A bi-level optimization heuristic for solving portfolio selection problem

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ABSTRACT

This research addresses the challenge of optimizing the selection of a stock portfolio within a scenario involving two distinct decision makers, each driven by their unique objective functions and constraints. Importantly, the choices made by one decision-maker impact the decisions of the other. The primary focus is a dual-level model: at the first level (leader), investment management firms' decisions are expressed as they strive to partake in trading profits through portfolio management. At the second level (follower), active investors in the capital market are examined, pursuing goals of maximizing returns and minimizing investment risk. A notable innovation lies in incorporating social criteria and implicit investor preferences into the considerations of portfolio management firms. Given that solving two-level optimization problems with variable variability at the follower level is recognized as a complex polynomial challenge, this research introduces an inventive algorithm based on exhaustive enumeration to tackle the proposed model. Numerical outcomes stemming from portfolio optimization, utilizing data from the Tehran Stock Exchange market and over-the-counter market, reveal that in the optimal scenario, Hafars 1, Khatrak 1, and Sep 1 companies collectively constitute the largest portion of the optimal portfolio, accounting for 0.09 percent of total assets. Furthermore, numerical analyses underscore the favorable performance of the proposed model and algorithm in addressing issues related to stock portfolio optimization. Thus, this model holds promise as a management tool for similar challenges.

Keywords: portfolio optimization, two-level programming, heuristic algorithm, Iran capital market.

1. Introduction

In recent years, there's been growing research interest in sustainable development within portfolio management, covering social, environmental, and economic aspects (Khan et al., 2020). However, the environmental dimension has been relatively neglected due to a lack of integration with investment culture, with most focus on socio-economic factors (Mohagheghi et al., 2016). Doaei and Saberfard (2021) and Mostafae Darmian and Doaei (2022) are expressed environmental criteria are less considered and it is practically impossible to extract comprehensive information regarding sustainable development from companies operating in the financial markets.

Company sustainability now involves economic trends, employment, social dimensions, and environmental commitments (Ruan, 2018). Challenges like global economic conditions, competition, and the 2020 COVID-19 pandemic have reshaped investment landscapes (Ferneini, 2020). This leads to less reliable decision-making criteria for portfolio selection, emphasizing the importance of assessing company stability (Talan & Sharma, 2019). Multiple decision-makers with distinct objectives exist in investment, challenging the assumption of a single decision-maker. Government policies, like those during the COVID-19 outbreak, influence stock performance (Kheybari et al., 2021). Collaborative decision-making based on bilateral policies is crucial (Stoilov et al., 2021), and it often involves a two-level planning structure with leaders and followers. In two-level optimization problems, investors aim to allocate capital across securities while minimizing risk and achieving expected returns (González-Díaz et al., 2021).

These problems are akin to competitive games in operations research, where outcomes depend on everyone's choices. Game theory helps establish rational criteria for policy or strategy selection (Osborne, 2004). Within the paradigm of game theory, policies comprise a set of decisions that an entity can undertake at specific decision junctures. Demonstrating prudence involves thorough contemplation of others before taking action, encompassing their objectives, preferences, and limitations, and subsequently selecting a course of action that minimizes adverse consequences.

The formulation of optimal policy selection criteria within game theory is founded on the following two assumptions:

- Firstly, all players are assumed to be rational.
- Secondly, players harness their full capabilities to achieve the best possible outcome when confronted with challenges (Li & Xu, 2009).

In the realm of game theory, games manifest in various types and are categorized into distinct classifications, of which the primary classification is zero-sum and non-zero-sum games. A zero-sum game denotes a scenario where one player's gain equates to another player's loss, and vice versa. This type of game unfolds within competitive and adversarial conditions (Cura, 2009).

This research delves into a competitive game involving two distinct decision-making tiers, each contributing to the formulation of a novel perspective within the realms of investment management and portfolio optimization. The problem at hand centers around two models, each encompassing dissimilar objective functions that effectively encapsulate the outlooks and determinations of the respective decision-making levels.

The first tier is dedicated to assessing investment quality and stock portfolio selection through a criterion that transcends risk and investment measures. This criterion inherently incorporates the social inclinations of investors within its framework. Conversely, the follower level scrutinizes active investors operating within the capital market, driven by an overarching ambition to maximize returns while minimizing investment risk.

The innovations stemming from this research can be delineated as follows:

1) Devising an optimization model grounded in a two-level decision-making framework, aimed at ascertaining the optimal investment magnitude and the count of selected stocks.
2) Incorporating risk metrics, returns, and the Sharpe coefficient as essential considerations.
3) Furnishing a solution methodology hinging on comprehensive enumeration.

