Distributionally Robust portfolio Optimization based on the Calmar ratio using the Wasserstein metric

Mona Beyranvand
PhD Candidate of Industrial Management, Department of Management, Dehaghan Branch, Islamic Azad University, Dehaghan, Iran.
monabeyranvand@gmail.com

Sayyed Mohammad Reza Davoodi
Assistant Professor. Department of Management ,Dehaghan Branch, Islamic Azad University, Dehaghan, Iran.
(Corresponding Author)
Smrdavoodi@ut.ac.ir

Mohammadreza Sharifi-Ghazvini
Assistant Professor. Department of Industrial Engineering,Dehaghan Branch, Islamic Azad University, Dehaghan, Iran
Sharifidocument@gmail.com

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ABSTRACT
the return distribution of a portfolio can vary over different periods due to the dynamic nature of financial markets, leading to portfolio instability. Distributionally Robust Portfolio Optimization (DRO) takes into account the uncertainty in the portfolio caused by changes in the distribution of portfolio returns. In this research, the objective of the portfolio model is to maximize the Calmar ratio, which is a measure of the reward-to-risk ratio. The calculation of this ratio depends on the distribution of portfolio returns. To robustly estimate the return distribution parameter, the research strategy considers all returns within a neighborhood of the empirical distribution of the portfolio. The Wasserstein metric is used to determine such distributions. The sample portfolio in this research consists of 8 indices or industries from the Tehran Stock Exchange over a weekly time horizon from 2011 to 2021. The test data is divided into 5 periods. To evaluate the DRO portfolio compared to a portfolio without this feature, the average of Calmar in each period is divided by their standard deviation. The results demonstrate that the DRO portfolio improves the Calmar ratio by 0.27. Furthermore, the minimum Calmar ratio in each of the 5 periods is higher for the DRO portfolio compared to the portfolio without this feature.

Keywords: Calmar ratio, Distributionally Robust Portfolio, Wasserstein metric, PSO.

1. Introduction

When an investor decides to divide his capital between several financial assets, the problem of choosing the optimal portfolio shows itself. The basic question is how to divide the initial capital among the assets so that the investor can achieve his goal. This goal is often to reach a certain minimum level of return or wealth while bearing the least possible risk, and Markowitz's portfolio model (mean-variance model) was the first to model such a problem (Zhang, 2022). There are different models of the optimal selection of the portfolio, which differ in the objective function, limitations, single-period or multi-period, the presence or absence of transaction costs, etc. After the closing of the portfolio, the price changes of the portfolio assets occur over time, and the portfolio achieves a return at the end of the investment time horizon. This return may be far away from the return that was expected in the theoretical stage due to the drastic price changes of several assets. What happened is a long distance between the prediction and the reality, and therefore we are facing a kind of instability in the selection of the portfolio. In our country, the capital market always faces a lot of risks, especially political risks, and changes in laws, which can cause the assets of investors' portfolios to face unexpected price changes and possibly lead to a decline. drastic changes in the value of the portfolio.

In mathematical programming, problems are usually optimized with the assumption that the parameters of the model are definite. However, in the real world, most of the data have uncertainty. The main premise of mathematical programming is model development based on explicitly defined data equal to a nominal value (Ji et al., 2022). However, in this type of model, the effect of data uncertainty does not affect the quality and feasibility of the answers. As a result, in real-world problems, a large number of constraints may be violated by changing one of the data, and the obtained solution may be suboptimal or even impossible. As a result of this discussion, the main question of creating a solution for the problem that is resistant to this uncertainty of the data, so to speak, these answers are stable and this type of optimization is called stable optimization.

Uncertainty can occur in the coefficients of the objective function, the technical coefficients of the constraints, or even the bounds of the decision variables. One of the reasons for this is the problem of accidents. In many cases, sampling methods and statistical estimates are used to estimate the appropriate coefficient. Although choosing an estimate for a parameter allows the use of optimization algorithms to calculate the optimal point, it faces the problem of instability or instability in some ways. In real conditions and at the time of running the model, the parameters that are randomly determined may deviate from their estimated value in terms of the size or smallness of the variance and challenge the previously calculated optimal solution. It is even possible that the previous answer no longer applies within the limits and is an unanswerable one. The return distribution of the portfolio is also calculated based on statistical estimates and plays a fundamental role in the calculation of returns and risk and finally the stability of the portfolio. Based on this, the current research aims to investigate the effect of uncertainty resulting from the distribution of portfolio returns on a portfolio selection model to maximize the Calmar ratio.

