price of the historical data of the actual financial markets. The results of the autocorrelation and Hurst exponent analysis of the model and the historical financial market data were compared using the Mann-Whitney test. The results of the Mann-Whitney test show that the model can effectively predict the behavior of the actual financial markets.

Keywords: Algorithmic Trading, Agent-based Modeling, Recurrent Neural Network/

1. Introduction

In this research, an attempt is made to make a comparative comparison of the efficiency of the combined model of recursive neural network and agent-based modeling in automating algorithmic trading strategies in regional and global financial markets and the Tehran Stock Exchange.

Financial markets are immensely complicated dynamic systems that incorporate the interactions of millions of individuals on a daily basis. Market participants vary immensely, both in terms of their trading objectives and in their beliefs on the assets they are trading. All of these participants compete with one another in an attempt to achieve their own personal objectives in the most efficient way possible. Traded assets may also be driven by latent factors, and agents must dynamically incorporate data into their trading decisions (Casgrain & Jaimungal, 2020).

Financial markets, through financial funds, allow funds to move from people who lack productive investment opportunities to people who have such opportunities. Financial markets are critical for producing an efficient allocation of capital (wealth, either financial or physical, that is employed to produce more wealth), which contributes to higher production and efficiency for the overall economy. In direct finance, borrowers borrow funds directly from financial markets by selling securities, and in indirect finance, a financial intermediary borrows funds from lender-savers and then uses these funds to make loans to borrower-spenders (Mishkin, 2007).

With the existence of financial markets, the lenders and borrowers are better off, as the lenders will benefit from the interest rates while the borrowers will increase their investment opportunities. Hence, financial markets play a crucial role in the economy as they are producing an efficient allocation of capital, which contributes to increased production and efficiency for the overall economy. Financial markets that are working efficiently improve the economic welfare of society (Ezzat, 2016).

Algorithmic trading is a computer program that takes and executes automated trading decisions in the stock market. Trading strategies play a vital role in algorithmic trading. The conventional wisdom is that the same trading strategy is not profitable for all stocks all the time. The selection of a trading strategy for the stock at a particular time instant is the major research problem in the stock market trading. An optimal dynamic trading strategy generated from the current pattern of the stock price trend can attempt to solve this problem. Reinforcement Learning can find this optimal dynamic trading strategy by interacting with the actual stock market as its environment. The representation of the state of the environment is crucial for performance (Chakole, Kolhe, Mahapurush, Yadav, & Kurhekar, 2021).

The study of algorithmic trading always starts with studying the optimal execution strategies. Instead of seeking speculative opportunities, optimal execution strategies focus on executing orders with minimized cost. From the perspective of individual investors, their orders would not significantly affect the market price, due to the relatively small volume of either buying or selling positions. However, orders from institutional investors are generally considered as the driving force of the market price, and generally lead to a gap (or in other words, execution cost) between the market price and the really captured price. The size of execution cost directly determines the profitability of an institutional investor (Liu, 2015).

The development of automated systems to perform intra day stock trading has gained a lot of popularity in recent years, as it allows non-expert users to become wealthy in the stock market without actively acting on it. Several machine learning algorithms have been proposed to do such a task, but a deeper study on reward-based classifiers performing a final decision of several reinforcements learning classifiers, according to ad-hoc combinations of decisions for specific markets has not been extensively explored so far. The research results support the hypothesis that this approach can tackle the uncertain and chaotic behavior of different stock markets through a flexible ensemble strategy. The strategy consists of the following components: some agents with different experiences in the market, diverse combinations of actions to be done by the trader, an ensemble strategy, done with different agreement thresholds, and multiple resolution data representing past price's behavior in a complementary fashion. Such properties could help design ad-hoc strategies to different markets and trading periods, with a high potential to be a recommended strategy to some different markets (Carta, Ferreira, Podda, Recupero, & Sanna, 2020).

Agent-based models allow researchers to model their ideas without engaging in mathematical complexities, and to perform experiments and analyses on factors that interact in the environment. Formally, agent-based modeling is a computational method that enables a researcher to create, analyze, and experiment with models composed of agents who interact within an environment (Gilbert, 2008).

By reviewing the previous research, it is clear that in the methods used in these researches, historical data are often used as input of the agent-based model to simulate financial market strategies. The use of historical data to simulate financial market strategies has the limitation that the simulation of the agents' behavior is based on these data, and the agents do not have the opportunity to identify the real behavior of the market and to predict events that may affect market prices at any moment. In addition, the simulation of historical data in two-sided markets does not provide the opportunity to make real bids for the selling price of shares, to offer the selling price based on the long-term trend of price changes, and to fill the selling side of the dynamic order book in the market. The use of intelligent agents trained with recurrent neural networks can help to overcome this limitation in simulating the behavior of agents on financial markets.

In this research, an agent-based modeling structure and the recursive neural network has been used in the design of a hybrid model. The agents are divided into two groups: intelligent agents and traditional agents. The group of intelligent agents is trained using a recurrent neural network and using historical data of time series and predicts the stock price. This price prediction is used by the intelligent agents as input to an agent-based model to compare this price with the prices recorded in the limited order book, and based on this, they place orders in the order book. The group of traditional agents is divided into three categories: Liquidity Consumers, Liquidity Providers, and Noise Traders. Traditional agents simulate the behavior of these three groups on the basis of the defined algorithms and, based on this, place orders in the Limited Order Book. The model has been run and the results reported. The autocorrelation and Hurst exponent tests have been run on the price time series of the model and the closing price time series of selected data from selected financial markets, and the results of these two groups have been compared using the Mann-Whitney U test.

The model is designed and coded using Netlogo software and Python programming tools (TensorFlow).

2. Background and related work

Due to the advances in technology and the rapid growth of high-frequency trading (HFT), advanced financial markets have substantially eliminated human intermediation in the trading process and replaced it with automated electronic limit order books that have allowed the growth of trading algorithms as one of the main investment tools. Some of the trading algorithms generated imitate the behavior of humans in the trading process, and over the last few years, these trading algorithms have considerably improved their speed to match the incidence of bid and ask orders (Manahov, Hudson, & Urquhart, 2019).

Artificial financial markets are models for studying the link between individual investor behavior and financial market dynamics. They are often computational models of financial markets and are usually comprised of a number of heterogeneous and bounded rational agents, which interact through some trading mechanism, while possibly learning and evolving. These models are built for studying agents' behavior, price discovery mechanisms, the influence of market microstructure, or the reproduction of the stylized facts of real-world financial time-series (e.g., fat tails of return distributions and volatility clustering). A similar bottom-up approach has been utilized in agent-based computational economics (ACE) - the computational study of economies modeled as evolving systems of autonomous interacting agents (Tesfatsion, 2006). Since agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad-hoc heuristics (Chan et al., 1999), such simulations are particularly suitable to test and generate various behavioral hypotheses (Lovric, 2011).

According to the different approaches for market modeling, previous studies can be roughly categorized into three types: traditional financial analysis, machine learning (ML) approaches, and deep learning (DL) approaches. In traditional financial analysis, mathematics is wildly adopted to recognize historical time series patterns and make predictions. The common models include the auto-regressive moving average (ARMA) model and the generalized auto-regressive conditional heteroskedasticity (GARCH) model. ARMA model contains auto-regressive (AR) and moving average (MA). Its generalization, AR-Integrated MA (ARIMA), becomes a popular method for time-series analysis in economics. GARCH model is frequently used for asset pricing, risk management, and volatility forecasting. The machine learning approach models the high-frequency limit order book using a support vector machine (SVM) with handcrafted features and shows the effectiveness in real-world data. The predicting direction of stock market prices is done with random forests and shows that the model is robust in predicting the future direction of the stock movement. With the development of deep learning approaches, recurrent neural network (RNN) is specifically designed to extract temporal information from raw sequential data. RNN variations, such as long short-term memory (LSTM) and gated recurrent unit (GRU) networks, have been proposed to mitigate the gradient vanishing problem and achieve state-of-the-art results in a variety of sequential data prediction problems (Li, Zheng, & Zheng, 2019).

Algorithmic trading is the computerized execution of financial instruments following pre-specified rules and guidelines, and it is classified by Kissell (2013) as follows: 1. Arrival price algorithm that optimizes a trading path to balance the trade-off between cost and risk at a user-specified level of risk aversion; 2. Implementation shortfall algorithm, which is similar to the arrival price algorithm, but incorporates real-time adaptation, i.e. the trading path of the implementation shortfall algorithm is updated by real-time data on every intraday trade while that of the arrival price algorithm is determined before trading and does not change during an intraday trade; 3. Black box algorithm that searches for profitable opportunities and makes investment decisions based on market signals (e.g. asset prices and trading volume) (Ha & Zhang, 2020).

Agent-based models are a powerful new paradigm for describing complex socio-economic systems. A timely issue for such models is the empirical estimates of these models. Analysis of available data is ultimately limited, however, with respect to counterfactual questions, such as the response of financial markets to rarely occurring shocks or the effects of alternative market rules and regulations. Answering such questions inherently required models that incorporate causal premises, specifically, assumptions as to how trading behavior is shaped by environmental conditions. Theoretical models can support such inference, and these also represent an important resource from the finance research literature. Trading in markets can be formulated as a game, and game-theoretic equilibrium concepts can be employed to characterize behavior in markets by rational agents. However, modeling algorithmic trading entails accommodating complex information and fine-grained dynamics, which often renders game-theoretic reasoning analytically intractable (Wellman & Wah, 2016).