Proceeding with the research's structure, the subsequent section delves into a comprehensive review of theoretical literature and analysis of preceding studies. Following this, the third segment outlines the proposed mathematical model and heuristic algorithm, setting the stage for the presentation of numerical outcomes in the fourth part. Finally, the fifth section encapsulates a summary of findings, accompanied by pertinent suggestions for future exploration.

Literature Review

Socially responsible investment (SRI), introduced in 2014 by Scholtens, integrates non-financial factors into investment decisions, reflecting both financial and ethical considerations. SRI investors often avoid industries like tobacco, alcohol, and gambling due to their negative social impact. Portfolio selection involves balancing risk and return, with the pursuit of higher returns typically carrying increased risk (Deng, Lin, & Lo, 2012). Markowitz's model in 1952 laid the foundation for portfolio selection, but it had limitations in assessing risk and suitability for long-term investments (Zanjirdar, 2020). Cesarone et al. (2019) introduced a hybrid approach combining simulation and optimization methods, highlighting the impact of risk management criteria. Castilho et al. (2019) used classical mean-variance analysis with machine learning to optimize stock portfolios, considering future returns and constraints. Galankashi et al. (2020) used the fuzzy analytic network process to rank stock portfolios, emphasizing factors like profitability, growth, market dynamics, and risk. Vuković et al. (2020) combined multi-criteria decision-making and modern portfolio theory for stock portfolio selection, revealing differences in stock rankings. Rezaei Nokandeh et al. (2020) introduced a hybrid model for stock portfolio selection, considering goals and constraints through data analysis, multi-criteria decision-making, and linear programming. Khoo et al. (2020) applied multi-criteria decision-making to select portfolios for renewable energy desalination systems, addressing data uncertainty. Stanković et al. (2020) discussed the limitations of modern portfolio theory and proposed modified models for portfolio management. Yoshino et al. (2021) explored stock portfolio selection in the context of the Covid-19 pandemic and sustainability goals, proposing a global pollution tax to drive desired share allocations. Goa et al. (2021) studied high-order Markov switching portfolio selection, considering capital gains tax and loss transportation effects, using a stochastically constructed model and particle swarm optimization algorithm. For an overview of noteworthy research within this field, refer to Table 1, which highlights key studies related to the subject matter.

Table 1. Summary of the related papers

References	Sustainability dimensions		Tool		Problem environment		Case study
	Social	Economic	Decisions making	Optimization	Bi-level	Single level	
González-Díaz et al., (2021)		✓		✓	✓		✓
Guo & Ching, (2021)				✓		✓	✓
Yoshino et al., (2021)	✓	✓		✓		✓	
Stoilov et al., (2021)		✓		✓	✓		✓
Kobayashi, Takano, & Nakata, (2020)		✓		✓	✓		✓
Jing, Xu, & Li, (2020)	✓			✓	✓		
Galankashi et al., (2020)		✓	✓			✓	✓
Vuković et al., (2020)		✓	✓			✓	✓
Rezaei & Vaez-Ghasemi, (2020)	✓	✓	✓	✓		✓	✓
Xu et al., (2020)		✓	✓			✓	✓
Benita, López-Ramos, & Nasini, (2019)		✓		✓	✓		✓
Cesarone et al., (2019)				✓		✓	✓
Castilho et al., (2019)				✓		✓	✓
Cao, (2019)				✓		✓	✓
García, González-Bueno, Oliver, & Tamošiūnienė, (2019)				✓		✓	✓
Wang, He, & Shi, (2019)				✓		✓	✓
X. Chen, Kelley, Xu, & Zhang, (2018)		✓		✓	✓		
Bilbao-Terol, Jiménez-López, Arenas-Parra, & Rodríguez-Uría, (2018)	✓	✓	✓			✓	✓
Kalashnikov, Kalashnykova, & Leal-Coronado, (2017)		✓		✓	✓		✓

Research gap analysis

The research landscape primarily emphasizes single-level paradigms, with limited exploration of multi-level optimization models due to their intricacies. Sustainable development, despite its significance, has received relatively modest attention, possibly due to its evolving prominence. The fusion of multi-level optimization and sustainable development offers transformative potential and unexplored research avenues in modern investment and portfolio management. There is a scarcity of studies, particularly in Persian journals, focusing on sustainable development dimensions. Notable work by Rezaei Nokandeh et al. (2019) combines mathematical optimization and decision-making techniques for stock selection tied to sustainable development criteria. This pioneering approach involves data analysis and the TOPSIS method, highlighting the need for more research that synthesizes multi-level optimization and sustainable development. This unexplored intersection of concepts presents an opportunity for innovative contributions to enrich the field.