2. Theoretical Background of Research

The parameters of portfolio optimization models are often estimated based on the statistical behavior of portfolio assets in historical data. The meaning of statistical behavior is the distribution of returns of the assets that make up the portfolio. When the asset return distribution is clear, the basic parameters of the portfolio such as return, variance, skewness, skewness, value at risk, expected drop, etc. can be calculated as quantiles or moments of the return distribution (Hu and Hong, 2013). Therefore, it can be said that the return distribution of a portfolio plays an essential role in modeling the portfolio and future expectations of its performance. When a parameter in an optimization model is associated with uncertainty (such as when it is estimated based on historical data), the optimization problem faces instability. This means that if the estimated parameters deviate from the actual values observed in the future or during the model performance test, there may be a significant difference between the expected results and the actual observed results. Regarding the issue of choosing the optimal portfolio, this can cause big losses.

Stabilizing the portfolio for one or more parameters means that the optimal solution is calculated in such a way that if the estimated parameters deviate from the expected value (within a certain deviation range), the portfolio does not face drastic changes. The general approach in stabilization is to consider scenarios for the parameter with certainty and the performance of the portfolio is evaluated in different scenarios. In this case, the most common approach in fortification is the worst-case scenario approach, based on which the optimal solution is selected in such a way that the greatest loss in different scenarios is minimized. In this case, the optimization problem appears in the form of a Max-Min model (Du et al., 2020).

The dynamics of the market can cause changes in the return of portfolio assets for different periods, and the distributional stable portfolio problem deals with the distribution of portfolio returns as a parameter with uncertainty. In the current research, the distribution of portfolio returns (not individual assets) is examined from the point of view of the empirical distribution. In this way, the distribution of the portfolio can deviate from its empirically estimated value. These deviations are the same scenarios that were discussed in stable optimization. For this purpose, it is necessary to measure the distance between two distributions (experimental distribution and distribution with deviation from the experimental distribution). For this purpose, the current study uses the K-L divergence distance criterion. By measuring the distance between two probability measures (one of which is the empirical distribution of portfolio returns), portfolio volatility is controlled by a neighborhood radius around empirical returns. In this way, the stabilization is controlled with higher confidence with a larger neighborhood radius around the empirical distribution.

So far, it has been stated that the research portfolio is a stable distribution type, and in the following, the objective function of the research portfolio model will be examined. Risk is defined as the uncertainty surrounding the future value of an asset or portfolio of financial instruments. Risk measurement and control are necessary for the survival and maintenance of a healthy and efficient financial system (Kobayashi et al., 2021). Risk-reward ratios are defined based on the functional division of the return or profitability of the portfolio on a measure of its risk. As an example, the Sharpe ratio is one of the most common risk-reward ratios, which is defined by dividing the return of the portfolio by its variability. This ratio of risk-return is measured in a unit of risk to determine how much return is added by adding one more unit of risk. Another common ratio is the Squid ratio, in which the average return of the portfolio is divided by the maximum loss, and the larger the Squid ratio, the more desirable it is for investors. The maximum drawdown measures the largest amount of decline in portfolio returns before a specified time. The objective function of the portfolio selection model of the current research is to maximize the Calmar ratio, which is dependent on the portfolio return distribution. In other words, changing the distribution of returns can change this ratio, and the current research aims to stabilize the portfolio model by considering the Calmar ratio with the distributional stabilization approach.