Agent-based modeling and simulation (ABMS) allow for a better understanding of how an aggregate-level outcome (e.g., product sales) may emerge as a result of the complex nature of the diffusion process. Such understanding will help decision-makers learn why a phenomenon happens and how they can more than manage it. In this ‘ground-up’ approach, researchers will seek to learn the behavior of the entities that constitute the system. Recent advances in computing technology and the availability of granulated data allow for such understandings. For example, in order to stimulate consumers, modelers should be able to describe the behavior of a consumer with regard to the focus of the study. The researcher then builds the agent-based model that explains the system. Using this system, researchers are able to examine various scenarios and improve their forecasts. Researchers should begin with simpler agent decision rules, and as they gain a better understanding of the system, they can build more complex agents who are more adaptive and may change their decision rules over time (Nejad, 2016).

In order to use macroeconomic agent-based models for policy, need to reduce the complexity of the ABM simulation to a less complex, more computationally tractable system. In other words, surrogate models or meta-modeling approaches need to be developed, that allow approximating or ’emulating’ the multi-dimensional nonlinear dynamics of the original system (van der Hoog, 2017).

In the financial forecasting literature, some studies used machine learning (ML) techniques to develop forecasting models, and technical analysis indicators have been used as inputs to these models to discover the hidden patterns and the relationships between them, in sequence, to forecast future price's movements and thus identify the best trading indicators. The study and analysis of the price time series using artificial intelligence (AI) techniques will potentially reduce the ambiguity associated with investment decision-making (Aloud, 2020).

A variety of artificial intelligence methods can be used within agents. The methods include logic programming, neural networks, advanced search techniques, distributed problem solving, and non-monotonic reasoning. Neural Networks Neural networks are particularly well suited for prediction and pattern recognition tasks. In principle, neural networks do essentially the same thing as a statistical regression. In practice, some problems seem to be more naturally addressed using neural networks than statistical methods. These problems include learning in complicated environments with large numbers of potential inputs of unknown quality, learning in situations with high ongoing rates of change, and recognizing patterns in data amidst substantial amounts of noise (North & Macal, 2007).

3. Structure of the model

3.1. Hybrid model structure

In this research, two groups of agents are used to perform model simulations. The first group of agents are the intelligent agents trained by a recurrent neural network.

The second group is the traditional agents, which make decisions based on traditional models and conventional market approaches. This group is divided into three categories: liquidity consumers, liquidity providers, and noise traders.

3.2. Intelligent agents

The first step involves predicting stock prices using a recurrent neural network and historical time series data. The recurrent neural network is utilized to train intelligent agents. Subsequently, the intelligent agents' price prediction is used as simulation input for the agent-based model to generate trading signals (i.e., buy, sell, or hold) and update the limit order book. The recurrent neural network was implemented with two linear and Long Short-Term Memory (LSTM) models.

In the linear model, a linear conversion is performed between the input and output of the model, and the model performs a set of independent predictions in successive time steps in which there is no interaction between the predictions in each time step.

Some studies, such as Nogales et al. (2002) and Jahandari et al. (2018), have used linear models for stock price forecasting.

LSTM is a special type of recurrent neural network that has been developed to solve the problems of these networks. A traditional recurrent neural network (if large enough) should theoretically be able to produce sequences of any complexity, but in practice, It can be seen that this network is incapable of storing information related to past inputs for the long term (Hochreiter, 2001).

This network consists of three gateways that control the data flow within it. These three gates are named forget gate, update gate (also known as the input gate), and output gate.

In addition to weakening the network's ability to model long-term structures, these "forgetfulness" causes these types of networks to be exposed to instability during sequence generation. The biggest feature of LSTM is the ability to learn long-term dependency that is not possible with recursive neural networks. To predict the next time step, it is necessary to update the values of the weights in the network, which requires preserving the information of the initial time steps. A recurrent neural network can only learn a limited number of short-term dependencies, but an LSTM network can learn these long-term dependencies correctly (Graves, 2013).

LSTM models have been used for stock price forecasting in some studies, such as Bao and Rao (2017).

The structure of a standard recurrent neural network compared to an LSTM network is shown in Figure 1.

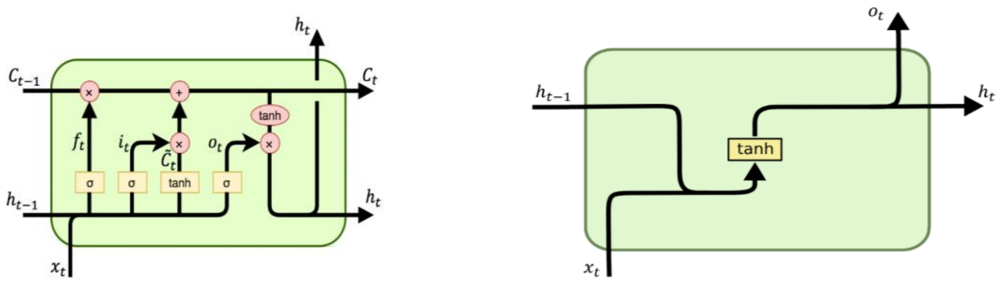

Figure 1- RNN network structure (right) vs. LSTM network structure (left)

3.3. Traditional agents

Traditional agents use the Monte Carlo simulation method to generate a trading signal and update a limited order book. Agents are allowed to place limited orders or market orders and could cancel previously registered orders. These agents are classified into three groups in the market: liquidity consumers, liquidity providers, and noise traders.

3.3.1. Liquidity consumers

This group is a large group of traders who take decisions about trading based on beliefs or stock portfolios that have been Rebalance to their needs. In real financial markets, this group includes investment institutions such as pension funds, banks, insurance companies, and other types of investors. This group creates large orders that want to operate with the least impact on the market and costs. The group buys or sells large stock orders in one day to minimize the impact of price and transaction costs. The probability that this group of agents is buying or selling is determined by equal probability.

3.3.2. Liquidity providers

This group of market participants try to profit from the difference between the buy and sell price through the supply of liquidity on both sides of the bid price and the asking price from the order book and maintain an almost neutral position during the trading day.

3.3.3. Noise traders

This group represents all other strategies on the market and can be viewed as speculative traders. The noise agents randomly decide on whether to buy or sell in each period with equal probability. Once decided, they randomly choose to place a market or limit order or to cancel an existing order (Oesch, 2014).

3.4. Validation of the agent-based model

Sensitivity analysis tests are used to test the validity and reliability of the model. Sensitivity analysis is the study of the influence of output variables from input variables of a statistical model, in other words, a method for changing the inputs of a statistical model in an organized (systematic) way to predict the effects of these changes on the output of the model. That is, how changes in independent variables in a given range can affect dependent variables (Saltelli et al., 2008).

4. Methodology

In this research, an agent-based model was developed in the first step. A sensitivity analysis test was used to test the validity and reliability of the model. Then, the model was run, and the price time series output of the model was stored. The autocorrelation and Hurst analysis tests were run on the price time series of the model and the results were reported. The autocorrelation analysis test was performed to calculate the degree of similarity between the model price time series and a lagged version of itself over successive time intervals. The Hurst exponent test measured the long-term memory of the price time series.

In the second step, information from 25 companies from the New York, Frankfurt, Tokyo, and Tehran stock exchanges were selected by data filtering method, which was extracted from the official websites of these exchanges, has been used. The data selected companies for New York, Frankfurt, and Tokyo is from 2013 to 2020 AD, and the Tehran Stock Exchange is from 1392 to 1399 Persian calendar (from March 2013 to March 2021 AD) that these companies have been most active in the field of buying and selling shares during the mentioned period. The autocorrelation and Hurst analysis tests were

performed on the price time series of these data, and the results were reported.
In the third step, to measure the ability of the model to simulate the actual financial markets, the Mann-Whitney test was used to compare the results of the autocorrelation test and Hurst's exponent for the model and the time series of the aforementioned financial markets.
The sensitivity analysis test, autocorrelation test, and Hurst's exponent test were performed by coding in Python software, and the necessary graphs were extracted. The Mann-Whitney U test was performed using SPSS software.

4.1. Development of the agent-based model

Netlogo software has been used to develop the agent-based model. Two types of agents have been used in this model. Since it is impossible to simulate two worlds simultaneously in the Netlogo software environment, other software features must be used to simulate the intelligent agents. To do this and to train the intelligent agents, Python software has been used. To create a link between these two software environments, the features of the Python extension are used in the Netlogo software environment. This extension makes it possible to call the Python software and to use the code written in the Python software in the Netlogo software environment. The model is coded in the Netlogo software environment using the Python extension and using the TensorFlow tools in the Python software environment. In the user interface of the Netlogo software, the possibility of initial settings and initialization of the model parameters is embedded for the users.

4.2. Intelligent and traditional agents

In the user interface of the Netlogo software, the option of selecting one of these two linear models and LSTM to choose how to train intelligent agents for users is embedded. The training of intelligent agents is performed using historical time series data. This price prediction is used as input to the agent-based model by the intelligent agents to compare this predicted price with the prices recorded in the limited order book and make a decision to place an order in the order book. Traditional agents make decisions and place orders based on the algorithms determined for each of the three categories of liquidity consumers, liquidity providers, and noise traders.

4.3. Initialize free parameters of the model

The initial values of the free parameters of the model are given in the Table 1. Using this set of parameters, the ability of the model to reproduce various statistical features is investigated.
If the free parameters of the model are chosen too far from the default values, much larger jumps and long periods where the price doesn't change at all can result.