Methodology

A key challenge in achieving optimal stock portfolio selection is determining the number of stocks to include, a critical decision variable. Previous research often overlooked or treated this variable as an initial input parameter. This study seeks a paradigm shift by integrating the number of shares as a dynamic decision variable. To capture nuanced aspects of investment quality, a comprehensive measure is needed, drawing from quality assessment criteria in relevant literature. The Sharpe ratio, introduced by Jing, Xu, and Li (2022) in the context of stock portfolio optimization, emerges as a suitable gauge of investment quality. This metric relates the number of stocks to their fitness, with a lower Sharpe ratio indicating higher investment quality. In an ideal scenario, a single stock would have maximum fitness, minimizing this ratio. However, the real world often requires a judicious selection of a mix of stocks, each with distinct advantages. Stock appropriateness depends on balancing return and risk, key criteria in stock selection literature. Figure 1 illustrates the interplay between the number of chosen shares (cardinality), return rate, and the reciprocal of the Sharpe criterion (SR). This visual representation highlights the complex relationship between these crucial factors, providing insight into the multifaceted decision-making involved in optimal portfolio selection.

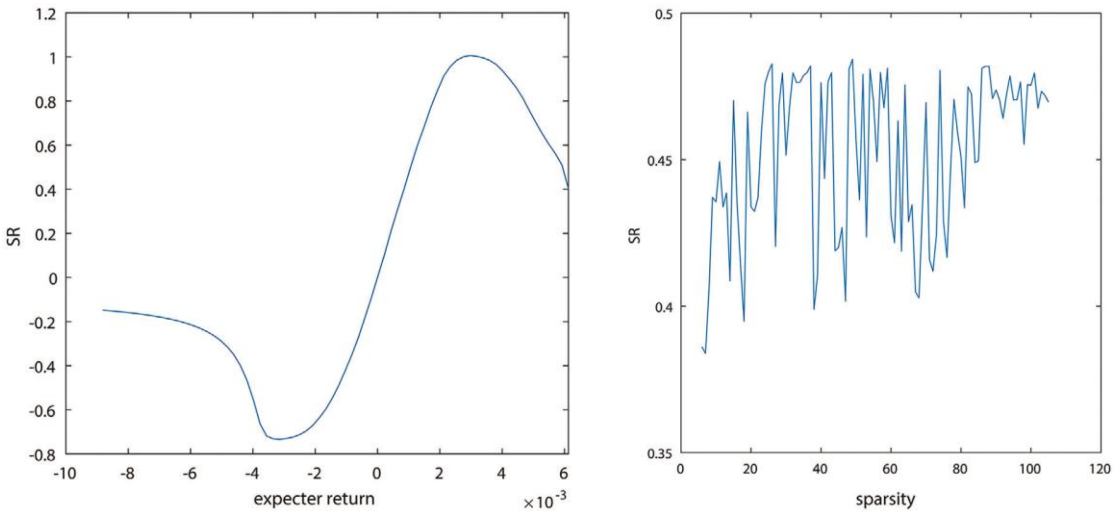

Figure 1- Analysis of the three behaviors of Sharpe's criterion, rate of return and number of shares relative to each other (adapted from (Jing, Xu, and Li, 2022))

Evidently, as return values ascend into positive territory, the inverse of the Sharpe ratio (SR) correspondingly surges, thereby signifying heightened investment quality. The rationale behind adopting the inverse of the Sharpe ratio lies in the desire to succinctly express figures with fewer decimal places, a pragmatic choice particularly pertinent to scenarios involving significant numerical magnitudes. Within this study, the core framework of the Sharpe criterion is harnessed, meticulously aligned with the foundation articulated in relation 1. This deliberate methodology ensures a rigorous and consistent approach, allowing for robust comparisons and informed decision-making in the context of optimal portfolio selection.

$$Sharpe = \frac{\rho}{\sqrt{\sum\left|return_i^{std} - risk_i^{std}\right|}} \qquad 1$$

Here, ρ symbolizes the count of stocks, while $return_i^{std}$ signifies the standardized representation of stock returns, and $risk_i^{std}$ denotes the standardized expression of stock risk. The process of standardization is employed to render the dimensions of return and risk comparable, facilitating an equitable scaling of these variables. By standardizing these metrics, we create a uniform platform for analysis and comparison, thus enhancing the interpretability and comparability of results.

Aiming to provide a comprehensive visualization, Figure 2 elucidates the interplay among all three criteria in a three-dimensional domain. This graphical representation serves as an insightful tool, affording a holistic perspective on the intricate dynamics weaving together the cardinality of selected stocks, standardized return values, and standardized risk values. The amalgamation of these dimensions within a three-dimensional framework offers a nuanced portrayal of the interrelationships and trade-offs intrinsic to optimal portfolio selection. This visualization serves as a powerful aid in deciphering the intricate web of variables that govern the decision-making process, fostering a deeper understanding of the underlying mechanisms that steer investment quality.