In the following, the background of the research conducted on the distributed stable portfolio is presented. Papageorgiou(2023) stated that There are numerous industrial settings in which a decision maker must decide whether to enter into long-term contracts to guarantee price (and hence cash flow) stability or to participate in more volatile spot markets. In this paper, we investigate a data-driven distributionally robust optimization (DRO) approach aimed at balancing this tradeoff. Unlike traditional risk-neutral stochastic optimization models that assume the underlying probability distribution generating the data is known, DRO models assume the distribution belongs to a family of possible distributions, thus providing a degree of immunization against unseen and potential worst-case outcomes. We compare and contrast the performance of a risk-neutral model, conditional value-at-risk formulation, and a Wasserstein distributionally robust model to demonstrate the potential benefits of a DRO approach for an "elasticity-aware" price-taking decision maker. Kirilyuk(2023) mentioned in his article that Polyhedral coherent risk measures and their worst-case constructions concerning the ambiguity set are considered. For the case of the discrete distribution and polyhedral ambiguity set, calculating such risk measures reduces to linear programming problems. The distributionally robust portfolio optimization problems based on the reward-risk ratio using worst-case constructions concerning the polyhedral ambiguity set for these risk measures and average return are analyzed. They are reduced to the appropriate linear programming problems. Kobayashi et al. (2023) This paper studies a distributionally robust portfolio optimization model with a cardinality constraint for limiting the number of invested assets. We formulate this model as a mixed-integer semidefinite optimization (MISDO) problem using the moment-based ambiguity set of probability distributions of asset returns. To exactly solve large-scale problems, we propose a specialized cutting-plane algorithm that is based on bilevel optimization reformulation. We prove the finite convergence of the algorithm. We also apply a matrix completion technique to lower-level SDO problems to make their problem sizes much smaller. Numerical experiments demonstrate that our cutting-plane algorithm is significantly faster than the state-of-the-art MISDO solver SCIP-SDP. We also show that our portfolio optimization model can achieve good investment performance compared with the conventional robust optimization model based on the ellipsoidal uncertainty set. Ji et al. (2022) studied the distributional robust optimization problem of a portfolio with a linearized stable tail adjusted return ratio, where the objective is to maximize the performance criterion of the said ratio in the worst case under data-driven uncertainty. To reflect the limitations of the Stock market, two constraints are considered, namely the purchase threshold and diversification constraints. The proposed problems are formulated into mixed integer linear programming problems and at the end of the validation results using the rolling horizon method, it shows the superior performance out of the sample of the research baskets. Hosseini Nodeh et al. (2022) consider the optimization of the portfolio with the assumption of the unknown distribution of asset returns in terms of distribution with a fuzzy stochastic dominance constraint. The objective is to maximize the expected return in the worst case and subject it to a fuzzy second-order stochastic dominant constraint. It has been shown that Wasserstein's fuzzy-based portfolio optimization can be reduced to a semi-definite program and second-order conical programming. The problems are investigated in depth using optimal solutions of optimization programs based on different settings. Zhang (2022) developed a robust distribution portfolio model under the condition that the return distribution of the risky asset is unknown, taking into account the risk aversion of investors and the uncertainty characterizing asset returns. Specifically, the goal is to find an optimal portfolio of assets that maximizes the worst-case utility level in Wasserstein size. This model is also formulated as a quadratic integer programming problem with cardinality constraints. In addition, a hybrid algorithm is proposed to improve the efficiency of the solution and make it more suitable for large-scale problems. Liu et al. (2021) proposed a high-moment consistent risk-averaged portfolio optimization model based on kernel density estimation (KDE) and phi divergence. To overcome the challenge of high dimensions, instead of the joint probability distribution of the asset return vector, The one-dimensional probability distribution of portfolio returns is considered. The researchers conducted some experimental tests with the false horizon approach and compared the performance of the optimal portfolio strategy obtained by the proposed model with three other strategies with four performance measures and their cumulative wealth curves. Results The empirical test shows that the quality of the portfolio strategy obtained by the proposed model is better in most cases. They also performed an empirical sensitivity analysis of the model parameters. Kobayashi et al.(2021) The main limitation is to limit the number of invested assets. This model is formulated as a mixed integer semidefinite optimization problem using the uncertainty set based on the moment of the asset return probability distribution. To solve the exact M For the large-scale problem, a specialized cutting plane algorithm is proposed, which is based on the reformulation of bilevel optimization, and the finite convergence of the algorithm is proved. Jalilian et al.,(2021) stated In this study, a bi-objective portfolio optimization model has been developed to determine the best sets of portfolios for a private investing company. Due to the lack of data on private assets, a simulation-based approach has been used to estimate the return of different assets as well as their correlations. A Covariance-Based Artificial Bee Colony is applied to solve the model. The results show that optimal portfolios consist of both high-risk and low-risk assets. Doo et al. (2020) modeled the portfolio selection problem with a robust distributed optimization method with uncertain return rate distribution information and based on a data-driven perspective. First, a mean-value portfolio optimization model under conditional risk with uncertainty is developed, where the uncertain distribution set used in the robust distribution model is an open Wasserstein sphere centered on the empirical distribution. Some numerical experiments have been carried out under different uncertainty sets, which show that the data-driven portfolio optimization method offers advantages over the unambiguous stochastic optimization method.

In research, Bardakci and Lagoa (2019) considered the portfolio optimization problem, which includes uncertainty in the probability distribution of asset returns. By estimating the mean and covariance matrix of asset returns, a class of acceptable distributions for returns is defined, and it is shown that worst-case loss risk optimization can be done numerically efficiently. The effectiveness of the proposed approach is demonstrated using academic examples. In research, Hu and Hong (2013) studied distributional robust optimization problems in which the uncertainty set of the probability distribution is defined by K-L divergence and it is shown that the resulting distributional robust min-max problems can be formulated as a one-layer convex minimization problem. be formulated They also considered the distributed robust problem where ambiguity is in the constraint and showed that fuzzy-constrained programs of the expected value type may be reformulated as a one-layer convex optimization problem. Finally, they show that the optimal solution of a likelihood minimization problem is also optimal for it.

in internal research so far.