Table 1. Standard setting for free parameters

Market parameters	Setting
Initial Price	100
Initial Spread	0.05
Tick Size	0.01
Agent group	Action probability
δ_c	0.10
δ_p	0.15
δ_n	0.55
δ_t	0.20
Liquidity Consumer parameters	Setting
h_{min}	1
h_{max}	100000
Liquidity Provider parameters	Setting
v_{min}	1
v_{max}	200000
v^-	1
ω	50

Market parameters	Setting
Noise Trader parameters	Setting
Order direction	Probability
buy or sell	0.5
Event Type	Probability
submit a market order	$\lambda_m = 0.03$
submit a limit order	$\lambda_l = 0.54$
cancel a limit order	$\lambda_c = 0.43$
Limit Order Type	Probability
crossing limit order	$\lambda_{crs} = 0.0032$
inside-spread limit order	$\lambda_{inspr} = 0.0978$
spread limit order	$\lambda_{spr} = 0.1726$
off-spread limit order	$\lambda_{offspr} = 0.7264$
Order Size Type	Parameters of Log-normal Distribution
market order size	$\mu_{mo} = 7 \quad \sigma_{mo} = 0.1$
limit order size	$\mu_{lo} = 8 \quad \sigma_{lo} = 0.7$
Limit Price Type	Parameters of Power-law Distribution
off-spread relative price	$xmin_{offspr} = 0.05 \quad \beta_{offspr} = 2.72$

4.4. Implementation of sensitivity analysis test

In this model, 20 input parameters and four output parameters are considered. The parameters related to the groups of traders and the range of initialization are given in Table 2.

Table 2. Input parameter ranges for global sensitivity analysis

Parameter	Symbol	Setting
Probability of Liquidity Providers acting	δ_p	[0.05 , 0.95]
Probability of Liquidity Consumers acting	δ_c	[0.05 , 0.95]
Probability of Noise Traders acting	δ_n	[0.05 , 0.95]
Probability of Trained Traders acting	δ_t	[0.05 , 0.95]
Liquidity Providers parameters		
Max order volume	v_{max}	$[10^3 , 10^6]$
Rolling mean period	ω	$[10 , 10^3]$
Liquidity Consumers parameters		[1 , 100]
Max order volume	h_{max}	$[10^3 , 10^6]$
Noise Traders parameters		
Market order probability	λ_m	[0 , 1]
Limit order probability	λ_l	[0 , 1]
Cancel order probability	λ_c	[0 , 1]
Market order size	μ_{mo}	[2 , 10]
Market order size	σ_{mo}	[0 , 1]
Limit order size	μ_{lo}	[2 , 10]
Limit order size	σ_{lo}	[0 , 1]
Off-spread relative price	$xmin_{offspr}$	[0 , 1]
Off-spread relative price	β_{offspr}	[0 , 1]
Crossing limit order	λ_{crs}	[0 , 1]
Inside-spread limit order	λ_{inspr}	[0 , 1]
Spread limit order	λ_{spr}	[0 , 1]
Off-spread limit order	λ_{offspr}	[0 , 1]

The output parameters of the sensitivity analysis are: Hurst's exponent of volatility (H), Median auto-correlations of mid-price returns (R(m)), Mean first lag autocorrelation term of the order-sign series (R(o)) and Concave Price Impact (β) (McGroarty, Booth, Gerding, & Chinthalapati, 2019).

As the model is stochastic (agents' actions are defined over probability distributions), there is inherent uncertainty in the range of outputs, even for fixed input parameters (Andersson & Britton, 2012).

4.5. Autocorrelation test

Autocorrelation is a mathematical representation of the degree of similarity between a given time series and a lagged version of itself over successive time intervals. It measures how the lagged version of the value of a variable is related to the original version of it in a time series.

The value of autocorrelation ranges from -1 to 1. A value between -1 and 0 represents negative autocorrelation. A value between 0 and 1 represents positive autocorrelation.

The analysis of autocorrelation helps to find repeating periodic patterns and gives information about the trend of a set of historical data, which can be used as a tool for technical analysis in the capital markets. A technical analyst can learn how the stock price of a particular day is affected by those of previous days through autocorrelation. Thus, he can estimate how the price will move in the future. If the price of a stock with strong positive autocorrelation has been increasing for several days, the analyst can reasonably estimate the future price will continue to move upward in the recent future days. The autocorrelation analysis only provides information about short-term trends. Therefore, it can only be applied to support trades with short holding periods (Broersen, 2006).

4.6. Hurst exponent test

The Hurst exponent is used as a measure of long-term memory of time series. It relates to the autocorrelations of the time series and the rate at which these decrease as the lag.

A variety of techniques exist for estimating the Hurst exponent. To estimate the Hurst exponent, one must regress to the rescaled range on the time span of observations. To do this, a time series of full length is divided into a number of shorter time series, and the rescaled range is calculated for each of the smaller time series.

The Hurst exponent fluctuates between zero and one. When the Hurst exponent is greater than 0.5, the data is exhibiting a strong long-term trend, and when H is less than 0.5, a trend reversal is more likely.

4.6.1. Trending: If the Hurst value range is between $0.5 < H < 1$ indicates persistence in time series. The higher the value of the Hurst exponent more the trendiness of the market structure. For values close to 1 the series is persistent.

4.6.2. Mean Reverting: If the Hurst value range is between $0 < H < 0.5$ indicate anti-persistence in the time series. The lower the value of the Hurst exponent more the mean-reverting behavior (trend reversal). For values close to 0, the series is anti-persistent.

4.6.3. Geometrical Brownian Motion: It explains the random walk with the unpredictability of the time series. If the Hurst Exponent value is $H = 0.5$, then the time series is expected to move in a random walk (Qian & Rasheed, 2004).

Geometric Brownian Motion is widely used to model stock prices in finance (Shehzad, Anwar, & Razzaq, 2023).

The Hurst exponent can be used in trend trading investment strategies. An investor would be looking for stocks that show strong persistence. These stocks would have an H greater than 0.5. An H less than 0.5 could be paired with technical indicators to spot price reversals (Marton & Cakir, 2022).

4.7. Measurement of the model's ability to simulate factual financial markets

To measure the ability of the model to simulate the factual financial markets, the Mann-Whitney test was used. The Mann-Whitney U Test is a non-parametric statistical test used to compare two samples or groups and assesses whether two sampled groups are likely to derive from the same population, and essentially asks; do these two populations have the same shape of their data? In other words, we want evidence as to whether the groups are drawn from populations with different levels of a variable of interest.

The hypotheses in a Mann-Whitney U Test are: The null hypothesis (H0) is that the two populations are equal. The alternative hypothesis (H1) is that the two populations are not equal (MacFarland & Yates, 2016).

Man-Whitney tests were run to compare the median of the autocorrelation and Hurst exponent data of the model and the autocorrelation and Hurst exponent data for each of the selected stocks in financial markets.

5. Results

5.1. The execution of the model

According to the initialization of the model parameters based on Table 1, the model has been executed with the initial data parameters for 2000 ticks to simulate 2000 days in the financial markets (almost similar to the data on the factual financial markets used in this research) and the price time series output of the model was saved. The global variance sensitivity analysis that is initialized with parameters based on Table 2 has been executed. The output of the sensitivity analysis test is shown in Figure 2.

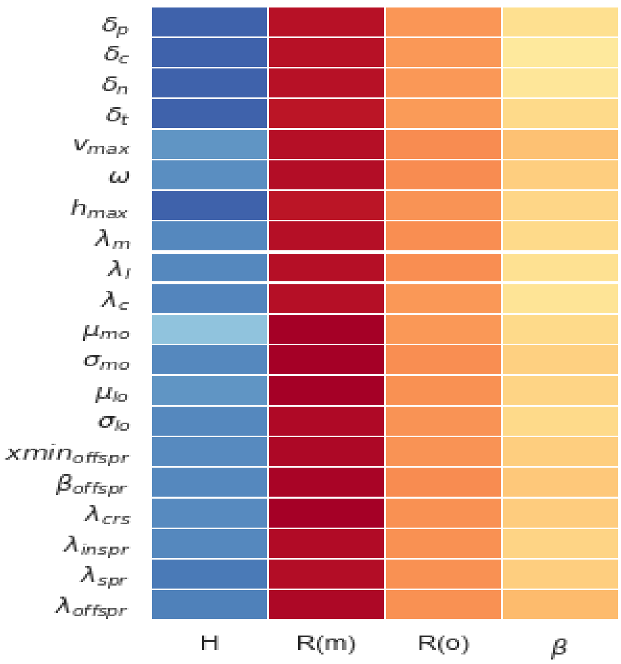

Figure 2. Heatmap of the global variance sensitivity

The global variance sensitivities clearly identify the probabilities of each of the agent groups acting as the most important input parameters for all outputs. This provides the optimal set of parameters. This is consistent with the findings of Booth (2016).

The autocorrelation and Hurst analysis tests were run on the trade price time series of the model. The results are shown in Table 3.

The price series graph of the model is shown in Figure 3.1 and the autocorrelation and Hurst exponent graphs of the price time series of the model are shown in Figures 3.2.

Table 3. Autocorrelation and Hurst exponent statistics for Agent-based model

	Stats	Min.	Q1	Median	Mean	Q3	Max.
Agent-based model	AC trade price	*-0.2462*	*-0.2048*	*0.0283*	*0.1145*	*0.3110*	*0.9929*
	H trade price	*0.2301*	*0.2508*	*0.2811*	*0.2906*	*0.3242*	*0.3814*

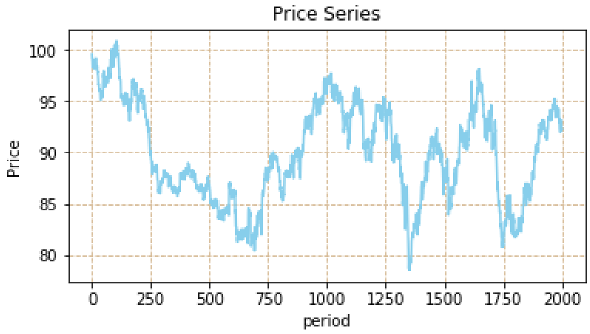

Figure 3.1- Price series graph for model

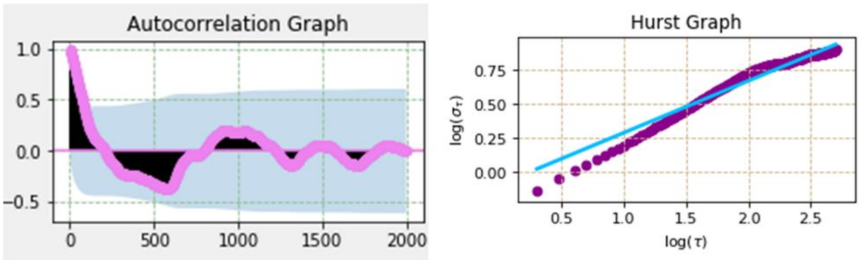

Figure 3.2- Autocorrelation & Hurst exponent graphs for model

5.2. The autocorrelation and Hurst exponent tests for selected samples

The autocorrelation and Hurst exponent tests were run on the closing price time series of selected stocks of New York, Frankfurt, Tokyo, and Tehran.