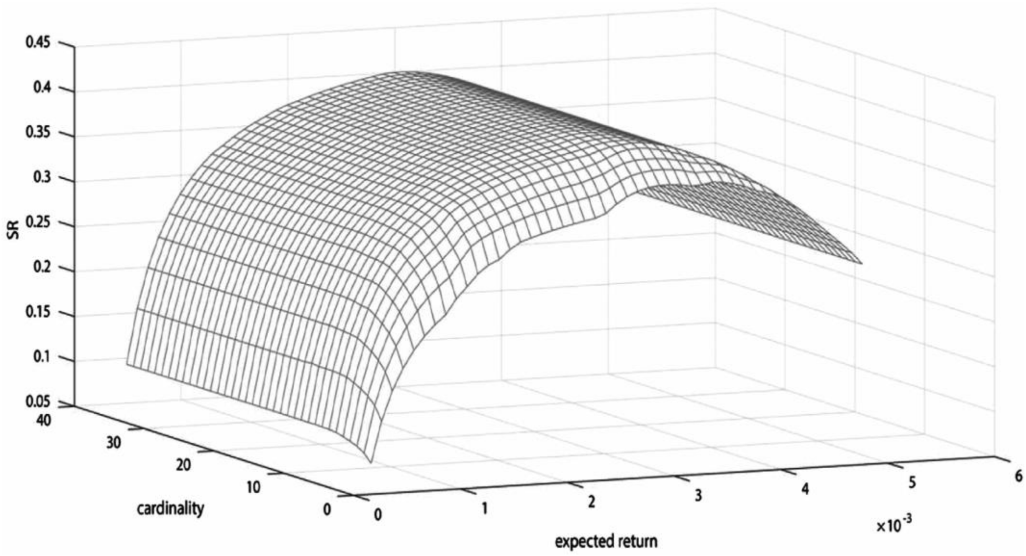

Figure 2- Interaction between the behavior of returns, Sharpe and the number of shares (adapted from (Jing, Xu, and Li, 2022))

It is imperative to underscore that the assessment of investment quality is confined to the purview of the initial model level, wherein decision-makers are chiefly concerned with this dimension. Conversely, the second level of the model is characterized by decision-makers who are inclined towards crafting investments that strike a harmonious balance between risk and return. Consequently, the formulation of the second-level model is outlined below.

x_i The amount of investment in the i-th company
β_i Risk in the i-th company
r_i Return on i-th firm
U_i The maximum amount of funds available for the i-th company

$$Min \sum_{i=1} \beta_i x_i \quad (2)$$

$$Max \sum_{i=1} r_i x_i \quad (3)$$

$s.t.$

$$\sum_{i=1} x_i = 1 \quad (4)$$

$$0 \leq x_i \leq U_i \quad (5)$$

The primary objective function is aimed at minimizing risk, while the secondary objective function strives to maximize returns. Constraint (3) ensures that the total proportion of shares acquired from all market companies sums up to 1. Additionally, Constraint (4) enforces a limitation on the share purchase percentage from each individual company, restricting it to a predetermined threshold.

However, in the context of the follower-level model (second level), a noteworthy challenge emerges due to the existence of dual objective functions. This duality inherently complicates the decision-making process, rendering the task of definitive answer selection intricate. Given the framework for resolving two-level quandaries, which necessitates establishing a recurring structure to derive ultimate solutions, the presence of two disparate objective functions adds considerable complexity. In response, this study employs three distinct methodologies for transforming multi-objective quandaries into single-objective ones. Specifically, the approaches utilized encompass: 1) the goal attainment method, 2) the comprehensive criterion method, and 3) the ideal programming method. These strategies are harnessed to consolidate the dual objective functions into a singular central function, thereby facilitating the formulation of a coherent two-level model.

Goal achievement Method

The goal achievement methodology stands as a potent instrument for extracting optimal compromise solutions within multi-objective contexts (Y.-L. Chen & Liu, 1994). Within this approach, a vector of weights denoted as v_1 and v_2 must be judiciously determined by the decision-maker to actualize the set goals, signifying the relative importance assigned to each objective. Subsequently, the minimum disparity between each objective function and its respective optimal value is computed using the following expressions:

$$Min\ G \quad (6)$$

$s.t.$

$$Z_1 - Z_1^* \leq v_1 G \quad (7)$$

$$Z_2^* - Z_2 \leq v_2 G \quad (8)$$

The technical coefficients, v_1 and v_2, hold pivotal significance in this context, serving as instrumental factors in the conversion process. It is essential to note that the summation of these coefficients equates to 1, reflecting a harmonious balance in their contribution. Anchored by these definitions, the follower-level model seamlessly metamorphoses into a unified single-objective model, encapsulated within the ensuing structure.