Therefore, the most important research gaps that have been noticed during the implementation of this research are announced as follows.

- No study has been observed in the field related to optimization studies of distribution-based portfolios based on Kalmar's ratio with Wassersten's metric.
- A distributive stable portfolio has not been discussed in any of the internal research.

3. Research model

In this section, the research model, i.e., stable distribution portfolio based on Calmar's ratio with Wasserstein's metric, is introduced. First, the distance or Wasserstein metric is introduced. Wasserstein distance or Kantorovich-Rubinstein metric is a distance function defined between probability distributions in a certain metric space M. Intuitively, if each distribution is considered as a mound of dirt stacked in M, the metric is the minimum cost of transforming one mound into another, which is the distance to be moved times the difference between the means of the two distributions. It (M, d) is a metric space and$p \geq 1$ ، $P_p(M)$ is defined as the space of all dimensions suchμ that the P-th moment of them is the terminus, i.e., there is a point x_0 that

$$\int_M \boldsymbol{d}\,(\boldsymbol{x}, \boldsymbol{x_0})^{\boldsymbol{p}} \boldsymbol{d\mu}(\boldsymbol{x}) < \infty. \qquad)1($$

I$\mu, \nu \in P_p(M)$ f, the p-th Wasserstein distance between μ, ν:

$$(2)\boldsymbol{W_p}(\boldsymbol{\mu}, \boldsymbol{\nu}) := \left(\inf_{\gamma \in \Gamma(\mu,\nu)} \int_{M \times M} \boldsymbol{d}\,(\boldsymbol{x}, \boldsymbol{y})^{\boldsymbol{p}} \boldsymbol{d\gamma}(\boldsymbol{x}, \boldsymbol{y})\right)^{\boldsymbol{1/p}}$$

It is defined $\Gamma(\mu, \nu)$ as the space of all sizes$M \times M$ whose marginal size functions are equalμ, ν. Under these conditions, it can be shown that

$$\boldsymbol{W_p}(\boldsymbol{\mu}, \boldsymbol{\nu}) = (\boldsymbol{inf}\, \boldsymbol{E}\,[\boldsymbol{d}(\boldsymbol{X}, \boldsymbol{Y})^{\boldsymbol{p}}])^{\boldsymbol{1/p}} \qquad)3($$

As an example for two atomic sizes $\mu_1 = \delta_{a_1}$ and$\mu_2 = \delta_{a_2}$, In real numbers, the Wasserstein distance is equal $W_p(\mu_1, \mu_2) = |a_1 - a_2|$.for two normal distributions $\mu_1 = \mathcal{N}(m_1, C_1)$ $\mu_2 = \mathcal{N}(m_2, C_2)$ $C_1, C_2 \in \mathbb{R}^{n \times n}$The Wasserstein distance is:

$$(4)\boldsymbol{W_2}(\boldsymbol{\mu_1}, \boldsymbol{\mu_2})^{\boldsymbol{2}} = \|\boldsymbol{m_1} - \boldsymbol{m_2}\|_2^2 + \textbf{trace}(\boldsymbol{C_1} + \boldsymbol{C_2} - \boldsymbol{2}(\boldsymbol{C_2^{1/2}} \boldsymbol{C_1} \boldsymbol{C_2^{1/2}})^{\boldsymbol{1/2}}).$$

which in normal one-dimensional form$W_1(\mu_1, \mu_2) = \int_{\mathbb{R}} |F_1(x) - F_2(x)|\, dx$ is. For discrete and finite sizes, $P, P_0$the Wasserstein distance becomes equation (5).