The autocorrelation test was performed on the closing price time series with 400 lags. Hurst's exponent test was also performed on the closing price time series with 20 lags. The descriptive statistics of the results of these two tests are reported separately in the tables for each stock market (Tables 4 to 7). The corresponding graphs for each market are also shown in Figures 4.1 to 7.2.

As can be seen from the tables and graphs, the markets in most cases in these parameters, despite their differences, follow a similar pattern in the long term which shows similar results to the study of ASEAN stock markets by Sharma and Wongbangpo (2002).

Table 4. Autocorrelation and Hurst exponent statistics (New York stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
New York stock exchange	Advanced Micro Devices Inc.	AMD	AC trade price	0.0309	0.1623	0.3072	0.3798	0.5573	0.9959
			H trade price	0.1831	0.4479	0.4820	0.4459	0.4973	0.5022
	Alibaba Group Holdings Ltd. ADR	BABA	AC trade price	0.0235	0.0951	0.2805	0.3676	0.5907	0.9973
			H trade price	0.1731	0.2753	0.3252	0.3108	0.3516	0.3914
	Alphabet Inc Class A	GOOGL	AC trade price	0.1835	0.3577	0.5044	0.5325	0.6956	0.9967
			H trade price	0.0699	0.1039	0.1432	0.1490	0.1495	0.3839
	Alphabet Inc Class C	GOOG	AC trade price	0.0672	0.2338	0.4151	0.4561	0.6630	0.9963
			H trade price	0.0237	0.0465	0.0969	0.1080	0.1178	0.3837
	Alphabet Inc Class C	AMZN	AC trade price	0.1592	0.3299	0.4867	0.5088	0.6371	0.9975
			H trade price	0.0946	0.1175	0.2335	0.2429	0.3449	0.4762
	AMC Entertainment	AMC	AC trade price	0.0216	0.2531	0.3509	0.4302	0.6227	0.9977
			H trade price	0.1741	0.2251	0.2746	0.3620	0.3677	0.9730

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
	Holdings Inc.								
	Apple Inc.	AAPL	AC trade price	0.1145	0.1731	0.2642	0.3681	0.5057	0.9914
			H trade price	0.2300	0.3276	0.3683	0.3581	0.3971	0.4153
	Bank of America Corp.	BAC	AC trade price	0.0773	0.2830	0.5165	0.5064	0.7154	0.9970
			H trade price	0.1503	0.3071	0.3210	0.3173	0.3391	0.3652
	BlackBerry Ltd.	BB	AC trade price	-0.2974	-0.1616	-0.0641	0.0729	0.2530	0.9922
			H trade price	0.1007	0.2325	0.2913	0.2697	0.3088	0.3330
	Boeing Co.	BA	AC trade price	-0.1101	0.1149	0.4077	0.4148	0.7117	0.9983
			H trade price	0.3860	0.4412	0.5009	0.4751	0.5055	0.5084
	DocuSign Inc.	DOCU	AC trade price	-0.2884	-0.2496	-0.1600	0.0090	0.1304	0.9942
			H trade price	0.1372	0.2092	0.2853	0.2848	0.3508	0.4352
	Facebook Inc.	FB	AC trade price	0.2018	0.2960	0.4710	0.5014	0.6438	0.9970
			H trade price	0.1614	0.1717	0.1934	0.2025	0.2305	0.2697
	Ford Motor Company	F	AC trade price	0.0686	0.3040	0.5004	0.5093	0.7175	0.9977
			H trade price	0.0127	0.0594	0.1111	0.2470	0.2659	0.9390

Table 4. Autocorrelation and Hurst exponent statistics (New York stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
New York stock exchange	General Motors Company	GM	AC trade price	-0.3252	-0.0994	-0.0458	0.0095	-0.0001	0.9893
			H trade price	0.0472	0.0862	0.1487	0.1868	0.2287	0.5873
	Intel Corporation	INTC	AC trade price	0.1394	0.3122	0.4558	0.5058	0.6968	0.9963
			H trade price	0.0068	0.0489	0.0927	0.0957	0.1398	0.2079
	JPMorgan Chase & Co.	JPM	AC trade price	0.1231	0.3499	0.5727	0.5528	0.7474	0.9968
			H trade price	0.1384	0.1553	0.2216	0.3003	0.3144	0.8021
	Micron Technology Inc.	MU	AC trade price	0.0897	0.1198	0.2304	0.3558	0.5655	0.9950
			H trade price	0.1565	0.2861	0.4627	0.3845	0.4723	0.4768
	Microsoft Corporation	MSFT	AC trade price	0.1117	0.2582	0.4440	0.4828	0.6777	0.9973
			H trade price	0.2716	0.5198	0.5793	0.5384	0.6023	0.6181
	Moderna Inc.	MRNA	AC trade price	-0.2876	-0.2237	-0.1514	-0.0010	0.1939	0.9911
			H trade price	0.0145	0.0886	0.1347	0.1346	0.1788	0.2442
	Netflix Inc.	NFLX	AC trade price	0.1712	0.3546	0.5088	0.5325	0.6808	0.9973
			H trade price	0.1355	0.1649	0.2489	0.2497	0.3289	0.3701
	Nio Inc Class A ADR	NIO	AC trade price	-0.1784	-0.1372	-0.0719	-0.0009	-0.0180	0.9873
			H trade price	0.1253	0.1872	0.2355	0.2464	0.3043	0.3945
	NVIDIA Corporation	NVDA	AC trade price	0.2172	0.2623	0.3062	0.4228	0.5338	0.9969
			H trade price	0.0949	0.2061	0.3640	0.3374	0.4540	0.5134
	Square Inc.	SQ	AC trade price	-0.0813	0.0656	0.1613	0.2392	0.3036	0.9934
			H trade price	0.0259	0.1008	0.1818	0.1807	0.2640	0.3368
	Tesla Inc.	TSLA	AC trade price	-0.0157	0.0057	0.0165	0.1453	0.1769	0.9875
			H trade price	0.1097	0.1691	0.2986	0.2866	0.3854	0.4429
	Zoom Video Communications Inc.	ZM	AC trade price	-0.3241	-0.2752	-0.1548	-0.0011	0.1566	0.9939
			H trade price	0.1912	0.4487	0.4685	0.4523	0.4831	0.4984

Table 4 shows the results of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the New York Stock Exchange that were the most active in the period from 2013 to 2020.

Figures 4.1 and 4.2 show the output graphs of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the New York Stock Exchange that were the most active in the period from 2013 to 2020.