Leader Model

$$Min\ Z_3 = G \quad (9)$$

$s.t.$

$$\sum_{i \in I} \beta_i \times x_i - Z_1^* \leq v_1 G \quad (10)$$

$$Z_2^* - \sum_{i=1} r_i x_i \leq v_2 G \quad (11)$$

$$\sum_{i=1} x_i = 1 \quad (12)$$

$$0 \leq x_i \leq U_i \quad (13)$$

$$G \in free\ in\ sign \quad (14)$$

Equation (9) encompasses the minimization of the disparity between the objective functions and their respective optimal values. Notably, owing to the akin numerical ranges of the presented objective functions, the necessity for normalizing them is obviated. As a result, the value of parameter G remains universally applicable, seamlessly accommodating both objective functions. Equations (10) and (11) quantitatively compute the weighted disparity between the objective functions and their optimal benchmarks. Constraint (14) underscores the inherent flexibility in the sign of the variable G.

In summation, the comprehensive two-level planning model devised in this study finds its embodiment in the subsequent formulation:

$$Min\ Sharpe = \frac{\rho}{\sqrt{\sum |r_i x_i - \beta_i \times x_i|}} \quad 15$$

$s.t.$

$$x_i \le M\ y_i \quad 16$$

$$y_i \le x_i \quad 17$$

$$\sum_{i \in I} y_i = \rho \quad 18$$

$$Min\ Z_3 = G$$

$s.t.$

$$\sum_{i \in I} \beta_i \times x_i - Z_1^* \le v_1 G \quad 19$$

$$Z_2^* - \sum_{i=1} r_i x_i \le v_2 G \quad 20$$

$$\sum_{i=1} x_i = 1 \quad 21$$

$$0 \le x_i \le U_i \quad 22$$

$$G \in free\ in\ sign \quad 23$$

Where,

y_i It is 1 if the i-th company's share is chosen for investment, and it is equal to zero otherwise.

Objective function (15) is dedicated to the minimization of the Sharpe coefficient, which directly corresponds to the enhancement of stock portfolio quality. Constraints (16) and (17) serve to determine the inclusion or exclusion of specific shares, while constraint (18) ensures that the chosen number of investment stocks adheres to the parameter ρ. It's noteworthy that other problem constraints have been previously elucidated.

LP-Metric Method

Employing the widely recognized LP-metric method, a comprehensive criterion approach tailored for addressing multi-objective models, the problem undergoes a transformative shift towards a single objective formulation. Given the inherent dissimilarity in scales between the two distinct objective functions, a preliminary normalization step is undertaken, as stipulated by the following relationship where Z_i^* signifies the optimal value for each respective objective function.

In the presented optimal model, the amalgamation of the two original objective functions yields the ensuing equation, ultimately converging into a solitary objective for the problem at hand. In the current study, it is assumed that the two distinct objective functions are designated as Z_1 and Z_2. Enacting the LP-metric method entails resolving the research model for each of these distinct objective functions individually. The LP-metric model for the objective function is articulated through the subsequent formulation.

$$Min\ Z_3 = \left[\alpha \frac{z_1 - z_1^*}{z_1^*}\right] + (1+\alpha)\frac{z_2 - z_2^*}{z_2^*} \quad 24$$

where 0 ≤ α ≤ 1 represents the range of weight coefficients. The specific weight coefficients for the various elements of the objective function are delineated in the preceding equation. Employing this meticulously crafted equation, the problem transforms into a well-defined target amenable to straightforward resolution. The application of this equation facilitates an expedient and efficient solution process, ushering in clarity and precision to the optimization endeavor.

$$Min\ Sharpe = \frac{\rho}{\sqrt{\sum |r_i x_i - \beta_i \times x_i|}} \quad 15$$

$s.t.$

$$x_i \le M\ y_i \quad 16$$

$$y_i \le x_i \quad 17$$

$$\sum_{i \in I} y_i = \rho \quad 18$$

$$Min\ Z_3 = \left[\alpha \frac{z_1 - z_1^*}{z_1^*}\right] + (1+\alpha)\frac{z_2 - z_2^*}{z_2^*} \quad 24$$

$s.t.$

$$z_1 = \sum_{i \in I} \beta_i \times x_i \quad 25$$

$$z_2 = \sum_{i=1} r_i x_i \quad 26$$

$$\sum_{i=1} x_i = 1 \quad 21$$

$$0 \le x_i \le U_i \quad 22$$

$$G \in free\ in\ sign \quad 23$$

Goal Programming method

Within the purview of the Goal Programming method, the aspiration is to bring each objective function's value closer to its ideal optimal state. To realize this, a pair of additional variables is introduced into the model, designed to capture the deviation from the optimal threshold. By incorporating these variables, the objective functions are effectively repurposed as constraints, tasked with the mandate of minimizing the divergence from their respective optimal benchmarks. This strategic augmentation empowers the optimization process, enabling the systematic alignment of objectives with their idealized goals. The inclusion of these deviation variables engenders a dynamic framework that finely tunes the attainment of optimal objectives while simultaneously accounting for inherent complexities and contingencies.