$$(5)\quad \boldsymbol{W}(\boldsymbol{P}, \boldsymbol{P_0}) = \inf_{\pi \geq 0} \Big(\sum_{i,j \in N} \boldsymbol{\pi_{i,j}}\, ||\boldsymbol{\xi_j} - \boldsymbol{\xi_i^0}|| : \begin{array}{l} \sum_{j \in N} \boldsymbol{\pi_{i,j}} = \boldsymbol{p_i^0}, \forall \boldsymbol{i} \in \boldsymbol{N} \\ \sum_{i \in N} \boldsymbol{\pi_{i,j}} = \boldsymbol{p_j}, \forall \boldsymbol{j} \in \boldsymbol{N} \end{array} \Big)$$

where the distribution$P$, obtains the value $\xi_j$with probability $p_j$and the distribution P_0obtains the valueξ_i^0 with probabilityp_i^0. After introducing the Wasserstein metric, Calmar's ratio will be introduced, which needs to be calculated. Calculate the maximum drop or decrease (MDD). The maximum drop or reduction of the worst consecutive loss in a specified period. The maximum loss from the portfolio by

$$MDD = -\min_{\tau\in(0,T)}\left(\min_{t\in(0,\tau)} r(t,\tau)\right) \quad (6)$$

It is measured as where $r(t,\tau)$ the efficiency is in the interval between t andτ. The maximum drawdown is the worst realized performance since the portfolio's inception over a given investment horizon. Therefore, assets with the lowest maximum drawdown are more attractive to investors. In addition, maximum drawdown also encodes information about the time evolution of a series of returns in contrast to other quantile-based risk measures. Therefore, the distribution of portfolio returns plays a fundamental role in calculating this amount of portfolio return distribution. Calamar ratio is the ratio of return to maximum reduction, ie:

$$Calmar = \frac{R}{MDD} \quad (7)$$

where R is the average realized return, and MDD is the maximum realized reduction of the equation over a given investment time horizon. Investors prefer assets with higher leverage ratios to assets with lower leverage ratios. After expressing the Wasserstein metric and the Calmar ratio, the stable distribution model of the research is defined to maximize the Calmar ratio in the form of Max-Min programming according to equation (8).

(8)

$$\max_{x} \inf_{P\in D(w)} Calmar(x,r) = \frac{E(x^t r)}{MDD(x,r)}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$\forall i: x_i \geq 0$$

where in (9)

$$D_w(P,P_0) = \{P \in P(\Omega) | W(P,P_0) < \theta\}$$

In relation (8), $x$the weight vector of the portfolio, $r$the return vector of the assets, $P_0$the empirical distribution of the portfolio, and the controlling parameterθ is the radius of the neighborhood from the empirical distribution. $D_W(P,P_0)$ also includes all measurements that are in the neighborhoodθ of the empirical distribution. To optimize the model (8), from algorithm of Particles Swarm Optimization or PSO is used. This algorithm consists of a mass of particles. Each particle is settled in a region of the search space. The value of the objective function for each particle shows the degree of fitness of that particle's location. Particles in the search area move at a certain speed. The speed of the particle (direction and amount of speed) is under two factors. One is the best experience that particle has ever had (the best amount of fitness it has ever had) and the active factor is the best experience that neighboring particles have ever had. And finally, the movement of the particles will converge toward the optimal point. To optimize the model (8), the combination of two-particle aggregation algorithms is used. The first PSO algorithm has the objective function of maximization, it produces a generation of solutions where each particle is a portfolio and its objective function is in the form (8)

$$f(x) = \min_{P\in D(w)} Kalmar(x,r) = \frac{E(x^t r)}{MDD(x,r)}$$

is. In the process of calculating the objective function of relation (10), there is a minimization model, which is also used for the recent minimization of a PSO algorithm, which is the decision variable of the size function, which is located in a neighborhood radius of the empirical distribution function. Finally, after optimizing the portfolio, its performance is evaluated on the test data.

4. Research findings

In this section, the formation of a distribution stable portfolio in the Tehran Stock Exchange is done practically. The portfolio of the research consists of 8 indices or industries from the Tehran Stock Exchange between 1390 and 1400. Using the index is the concept of forming a diverse portfolio of s in that industry. As an example, using the car index as an asset means that the subset of this index is purchased diversely (proportionate to their weight in the index). The time horizon of the portfolio is one week (it is closed for one week) and 5 working days are considered each week. Descriptive statistics related to 594 weekly asset returns are presented in Table (1).

594 weekly results of the research were divided into 304 data for training and optimization of research models and 290 data for testing and checking the stable performance and profitability of the models. The research model uses the Wasserstein metric to specify the sizes or distributions that are within the appropriate radius of the experimental distribution. To determine the radius of the sphere around the experimental size, the training data were divided into 5 categories, and

the radius of the sphere equal to one-tenth of the largest distance between the 5 categories was selected, which is equal to 0.002. Based on the research method in the previous section, the optimal portfolio was calculated using the combination of two-particle aggregation algorithms to optimize the Max-Min model. With the help of the cumulative particle algorithm with 1000 repetitions and 200 particles, the optimal stable distribution basket with the objective function of maximizing the squid ratio in the model (8) was calculated in the form of a table (2).