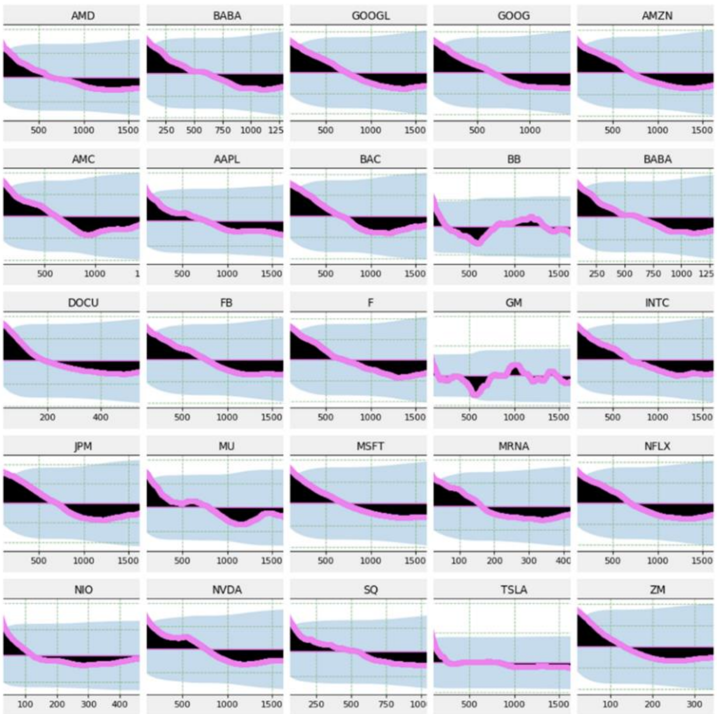

Figure 4.1- Autocorrelation graphs for New York stock exchange

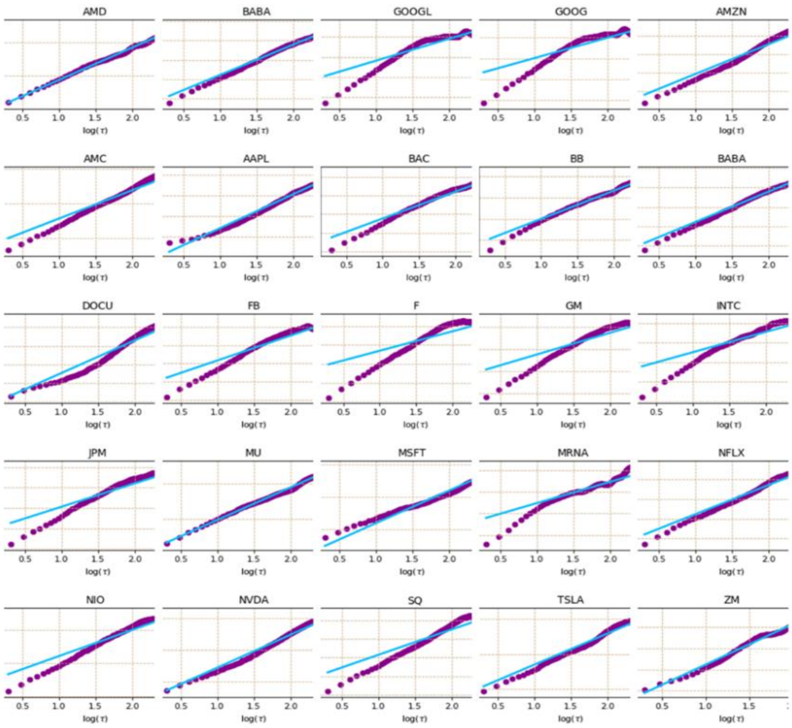

Figure 4.2- Hurst exponent graphs for New York stock exchange

Table 5. Autocorrelation and Hurst exponent statistics (Frankfurt stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
Frankfurt stock exchange	Adidas AG	ADSGn	AC order signs	0.1452	0.3386	0.5985	0.5701	0.7770	0.9979
			H order signs	0.0498	0.2163	0.2277	0.2279	0.24782	0.3150
	Allianz SE VNA O.N.	ALVG	AC order signs	0.1336	0.3063	0.4529	0.4818	0.6689	0.9963
			H order signs	0017	0.0483	0.1031	0.1961	0.2000	0.7863
	BASF SE NA O.N.	BASFn	AC order signs	-0.4446	-0.3302	-0.2274	-0.0463	0.2089	0.9937
			H order signs	0.0081	0.2643	0.2810	0.2738	0.3112	0.3767
	Bayer AG NA	BAYGn	AC order signs	-0.0461	0.0669	0.2726	0.3452	0.6015	0.9966
			H order signs	0.1147	0.4034	0.4335	0.4026	0.4500	0.4618
	Bayerische Motoren Werke AG	BMWG	AC order signs	-0.1501	-0.1003	0.1105	0.1997	0.4486	0.9949
			H order signs	0.2988	0.3024	0.3105	0.3196	0.3337	0.3675
	Daimler AG NA O.N.	DAIGn	AC order signs	-0.1824	-0.0087	0.2010	0.2498	0.4531	0.9965
			H order signs	0.3271	0.3295	0.3413	0.3545	0.3757	0.4283
	Delivery Hero AG	DHER	AC order signs	0.1441	0.3165	0.5610	0.5408	0.7401	0.9919
			H order signs	0.4964	0.5488	0.5938	0.5960	0.6490	0.6857
	Deutsche Bank AG NA O.N.	DBKGn	AC order signs	0.0973	0.3134	0.5018	0.5308	0.7603	0.9982
			H order signs	0.0358	0.2450	0.2913	0.2670	0.3285	0.3652
	Deutsche Post AG NA O.N.	DPWGn	AC order signs	-0.0431	0.0133	0.0989	0.2086	0.2980	0.9947
			H order signs	0.2721	0.2780	0.2879	0.3047	0.3178	0.3935
	Deutsche Telekom AG Na	DTEGn	AC order signs	-0.2054	-0.0316	0.2101	0.2353	0.4515	0.9945
			H order signs	0.3524	0.3570	0.3597	0.3599	0.3609	0.3738
	Deutsche Wohnen	DWNG	AC order signs	0.1434	0.3624	0.5434	0.5555	0.7421	0.9980

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
	AG		H order signs	0.1314	0.2384	0.2538	0.2580	0.2859	0.3479
	E.ON SE	EONGn	AC order signs	-0.1906	0.0058	0.3958	0.3691	0.6883	0.9964
			H order signs	0.4502	0.4776	0.4947	0.4967	0.5090	0.5674
	Fraport AG	FRAG	AC order signs	-0.2464	0.0243	0.1141	0.2224	0.4177	0.9974
			H order signs	0.1671	0.3312	0.3716	0.3589	0.4147	0.4704

Table 5. Autocorrelation and Hurst exponent statistics (Frankfurt stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
Frankfurt stock exchange	HelloFresh SE	HFGG	AC order signs	-0.2615	-0.1787	0.0596	0.1834	0.5454	0.9919
			H order signs	0.5080	0.6292	0.7064	0.6681	0.7187	0.7273
	Infineon Technologies AG NA O.N.	IFXGn	AC order signs	0.1548	0.2830	0.4398	0.4856	0.6635	0.9960
			H order signs	0.3600	0.3649	0.3767	0.3865	0.3967	0.4537
	Linde PLC	LINI	AC order signs	-0.0477	0.1865	0.3683	0.4020	0.6156	0.9921
			H order signs	0.3201	0.3337	0.3610	0.3650	0.3918	0.4216
	Merck KGaA	MRCG	AC order signs	0.0014	0.0790	0.2241	0.3061	0.4975	0.9946
			H order signs	0.2803	0.2908	0.3116	0.3203	0.3406	0.3963
	Muench. Rueckvers. VNA O.N.	MUVGn	AC order signs	0.0624	0.1612	0.3319	0.3807	0.5979	0.9944
			H order signs	0.0306	0.0583	0.0922	0.1025	0.1389	0.2172
	Porsche Automobil Holding SE	PSHG_p	AC order signs	-0.4020	-0.2135	0.0490	0.0612	0.1856	0.9951
			H order signs	0.2095	0.2236	0.2479	0.2617	0.2892	0.3666
	RWE AG ST O.N.	RWEG	AC order signs	-0.2655	-0.1252	0.2080	0.2536	0.5719	0.9963
			H order signs	0.5032	0.5379	0.5665	0.5597	0.5795	0.6126
	SAP SE	SAPG	AC order signs	0.1345	0.3009	0.5403	0.5326	0.7492	0.9975
			H order signs	0.0526	0.0664	0.0925	0.1054	0.1370	0.2264
	Siemens AG Class N	SIEGn	AC order signs	-0.3028	-0.2020	0.0266	0.1058	0.3736	0.9934
			H order signs	0.3313	0.3390	0.3549	0.3606	0.3690	0.4249
	Volkswagen AG VZO O.N.	VOWG_p	AC order signs	-0.2987	-0.1436	0.0789	0.1013	0.1691	0.9946
			H order signs	0.1800	0.1961	0.2197	0.2369	0.2597	0.3501
	Vonovia SE	VNAn	AC order signs	0.1206	0.2758	0.4423	0.4804	0.6615	0.9968
			H order signs	0.0113	0.0145	0.0297	0.0489	0.0679	0.1586
	Zalando SE	ZALG	AC order signs	-0.0842	0.0908	0.1362	0.2532	0.4005	0.9923
			H order signs	0.0761	0.2365	0.3083	0.2929	0.3677	0.4462

Table 5 shows the results of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the Frankfurt Stock Exchange that were the most active in the period from 2013 to 2020.

Figures 5.1 and 5.2 show the output graphs of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the Frankfurt Stock Exchange that were the most active in the period from 2013 to 2020.

Table 6 shows the results of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the Tokyo Stock Exchange that were the most active in the period from 2013 to 2020.

Figures 6.1 and 6.2 show the output graphs of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the Tokyo Stock Exchange that were the most active in the period from 2013 to 2020.

Table 7 shows the results of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the Tehran Stock Exchange that were the most active in the period from 1392 to 1399 Persian calendar.