$$Min\ d_1^+ + d_2^+ \quad 27$$
$$s.t.$$
$$Z_1 + d_1^- - d_1^+ = Z_1^* \quad 28$$
$$Z_2 + d_2^- - d_2^+ = Z_2^* \quad 29$$
$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0 \quad 30$$

Leveraging the equation detailed above, the presented problem assumes a form conducive to straightforward resolution. This formulation facilitates a streamlined approach to solving the complex optimization challenge at hand, offering a pragmatic and efficient pathway to achieving optimal outcomes.

$$Min\ Sharpe = \frac{\rho}{\sqrt{\sum|r_i x_i - \beta_i \times x_i|}} \quad 15$$
$$s.t.$$
$$x_i \leq M\ y_i \quad 16$$
$$y_i \leq x_i \quad 17$$
$$\sum_{i \in I} y_i = \rho \quad 18$$
$$Min\ Z_3 = d_1^+ + d_2^+ \quad 27$$
$$s.t.$$
$$Z_1 + d_1^- - d_1^+ = Z_1^* \quad 28$$
$$Z_2 + d_2^- - d_2^+ = Z_2^* \quad 29$$
$$\sum_{i=1} x_i = 1 \quad 21$$
$$0 \leq x_i \leq U_i \quad 22$$
$$G \in free\ in\ sign \quad 23$$
$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0 \quad 30$$

Solution Method

Resolving two-level models with discrete variables at the follower level remains a challenging problem in operations research (Gao, Wu, & Sun, 2005). Innovative methodologies are required to find optimal solutions (Saffarian, etal., 2021). This research introduces a pioneering iterative approach based on exhaustive enumeration. The algorithm begins by considering all potential solutions proposed by the leader level, forming an initial set of answers. This set represents a wide range of strategies from the leader, providing choices for the follower. The follower level then solves its unique problem for each of these potential answers, creating distinct responses for each leader-proposed solution. Essentially, in response to the leader's strategies, the follower generates a corresponding set of mutual decisions. In the final phase, each follower response is incorporated into the leader's model, and the resulting objective function value is calculated. Among the outcomes obtained, the response leading to the most favorable result for the leader is identified as the optimal solution. The following sections provide a detailed explanation of this algorithm's components, offering a thorough understanding of its operation and implications.

3-4-1- Generation of Leader Strategies

The cardinal significance of the leader's set of justified answers lies in its role as the pivotal decision-making strategies for addressing the follower's predicament. Notably, the precision and efficacy of these strategies profoundly impact the breadth of responses that the follower can furnish. Consequently, an intricate web of strategies enhances the expansive domain within which the leader's decision-making process unfolds.

In the context of this research, the comprehensive counting technique emerges as the chosen approach, diligently exploring the entire spectrum of potential strategies. This methodological choice reflects a commitment to exhaustively probe and evaluate all conceivable avenues. Notably, when the number of shares is denoted as |I|, signifying the aggregate available for investment, the multitude of potential choices from the subsets $\{1, \dots, I\}$ can be rigorously computed. This quantification is succinctly captured through the utilization of formula (31), enabling a systematic and thorough exploration of the decision space.

$$i \in \{1, \dots, I\}$$
$$LSS = \left\{\binom{|I|}{i}\right\}$$
$$|LSS| = \sum_{i \in \{1, \dots, I\}} \binom{|I|}{i} \quad 31$$

Illustratively, consider a scenario where there are three shares available for investment. In this case, the investor can potentially explore seven distinctive investment configurations. The efficiency of solving the follower model with commercially available solvers, which boasts rapid execution times, enables a comprehensive examination of all feasible investment scenarios.

However, a pivotal challenge surfaces in the comprehensive design of diverse investment scenarios, centering on the determination of the allocation of investment across each share. In essence, within every distinct amalgamation of stock selection, an infinite array of investment distributions can conceivably be allocated. This arises from the intrinsic nature of $x_i \in \mathcal{R}^+$, signifying the unbounded nature of potential investment values. Consequently, the exhaustive enumeration of all plausible investment magnitudes for the leader remains a formidable undertaking.

In an astute resolution to this conundrum, the optimal investment distributions are ascertained for each distinct stock selection portfolio through the implementation of a sophisticated neighborhood search algorithm. In this algorithmic scheme, the fitness function is aligned with the follower's objective function, thereby guiding the search towards optimal investment allocations. This strategic utilization of a neighborhood search algorithm synergizes the pursuit of optimal solutions with computational efficiency, paving the way for a systematic and nuanced exploration of investment scenarios. Please find the process of local search algorithm for optimal investment allocation in the supplementary material. Data collection process in presented in the supplementary material file.