Then, the performance of the optimal portfolio was collected on the test weeks, and the performance of the weekly return of the distributed stable model on the test weeks was presented in the graph (1).

The test data was divided into 5 periods, the length of each period was 58 weeks, and in each period, the proportion of squid was calculated, and the result is presented in the graph (2).

To check the stable performance of the research model in the test data, two criteria are used. To calculate the first criterion, two quantities are calculated. The first quantity is the average squid

ratios obtained in 5 test data groups (the size of each group is 58 weeks). The second quantity is the standard deviation of the squid ratios obtained in 5 groups of test data, which indicates the amount of data dispersion around the average. The simultaneous consideration of the two criteria of profitability and stability of the Calmar ratio can be found in the result of dividing the last two quantities, which forms the first criterion. A higher value of this ratio indicates that a higher Calamar ratio has been achieved while bearing less risk. The second criterion is the value of the lowest squid ratio obtained in 5 groups of test data. The performance of the robust distribution model with the Wasserstein distance criterion is presented in Table (3).

After examining the stable distribution model for the squid ratio, the maximization model of the squid ratio without distribution stability property was calculated. In this case, only one distribution was used and that is the experimental data distribution. The optimal basket in this model was calculated by the cumulative particle algorithm according to table (4).

As before, the test data was divided into 5 periods, the length of each period was 54 weeks, and the proportion of squid was calculated in each period, and the result is presented in the graph (3).

Then, in each period, the proportion of squid was calculated, and the result is presented in the graph (4).

Finally, the performance of the model without stable distribution property by maximizing the Calmar ratio is presented in Table (2).

The comparison of the performance table of the stable distribution model in Table (3) with the model without this feature in Table (4) shows that the distribution stable basket improves the division of the mean by the standard deviation of the squid ratio in 5 periods by 0.27 and in addition, the minimum squid ratio in 5 periods in the stable distribution basket is more than the basket without this feature.

Table (1)- Descriptive statistics of weekly return of portfolio assets

statistical indicators / asset	Average	Median	maximum	Minimal	standard deviation	Statistic Jarque-Bera	probability value
1. k- non-metal	0.007	0.001	0.150	-0.150	0.039	301.666	0.000
2. k- metal	0.009	0.000	0.150	-0.136	0.047	77.510	0.000
3-Cement	0.009	0.000	0.150	-0.136	0.047	77.510	0.000
4-drug	0.010	0.002	0.150	-0.134	0.036	656.248	0.000
5-f oil	0.010	0.002	0.150	-0.150	0.054	38.523	0.000
6-Machinery	0.009	0.003	0.150	-0.150	0.038	206.295	0.000
7-Sugar	0.011	0.003	0.150	-0.112	0.049	59.542	0.000
8-automobile	0.008	0.001	0.150	-0.150	0.055	24.493	0.000

Table (2)- optimal distribution stable portfolio

Asset number	1	2	3	4	5	6	7	8
Weight	0/3973	0/2400	0/0105	0/0890	0/1674	0/0000	0/0000	0/1481