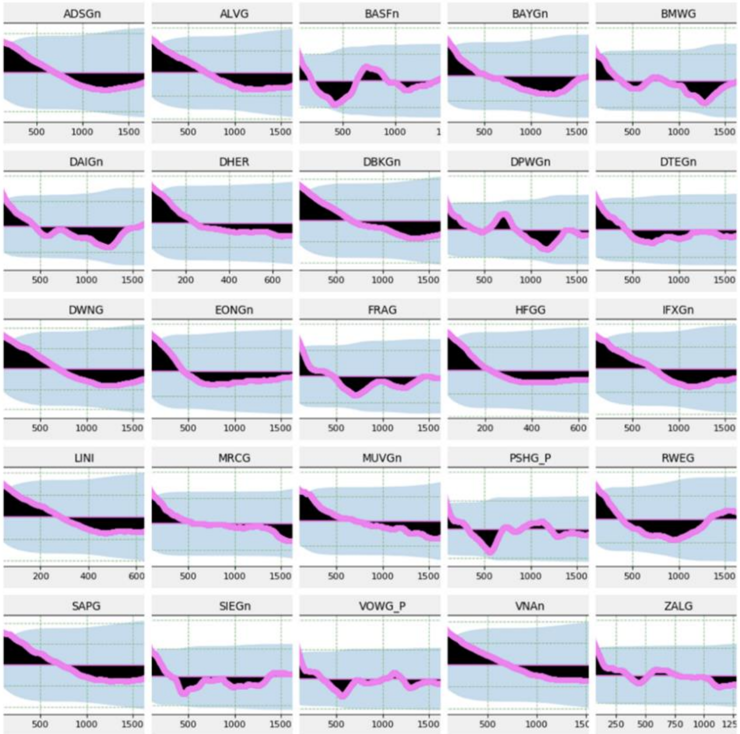

Figure 5.1- Autocorrelation graphs for Frankfurt stock exchange

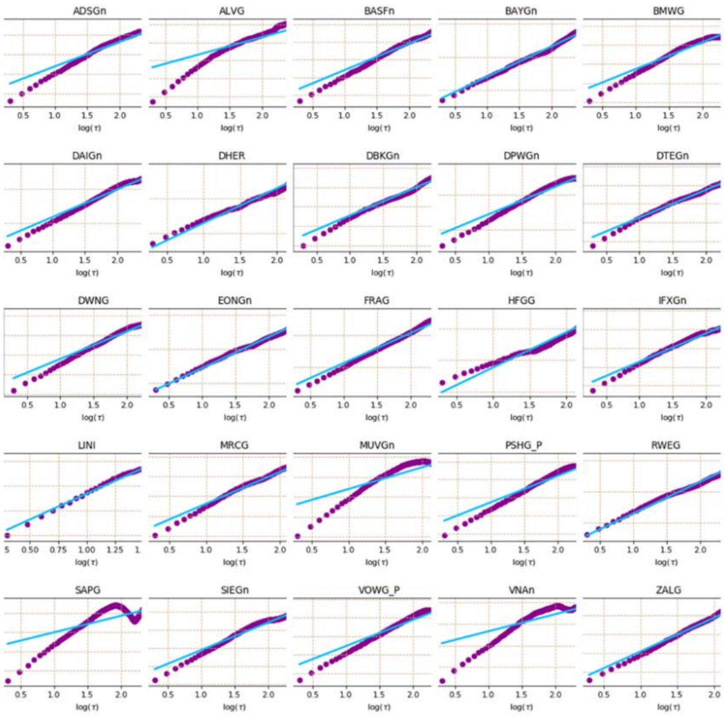

Figure 5.2- Hurst exponent graphs for Frankfurt stock exchange

Table 6. Autocorrelation and Hurst exponent statistics (Tokyo stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
Tokyo stock exchange	Daiichi Sankyo Co., Ltd.	4568	AC trade price	-0.0639	-0.0290	0.0122	0.1351	0.1859	0.9917
			H trade price	0.0815	0.1404	0.1906	0.1942	0.2438	0.3181
	East Japan Railway Co.	9020	AC trade price	-0.0174	0.0052	0.1534	0.2888	0.5610	0.9926
			H trade price	0.1318	0.3402	0.3743	0.3446	0.3926	0.4102
	Eisai Co., Ltd.	4523	AC trade price	0.0632	0.0839	0.1892	0.3307	0.5528	0.9945
			H trade price	0.1094	0.1788	0.2372	0.2366	0.2968	0.3503
	Fast Retailing Co., Ltd.	9983	AC trade price	0.1095	0.2961	0.3992	0.4491	0.5800	0.9931
			H trade price	0.1979	0.2238	0.2353	0.2491	0.2687	0.3391
	Hitachi Ltd	6501	AC trade price	-0.4250	-0.3264	-0.0172	0.0794	0.4348	0.9923
			H trade price	0.2136	0.3445	0.3583	0.3479	0.3721	0.4087
	Japan Airlines Co Ltd.	9201	AC trade price	-0.2156	-0.0324	0.2519	0.3026	0.6239	0.9956
			H trade price	0.4325	0.5244	0.5368	0.5253	0.5434	0.5493
	JFE Holdings, Inc.	5411	AC trade price	-0.1995	-0.0466	0.2103	0.2815	0.6122	0.9962
			H trade price	0.3581	0.4155	0.4228	0.4194	0.4338	0.4508
	Keyence	6861	AC trade price	0.0287	0.1421	0.3335	0.3856	0.6038	0.9936
			H trade price	0.3254	0.3784	0.3817	0.3763	0.3833	0.3972
	Lasertec Corp.	6920	AC trade price	0.0870	0.1508	0.3403	0.4050	0.6720	0.9951
			H trade price	0.3253	0.4862	0.5013	0.4826	0.5132	0.5207
	M3 Inc.	2413	AC trade price	0.0962	0.1551	0.2776	0.3531	0.4882	0.9925
			H trade price	0.1858	0.2984	0.3449	0.3478	0.4060	0.4885
	Mitsubishi UFJ Financial Group Inc.	8306	AC trade price	-0.0900	-0.0695	-0.0156	0.1952	0.4609	0.9945
			H trade price	0.0337	0.2075	0.2481	0.2404	0.2984	0.3778
	Nidec Corp.	6594	AC trade price	0.2988	0.4168	0.5352	0.5546	0.6481	0.9957
			H trade price	0.2008	0.2150	0.2404	0.2616	0.3002	0.3787
	Nintendo Co Ltd.	7974	AC trade price	0.3243	0.4550	0.5704	0.6025	0.7400	0.9966
			H trade price	0.1493	0.2623	0.2987	0.2924	0.3403	0.3876

Table 6. Autocorrelation and Hurst exponent statistics (Tokyo stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
Tokyo stock exchange	Nippon Steel Corp.	5401	AC trade price	-0.0887	0.0008	0.0847	0.2263	0.4222	0.9947
			H trade price	0.1976	0.2266	0.2470	0.2627	0.2891	0.3707
	Nippon Yusen K.K	9101	AC trade price	0.1748	0.3712	0.4990	0.5210	0.6464	0.9961
			H trade price	0.3029	0.3049	0.3096	0.3288	0.3498	0.3932
	Pharma Foods International Co Ltd.	2929	AC trade price	0.0279	0.0419	0.1542	0.2202	0.3383	0.9922
			H trade price	0.2855	0.2877	0.2900	0.2954	0.2974	0.3394
	Recruit Holdings Co Ltd.	6098	AC trade price	0.2332	0.4501	0.6693	0.6172	0.7579	0.9970
			H trade price	0.1425	0.1553	0.1730	0.1869	0.1985	0.2909
	Renesas Electronics Corp.	6723	AC trade price	-0.2955	-0.1771	0.1650	0.2023	0.5228	0.9943
			H trade price	0.3284	0.4302	0.4396	0.4333	0.4539	0.4685
	Softbank Group Corp.	9984	AC trade price	0.2000	0.2320	0.2705	0.3663	0.4620	0.9862
			H trade price	0.0361	0.1212	0.1502	0.1533	0.1942	0.2671
	Sony Corp.	6758	AC trade price	0.3296	0.4486	0.6093	0.6086	0.7473	0.9962
			H trade price	0.0852	0.1357	0.1744	0.1798	0.2188	0.2920

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
	Sumitomo Mitsui Financial	8316	AC trade price	-0.1004	-0.0176	0.0284	0.1957	0.4092	0.9930
			H trade price	0.0846	0.1379	0.1847	0.1890	0.2355	0.3063
	SymBio Pharmaceuticals Ltd.	4582	AC trade price	0.0500	0.1994	0.2970	0.3858	0.5589	0.9927
			H trade price	0.3255	0.3283	0.3300	0.3361	0.3402	0.3664
	Tokyo Electron Ltd.	8035	AC trade price	0.2869	0.3683	0.5491	0.5595	0.7371	0.9958
			H trade price	0.1591	0.3446	0.3691	0.3466	0.3823	0.4103
	Totoku Electric Co Ltd	5807	AC trade price	-0.0424	0.0509	0.3673	0.3979	0.7162	0.9969
			H trade price	0.3249	0.4917	0.5212	0.5036	0.5491	0.5623
	Toyota Motor Corp.	7203	AC trade price	-0.2053	-0.1761	-0.0613	0.1041	0.3211	0.9895
			H trade price	0.0748	0.1874	0.2292	0.2312	0.2900	0.3623