Findings and analysis

The subsequent phase of the research delves into the numerical domain, embarking on a rigorous scrutiny of the efficacy of the follower level model—essentially a multi-objective optimization conundrum. This scrutiny, however, is conducted through the lens of diverse single-targeting methodologies inherent to the proposed mathematical model. It is paramount to acknowledge that each of the envisaged methodologies for the conversion of the mathematical model from its multi-objective iteration to a single-objective semblance proffers its unique modus operandi. This segment, therefore, endeavors to meticulously investigate their respective performances, employing a tapestry of statistical techniques as the evaluative apparatus.

To orchestrate this meticulous evaluation, a corpus of 30 distinct numerical instances is meticulously crafted. These instances encompass a medley of randomized amalgamations of the active companies traversing both the stock and over-the-counter markets. The ensuing results, derived from the incisive resolution of the model, are meticulously outlined and elucidated. It is of import to underscore that this specific segment is primed for the meticulous assessment of methodological efficacy. Regrettably, it refrains from delving into the practical instantiation of model outcomes in real-world scenarios or the concretization of investment directives.

The numerical results chapter thus serves as a crucible wherein the proposed methodologies are subjected to rigorous scrutiny, shedding light on their respective strengths and shortcomings. By traversing this analytical terrain, the research engenders a discerning perspective on the manifold techniques employed for converting the multi-objective model into a single-objective framework.

Table 2- Numerical results of solving the model using different methods

Ins	Portfolio size	LP_Metric			Goal Programming			Goal Attainment		
		Main	Obj1	Obj2	Main	Obj1	Obj2	Main	Obj1	Obj2
1	30	0.001671497	1.6279	1.7549	7.014384589	1.6305	1.7469	9.524801439	1.6287	1.7512
2	32	0.042309066	2.7516	2.3892	219.1938428	2.7582	2.3863	413.6545557	2.7466	2.4062
3	34	0.000486245	2.3806	1.9144	3.025393886	2.3818	1.9139	4.287604492	2.3810	1.9141
4	36	0.004802334	1.8527	1.2716	76.61352095	1.9257	1.2618	109.9925269	1.8844	1.2673
5	38	0.003092067	2.4572	1.9873	37.01683732	2.4855	1.9824	184.6848383	2.4699	1.9916
6	40	0.003987622	3.1383	2.7760	31.41159713	3.1490	2.7714	439.7743835	3.1444	2.7933
7	42	0.001980623	2.9232	5.4264	23.18053546	2.9388	5.4162	30.99165163	2.9327	5.4176
8	44	0.001525611	3.0306	3.6055	0	3.0268	3.6211	0.000302718	3.0268	3.6211
9	46	0.012263313	2.4400	2.6946	71.60140004	2.4532	2.6899	159.7139618	2.4445	2.6976
10	48	0.012077136	2.8253	2.6713	63.40770845	2.8289	2.6732	214.4292116	2.8247	2.6837
11	50	0.005259108	2.4708	2.2727	36.15342929	2.4851	2.2723	133.9603989	2.4764	2.2789
12	52	0.011683378	2.5315	2.6733	75.30961062	2.5536	2.6689	178.2038778	2.5399	2.6776
13	54	0.005444453	2.6906	2.5572	35.18922524	2.7015	2.5576	165.0486596	2.6947	2.5657

Ins	Portfolio size	LP_Metric			Goal Programming			Goal Attainment		
		Main	Obj1	Obj2	Main	Obj1	Obj2	Main	Obj1	Obj2
14	56	0.00648869	2.5782	2.8103	49.92275982	2.6010	2.8054	173.9550657	2.5881	2.8140
15	58	0.006041708	2.7195	3.1045	40.57523956	2.7337	3.1035	135.3305154	2.7253	3.1102
16	60	0.005858644	2.7240	2.8315	37.83623587	2.7358	2.8316	172.5404697	2.7286	2.8401
17	62	0.008056213	2.6774	3.1731	51.69049855	2.6922	3.1698	152.1663934	2.6834	3.1773
18	64	0.006323268	2.7340	2.8880	37.83604814	2.7434	2.8899	139.067993	2.7371	2.8967
19	66	0.007990373	2.5937	2.7633	51.4785689	2.6092	2.7611	158.5154515	2.5997	2.7689
20	68	0.008230933	2.6817	2.8329	50.78384278	2.6947	2.8320	163.2517289	2.6864	2.8400
21	70	0.006513638	2.6390	2.7080	42.74287286	2.6530	2.7071	158.0485382	2.6444	2.7149
22	72	0.007857848	2.6316	2.8665	51.78619901	2.6482	2.8640	159.9245101	2.6382	2.8718
23	74	0.00661007	2.6886	2.8269	42.39280907	2.7015	2.8265	156.1732021	2.6935	2.8342
24	76	0.00705364	2.6475	2.8542	46.8357063	2.6630	2.8522	161.6176615	2.6537	2.8602
25	78	0.007167668	2.6829	2.9352	45.98576334	2.6967	2.9340	152.5045721	2.6882	2.9415
26	80	0.006788186	2.6728	2.8296	43.58685778	2.6861	2.8289	156.923984	2.6779	2.8366
27	82	0.007592123	2.6554	2.8936	48.87820524	2.6700	2.8917	157.8434716	2.6611	2.8995
28	84	0.007032269	2.6792	2.8449	44.60091417	2.6922	2.8443	154.8304986	2.6841	2.8519
29	86	0.007261154	2.6456	2.8229	47.12990347	2.6602	2.8214	158.2434451	2.6513	2.8291
30	88	0.007393152	2.6664	2.8604	47.36537673	2.6805	2.8591	157.5387755	2.6718	2.8669