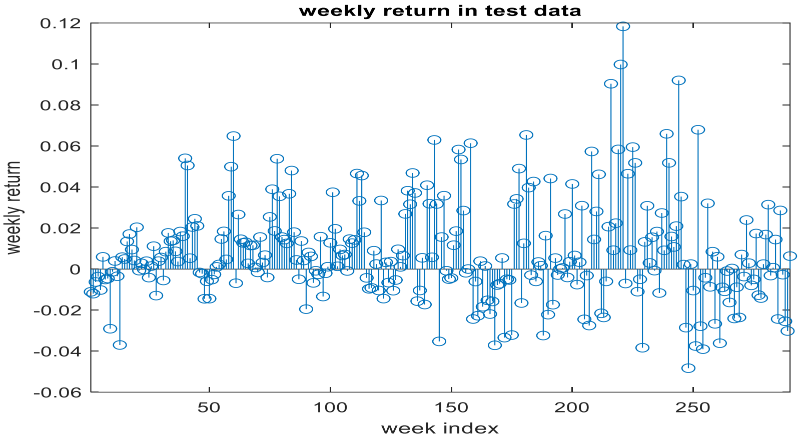

Diagram (1)- Weekly returns obtained during the test period

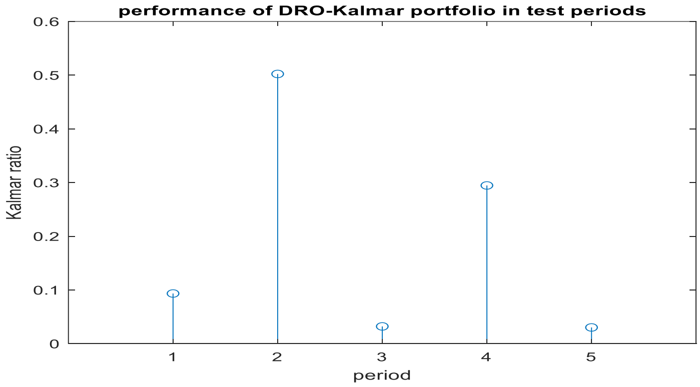

Diagram (2)- Squid ratio in 5 groups created in test data

Table (3): Performance of the research model

Value	performance measure
0.1906	Average Calmar ratio in the 5 test periods
0.2051	The standard deviation of the Calmar ratio in the 5 periods
0.9293	Dividing the average by the standard deviation of the Rachev Ratio in 5 periods
0.0303	The lowest Calmar ratio in the 5 test periods

Table (4): The optimal portfolio does not have stable distribution properties

Asset	1	2	3	4	5	6	7	8
Value	0/1844	0/0909	0/1307	0/0206	0/2156	0/1297	0/1725	0/1340

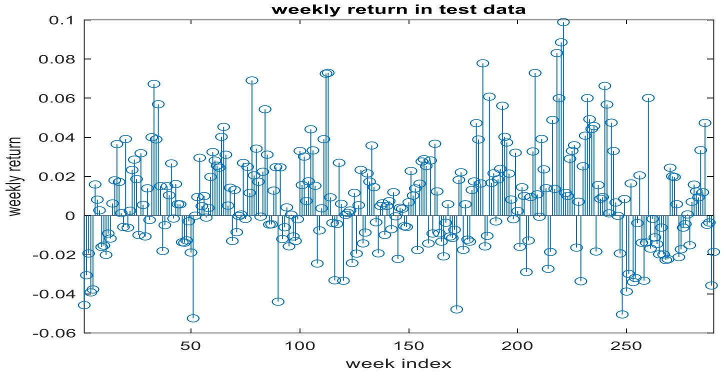

Diagram (3)- Weekly returns obtained during the test period

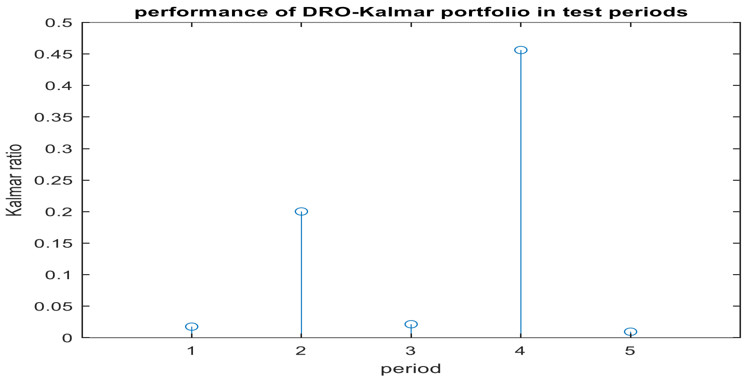

Diagram (4)- Squid ratio in 5 groups created in test data

Table (5)- Performance of the basket without distributional stability

Value	performance measure
0.1411	Average Calmar ratio in the 5 test periods
0.1934	The standard deviation of the Calmar ratio in the 5 periods
0.7292	Dividing the average by the standard deviation of the Rachev Ratio in 5 periods
0.0096	The lowest Calmar ratio in the 5 test periods

5- Conclusions and suggestions

One of the major challenges in optimal portfolio selection models is the statistical estimation of the model parameters, which provides the basis for the instability of the expected results and may lead to large losses. Robust programming is an effort to control the consequences of choosing a value instead of a parameter with uncertainty.

The distribution-based portfolio model is a financial modeling method that tries to evaluate and optimize stock portfolios by considering the distribution of the portfolio as an uncertain parameter. The main goal of this model is to increase investment profits while keeping the risk at a minimum. The distribution stable stock portfolio model is designed based on Kalmar's ratio. Kalmar's ratio is the risk-adjusted return ratio, which shows how much return can be expected for each unit of risk one is willing to bear. By maximizing the Kalmar ratio, the investment profit per unit of risk is maximized. To use the distribution-based portfolio model, the distribution of the portfolio must be accurately modeled. For this purpose, the Wasserstein metric is used, which determines the probabilities in the neighborhood of the distribution. By adjusting the neighborhood radius, the stability of the stock portfolio can be controlled. To evaluate the performance of the distributed stable stock portfolio model, two criteria are used. The first criterion includes the average proportion of squid in the test data groups, and the second criterion includes the standard deviation of the proportion of squid in each test data group. According to the results obtained in the research, the use of the distributed stable stock portfolio model is highly recommended as an effective and sustainable method for stock selection. This model increases the investment performance and reduces the uncertainty in the model results by optimizing the investment profit per risk unit.