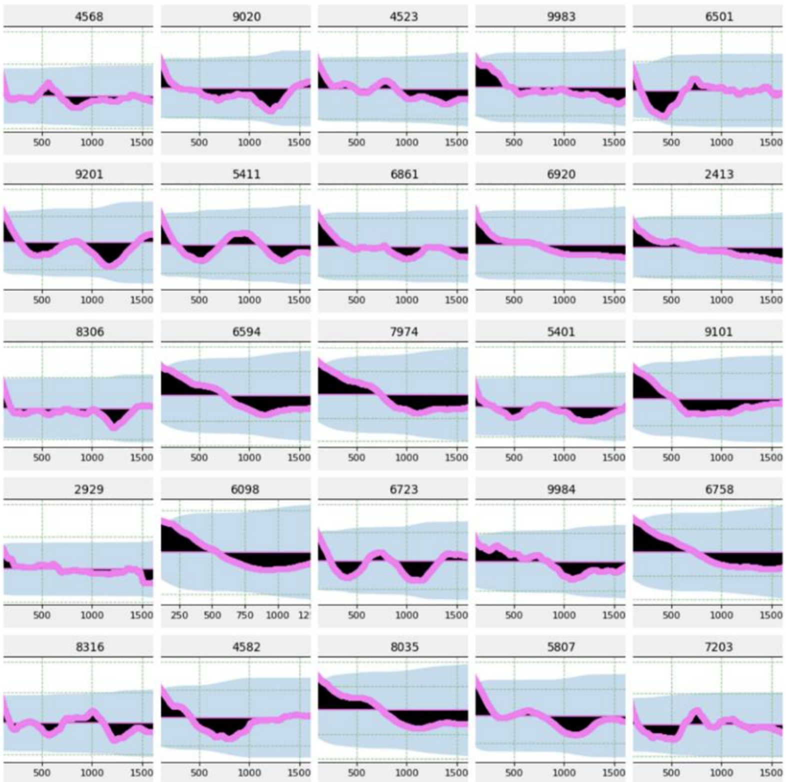

Figure 6.1- Autocorrelation graphs for Tokyo stock exchange

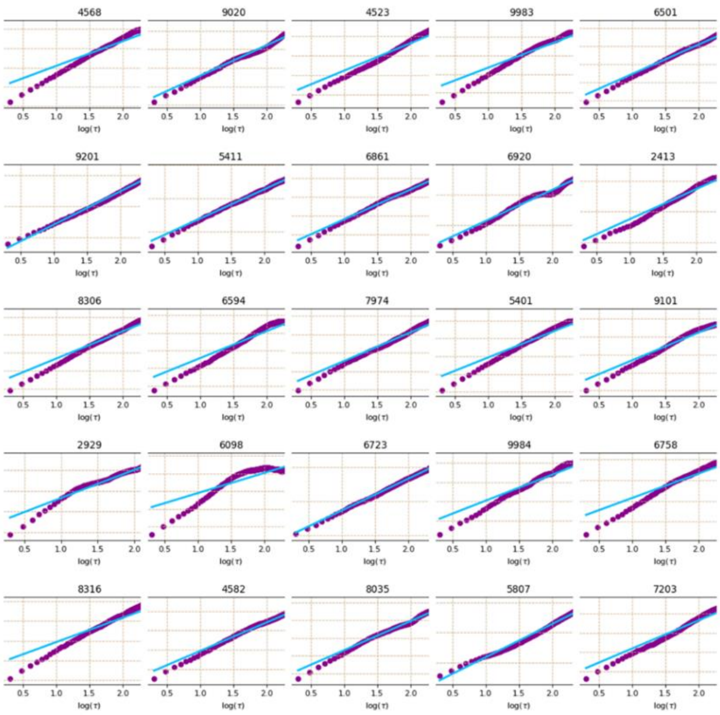

Figure 6.2- Hurst exponent graphs for Tokyo stock exchange

Table 7. Autocorrelation and Hurst exponent statistics (Tehran stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
Tehran stock exchange	Telecommunication Company of Iran	akhaber	AC trade price	-0.0358	0.0078	0.1377	0.2914	0.5617	0.9967
Tehran stock exchange	Telecommunication Company of Iran	akhaber	H trade price	0.2120	0.2280	0.2431	0.2623	0.2892	0.3594
Tehran stock exchange	Asia Insurance Co.	asia	AC trade price	-0.0116	0.0007	0.0891	0.2372	0.4363	0.9950
Tehran stock exchange	Asia Insurance Co.	asia	H trade price	0.0360	0.0618	0.0862	0.1074	0.1413	0.2437
Tehran stock exchange	Parsian Oil and Gas Development Group	parsan	AC trade price	-0.0046	0.0505	0.1903	0.3250	0.5890	0.9976
Tehran stock exchange	Parsian Oil and Gas Development Group	parsan	H trade price	0.3107	0.3117	0.3229	0.3382	0.3507	0.4205
Tehran stock exchange	Tamin Petroleum & Petrochemical investment Co.	tapico	AC trade price	-0.0335	-0.0069	0.0941	0.2702	0.5375	0.9975
Tehran stock exchange	Tamin Petroleum & Petrochemical investment Co.	tapico	H trade price	0.2319	0.2464	0.2651	0.2904	0.3225	0.4138
Tehran stock exchange	IRISL Group	hakashti	AC trade price	-0.0260	-0.0030	0.0794	0.2412	0.4589	0.9977
Tehran stock exchange	IRISL Group	hakashti	H trade price	0.0686	0.0786	0.1086	0.1322	0.1586	0.2920
Tehran stock exchange	SAIPA Group	Khasapa	AC trade price	-0.1091	-0.0061	0.1448	0.2008	0.3594	0.9920
Tehran stock exchange	SAIPA Group	Khasapa	H trade price	0.1191	0.1439	0.1759	0.1724	0.1902	0.2391
Tehran stock exchange	Iran Khodro Co.	khodro	AC trade price	-0.0669	-0.0331	0.0585	0.1430	0.2666	0.9875
Tehran stock exchange	Iran Khodro Co.	khodro	H trade price	0.0068	0.0206	0.0374	0.0522	0.0767	0.1448
Tehran stock exchange	MAPNA Group	rampna	AC trade price	-0.1177	-0.0719	0.0847	0.2092	0.4234	0.9976
Tehran stock exchange	MAPNA Group	rampna	H trade price	0.1254	0.1284	0.1505	0.1855	0.2360	0.3330
Tehran stock exchange	Bandar Abbas Oil Refining co.	shabandar	AC trade price	0.1081	0.2005	0.2709	0.3695	0.4907	0.9971
Tehran stock exchange	Bandar Abbas Oil Refining co.	shabandar	H trade price	0.1399	0.1466	0.1712	0.2087	0.2466	0.4154
Tehran stock exchange	Isfahan Oil Refining Co.	shepna	AC trade price	0.0109	0.1222	0.2569	0.3264	0.4673	0.9975
Tehran stock exchange	Isfahan Oil Refining Co.	shepna	H trade price	0.2237	0.2282	0.2440	0.2782	0.3082	0.4635
Tehran stock exchange	Fars Construction	sefars	AC trade price	-0.0820	-0.0646	0.1256	0.2198	0.3824	0.9979

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
	and Development Co.		H trade price	0.1063	0.1099	0.1341	0.1635	0.1837	0.3322
	Persian Gulf Petrochemical Industries	fars	AC trade price	-0.0406	0.0045	0.1481	0.2524	0.4222	0.9957
			H trade price	0.1544	0.1580	0.1805	0.2025	0.2278	0.3225
	Khouzestan Steel co.	fakhouz	AC trade price	0.0103	0.0492	0.0996	0.2717	0.4510	0.9961
			H trade price	0.1761	0.1870	0.2069	0.2270	0.2422	0.3487

Table 7. Autocorrelation and Hurst exponent statistics (Tehran stock exchange)

Financial market	Company name	Stock symbol	Stats	Min.	Q1	Median	Mean	Q3	Max.
Tehran stock exchange	National Iranian Copper Industries Co.	fameli	AC trade price	0.0310	0.0569	0.1382	0.2941	0.4886	0.9975
			H trade price	0.1032	0.1101	0.1427	0.1672	0.1923	0.3242
	Mobarakeh Steel Company	foolad	AC trade price	0.1151	0.1337	0.1961	0.3568	0.5709	0.9973
			H trade price	0.1418	0.1606	0.1752	0.2064	0.2371	0.3563
	Chadormalu Mining and Industrial Co.	kachad	AC trade price	0.0386	0.1019	0.2658	0.3715	0.6367	0.9920
			H trade price	0.3707	0.4611	0.4763	0.4618	0.4769	0.4880
	Golgohar Mining & Indudtrial Co.	kagal	AC trade price	0.0894	0.1654	0.3187	0.4233	0.6953	0.9958
			H trade price	0.4173	0.4513	0.4629	0.4618	0.4780	0.4923
	Omid Investment Management group Co.	vaomid	AC trade price	0.0276	0.0615	0.1624	0.3325	0.6104	0.9982
			H trade price	0.2375	0.3407	0.3724	0.3755	0.4211	0.4830
	Mellat Bank	webmellat	AC trade price	-0.0524	0.0244	0.1487	0.2274	0.3462	0.9911
			H trade price	0.0022	0.0056	0.0273	0.0513	0.0805	0.1851
	Oil Industry Investment Co.	vanaft	AC trade price	0.0271	0.1286	0.3320	0.4074	0.6799	0.9970
			H trade price	0.4117	0.4227	0.4414	0.4419	0.4584	0.4767
	Pasargad Bank	vapasar	AC trade price	-0.0488	0.0074	0.1626	0.3027	0.5866	0.9965
			H trade price	0.2887	0.4007	0.4056	0.4090	0.4291	0.4906
	Tejarat Bank	vatejarat	AC trade price	-0.1229	-0.0669	0.0111	0.1835	0.3981	0.9960
			H trade price	0.0185	0.0881	0.1525	0.1615	0.2300	0.3267
	Ghadir Investment Co.	vaghadir	AC trade price	-0.0515	-0.0032	0.1414	0.2921	0.5849	0.9977
			H trade price	0.2826	0.3122	0.3275	0.3438	0.3674	0.4383
	Mining and Metals Development Investment Co.	vamaaden	AC trade price	0.0861	0.1342	0.2430	0.3824	0.6241	0.9931
			H trade price	0.3026	0.3110	0.3263	0.3416	0.3668	0.4107
	Mobile Telecommunicatio n Company of Iran	hamrah	AC trade price	-0.0093	0.0496	0.1038	0.2427	0.3741	0.9961
			H trade price	0.1010	0.1106	0.1306	0.1521	0.1717	0.2882

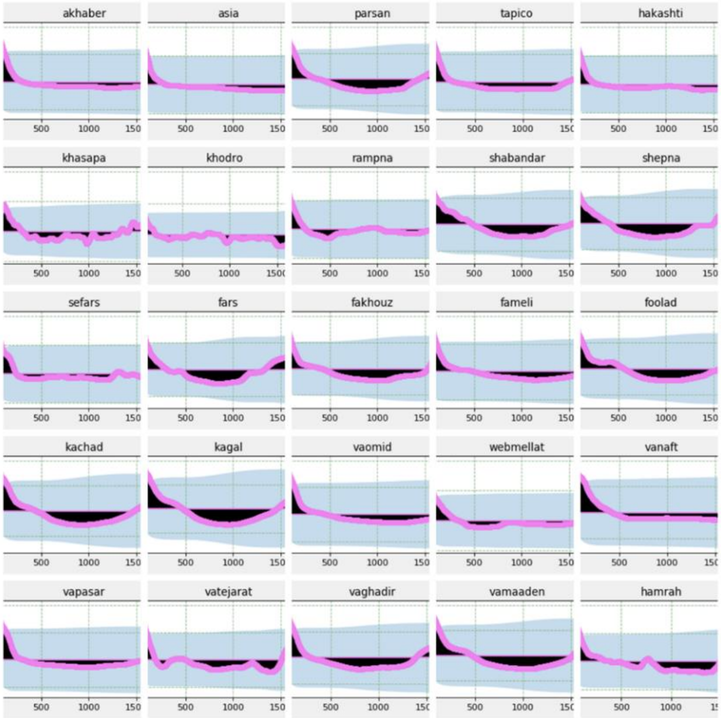

Figure 7.1- Autocorrelation graphs for Tehran stock exchange

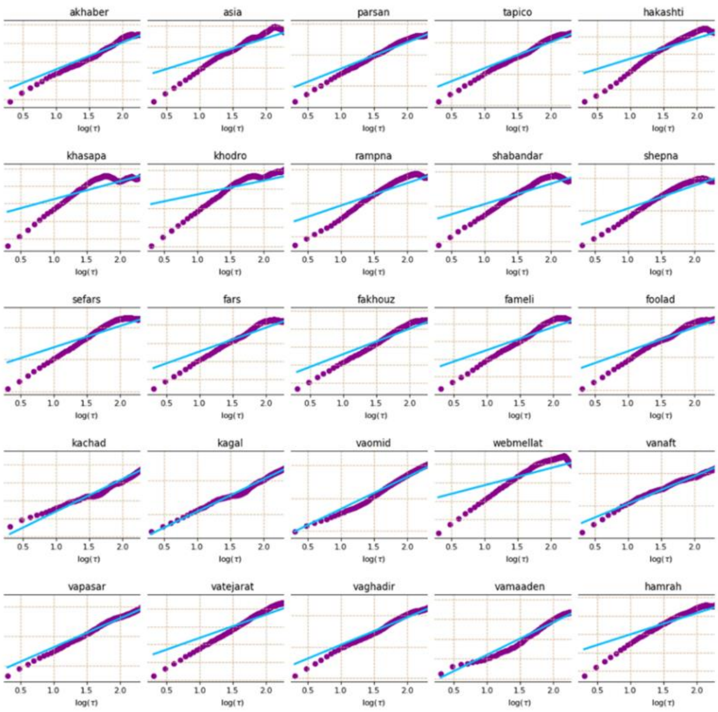

Figure 7.2- Hurst exponent graphs for Tehran stock exchange

Figures 7.1 and 7.2 show the output graphs of the autocorrelation and Hearst's exponent tests for 25 selected stocks on the Tehran Stock Exchange that were the most active in the period from 1392 to 1399 Persian calendar.

5.3. Mann-Whitney U Test for data samples

5.3.1. Mann-Whitney U Test for autocorrelation data samples

A data panel with two groups was created to compare the median of the autocorrelation data of the model and the autocorrelation data of each of the selected shares in financial markets. In the first group the data of the first 10 lags of autocorrelation output of the model, and in the second group the data of the first 10 lags of the autocorrelation data related to each of the selected shares of financial markets were placed. The results of the Man-Whitney test on the two groups are shown in Table 8.

Null Hypothesis: The distribution of the Rank of Ac is the same across categories of groups.

The result of the Mann-Whitney test comparing the autocorrelation of the time series of the model price with the panel data of the autocorrelation of the time series of the closing price of the selected stocks of the selected financial markets shows that the distribution of the rank of the autocorrelation is the same in both groups. Therefore, it can be concluded that these two populations have the same shape of their data, and thus the model can simulate the behavior of financial markets well. Jirasakuldech et al. (2011) and Zheng & Chen (2009) used similar methods in their research. These methods included the use of time series autocorrelation analysis and the non-parametric Mann-Whitney test for comparing the target population.

Table 8. Independent-Samples Mann-Whitney U Test Summary

Financial market	Mann-Whitney U	Wilcoxon W	Test Statistic	Standard Error	Standardized Test Statistic	Asymptotic Sig. (2-sided test)	Decision
New York stock exchange	1693.000	33068.000	1693.000	233.184	1.900	0.057	Retain the null hypothesis
Frankfurt stock exchange	1643.000	33018.000	1643.000	233.184	1.685	0.092	Retain the null hypothesis
Tokyo stock exchange	1441.000	32816.000	1441.000	233.184	0.819	0.413	Retain the null hypothesis
Tehran stock exchange	1657.000	33032.000	1657.000	233.184	1.745	0.081	Retain the null hypothesis

5.3.2. Mann-Whitney U Test for Hurst data samples

Also, for comparing the median of the Hurst exponent between two groups of the model and selected shares in financial markets a data panel with two groups was created. In the first group data of the 20 lags of the Hurst output of the model and in the second group data of the 20 lags of the Hurst output of selected shares in financial markets related to each of the selected shares of financial markets were placed. The results are shown in Table 9.

Null Hypothesis: The distribution of the Rank of Hurst is the same across categories of groups.

Table 9. Independent-Samples Mann-Whitney U Test Summary

Financial market	Mann-Whitney U	Wilcoxon W	Test Statistic	Standard Error	Standardized Test Statistic	Asymptotic Sig. (2-sided test)	Decision
New York stock exchange	4551.000	129801.000	4551.000	658.913	-0.681	0.496	Retain the null hypothesis
Frankfurt stock exchange	5960.000	131210.000	5960.000	658.913	1.457	0.145	Retain the null hypothesis
Tokyo stock exchange	5535.000	130785.000	5535.000	658.913	0.812	0.417	Retain the null hypothesis
Tehran stock exchange	3787.000	129037.000	3787.000	658.913	-1.841	0.066	Retain the null hypothesis

Also, the result of the Mann-Whitney test comparing the Hurt exponent of the time series of the model price with the panel data of the Hurt exponent of the time series of the closing price of the selected stocks of the selected financial markets shows that the distribution of the rank of the long-term memory is the same in both groups. Therefore, it can be concluded that these two populations have the same shape of their data, and thus the model can accurately simulate the behavior of financial markets. Martinez et al. (2018) and Freund and Pagano (2000) used similar methods in their research, employing time series Hurst exponent analysis and non-parametric Mann-Whitney test to compare the target population.

6. Conclusion

In this research, agent-based modeling and machine-learning methods have been used to design the hybrid model. The purpose of implementing the designed model is to compare the efficiency of using the model in automating algorithmic trading strategies in global financial markets. The model has been designed and executed, and the global variance sensitivity analysis has been performed to validate the model. The autocorrelation and Hurst exponent analysis tests were performed on the model's trading price time series.

In addition, the autocorrelation and Hurst's exponent analysis tests were performed on the historical data of the selected stocks of New York, Frankfurt, Tokyo and Tehran financial markets.

The Mann-Whitney U test was performed on the autocorrelation and Hurst's exponent data of the model and data panel of the selected stocks of each market to compare two samples or groups and to assess whether the two sampled groups are likely to come from the same population. The Mann-Whitney test showed that the model can effectively predict the behavior of actual financial markets.

Agent-based models consider systems at a decentralized level. Such a level of detail includes the description of several agent attributes and behaviors and their interaction with the environment. In this model, agents learn and exhibit strategic behavior, and they attempt to simplify realities. Real-world market participants exhibit irrational behavior, subjective choices, and complex psychology. However, the agents are still able to generate realistic macro-level stylized facts. Constructing such agents requires a very different kind of data than traditional top-down, mathematical models. You need to know how market participants behave, interact, form relationships, and make decisions. Agent-based models can provide a simulation environment in which market dynamics are emergent phenomena that arise from the interactions of behaviors that occur on different time scales. Regulation of algorithmic trading is not a simple matter of yes or no. Stable markets can arise from environments dominated by high-frequency algorithmic strategies. It is the interactions of these strategies that are particularly important.

When the market has a very high proportion of noise traders, 90% or more, the permanent impact is actually larger than the temporary part. This can be explained by two facts: First, markets with only noise agents will follow a random walk and only exhibit a permanent impact. Second, the liquidity providers are not able to create a price-efficient market, and the autocorrelation of returns can be seen in the increasing temporal market impact function. This is also why the difference between the temporary and permanent impact is the largest when there are much more liquidity consumers than liquidity providers.

Policymakers should take note that focusing efforts on the prevention of malicious behaviors and guiding regulation in that direction may not prove to be fruitful. Instead, regulators should focus on understanding how the interactions of market participants can lead to unexpected systemic behaviors.

Considering the diversity of the financial markets and the stocks selected in this research, it can be concluded that the model can provide a suitable model for automated algorithmic trading strategies. Given the need for investors to use the latest financial tools to compete in investing in the financial markets, it seems necessary to develop models that can make algorithmic trading possible for all users of these markets .

Agent-based modeling, combined with machine learning methods for training intelligent agents, can be a suitable method for developing models that can predict the behavior of other agents in the market and, based on those predictions, make an appropriate buy, sell, or hold decision and quickly replace those orders in the order book. Such a model allows regulators to understand the effects of algorithms on market dynamics. It also allows trading firms to optimize proprietary algorithms.

The execution of algorithmic trading compared to the old methods of market analysis and order placement using human factors can create many relative advantages for the users of these tools. The first advantage of these tools is the high speed of these agents in making decisions and placing orders in the market order book. Another advantage of these tools compared to human agents is the elimination of personal judgments such as doubt, fear, and greed that exist in human agents and disrupt the decision-making process.

7- Research limitations and areas for future research

Using large samples to train recurrent neural networks with the TensorFlow tool requires appropriate hardware. For large samples, TensorFlow workstations require a multi-core Intel processor, high-specification NVIDIA graphics cards, large memory RAM, air cooling for the GPU, and water cooling for the CPU.

Using other methods such as other types of neural networks, metaheuristic algorithms, and Support vector machines to train intelligent agents and comparing the efficiency of these methods in the performance of intelligent agents can lead to the discovery of more efficient methods for training intelligent agents. Also, using a diverse statistical population in other financial markets in developed and developing countries with larger samples and comparing the results of model performance in simulating the performance of these markets can be effective in optimizing algorithmic trading strategies.

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