The distribution stable portfolio model that was examined in the current research follows the stabilization of the portfolio by considering the distribution of the portfolio as an uncertain parameter. The research model is designed to maximize the Calamar ratio to maximize the investment profit per unit of risk. In other words, the objective of this portfolio model is to maximize risk-adjusted returns. The calculation of the Calmar ratio with two parameters of return and maximum loss is dependent on the distribution of the portfolio, and changes in the distribution of the portfolio return can affect the expected results. Therefore, in its theoretical part, the current research designed the distribution-stable optimal portfolio with Calmar's ratio, which uses Wasserstein's metric for distribution-stable modeling. This metric was used to specify the probability measures in the neighborhood of the empirical distribution, which controls the neighborhood radius of the degree and the degree of stabilization. To check the stable performance of the research model in the test data, two criteria are used. To calculate the first criterion, two quantities are calculated. The first quantity is the average squid ratios obtained in 5 test data groups (the size of each group is 58 weeks). The second quantity is the standard deviation of the squid ratios obtained in 5 groups of test data, which indicates the amount of data dispersion around the average. The simultaneous consideration of the two criteria of profitability and stability of the Calmar ratio can be found in the result of dividing the last two quantities, which forms the first criterion. A higher value of this ratio indicates that a higher Calamar ratio has been achieved while bearing less risk. The second criterion is the value of the lowest squid ratio obtained in 5 groups of test data.

Lee (2023) introduced a type of stabilization of the general portfolio optimization model called Kelly-Wassertein portfolio optimization. Ji et al. (2022) studied the distributional robust optimization problem of a portfolio with an adjusted return ratio with a linearized stable tail, in which the objective is to maximize the performance criterion of the said ratio in the worst case under uncertainty and guidance based on optimization data. Node et al. (2022)) portfolio optimization under the assumption of unknown distribution considers asset returns in terms of distribution with a fuzzy stochastic dominance constraint. Zhang (2022) created a distributional stable portfolio model by considering the risk aversion of investors and the uncertainty characterizing asset returns under the condition that the return distribution of the risky asset is unknown. Liu et al. (2021) proposed a high-moment consistent risk-average portfolio optimization model based on kernel density estimation (KDE) and phi divergence. To overcome the challenge of high dimensions, instead of the joint probability distribution of the asset return vector, The one-dimensional probability distribution of portfolio returns is considered. In a research, Kobayashi et al. (2021) studied a distributed robust portfolio optimization model with a main constraint to limit the number of invested assets. This model uses an uncertainty set based on The moment of the asset return probability distribution is formulated as a mixed integer semidefinite optimization problem. Doo et al. (2020) modeled the portfolio selection problem with a robust distribution optimization method with uncertain return rate distribution information and based on a data-driven perspective. First A mean-value portfolio optimization model under conditional risk with uncertainty is developed. Bardaki and Lagua (2019) in research considered the portfolio optimization problem that includes uncertainty in the probability distribution of asset returns. By estimating the mean matrix the covariance of asset returns defines a class of acceptable distributions for returns. Hu and Hong (2013) studied distributional robust optimization problems in a research where the uncertainty set of the probability distribution is defined by K-L divergence, but this research discussed distributional robust portfolio optimization based on the Kalmar ratio with the Wasserstein metric.

Future researchers are also suggested:

1) Use other metrics to measure the distance between two probability sizes, such as the sum of changes metric or the Hellinger metric, for distribution stabilization in the selection of stocks and compare the results.
2) Instead of the empirical distribution as the center of the sphere in stabilizing the distribution, use parametric distributions such as normal or log-normal and compare the results with the experimental distribution.
3) In addition to the spherical neighborhood radius, box, and ellipse neighborhood radii should also be used in stable optimization and compare the results.

The performance of the stock portfolio depends on its constituent assets. Therefore, in generalizing the results of the current research to a stock portfolio with content other than the assets in the current research portfolio, while optimizing the stock portfolio, it is necessary to evaluate its profitability performance in out-of-sample data, which is the subject of this The research is presented as a limitation of the research.

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