Upon perusal, it becomes apparent that discerning a conspicuous disparity in the first objective function's values across different methodologies proves rather elusive. Nonetheless, a broader perspective reveals that the comprehensive benchmark methodology consistently furnishes lower values in this context. Analogously, an analogous observation is deduced from the second objective function, where a semblance of uniformity prevails amongst all methodologies. Notably, definitively asserting the supremacy of a particular methodology is a task fraught with complexity. The numerical values attributed to both the first and second objective functions fail to exhibit substantial divergence, rendering the identification of an optimal approach challenging. As such, recourse to statistical methods to discern trends through the comparison of average values among the assorted methodologies emerges as imperative.

Statistical Comparison of Solution Methods for the Multi-Objective Mathematical Model

This section employs ANOVA (Analysis of Variance) analysis, utilizing a one-way model, to facilitate a comprehensive comparison of the mean performance across three distinct methodologies: 1) Comprehensive Criteria Method, 2) Goal Achievement Method, and 3) Ideal Planning Method. The primary objective is to assess these methodologies from a statistical standpoint. The analysis is founded on the outcomes derived from solving 30 randomly generated numerical instances, and the ensuing results are elucidated herewith.

It is imperative to underscore that, in the context of these analyses, a null hypothesis of equal means is posited. Conversely, an alternative hypothesis of unequal means is considered. This enables a robust evaluation of the average performance across the three methodologies, ultimately discerning potential disparities and trends in their effectiveness.

$$\begin{cases} \mu_1 = \mu_2 = \mu_3 & \textit{null hypothesis} \\ \mu_1 \neq \mu_2 \neq \mu_3 & \textit{alternative hypothesis} \end{cases}$$

Please find the Performance Comparison Based on the First Objective Function, Performance comparison from the perspective of the second objective function and Performance comparison in terms of solution time in the supplementary material file.

Considering the entirety of the analysis provided in the supplementary material file, it becomes apparent that the comprehensive criterion method stands out as the most favorable choice among the three methods. Its consistent and superior performance, both in terms of objective function results and solution times, makes it a compelling option for decision-makers seeking optimal investment strategies within manageable time frames.

The comprehensive criterion method's multifaceted advantage, encompassing accuracy, computational efficiency, and ease of implementation, positions it as a robust solution for addressing complex multi-objective portfolio optimization challenges. While each method has its strengths, the comprehensive criterion method's comprehensive and well-balanced performance makes it a promising candidate for practical applications in investment management and portfolio optimization.

Case study

In this section of the research, a case study is conducted to investigate the problem of stock portfolio selection within the Iran capital market as Tehran stock exchange and Farabourse stock exchange market. As previously highlighted, the primary objective of this study is to ascertain the optimal number of shares and the corresponding optimal investment amounts for each share. It's worth noting that the model's output reports the percentage of investment allocated to different stocks. Given the extensive dataset comprising risk and stock return information, the relevant data is presented in the attached table.

Similar to the numerical examples explored in earlier sections, the case study is tackled employing optimization software on a personal system equipped with a processing power of 32 GHz and an available random memory of 16 GB. A pivotal aspect of addressing this case study, which encompasses a substantial number of stocks, is devising optimal strategies at the leader level for use in the algorithm aimed at resolving the follower-level problem. It's important to acknowledge that considering all possible states for determining the leader's strategies is an impractical endeavor due to the staggering magnitude of potential states, quantified by the relation (1.5983×10147). This astronomical number of strategies, with a time requirement of approximately (5.0683×10139) years for computation at one second per state, is beyond the realm of feasibility. Hence, it becomes imperative to derive a limited set of leader strategies and address the follower model's resolution through the algorithm outlined in the third chapter. However, constraining the selection of leader strategies cannot be arbitrary; a systematic and well-defined approach is essential. To address this need, a systematic repetitive structure based on numerical arrays is formulated to facilitate the strategy selection process. This structured procedure is elucidated as follows.

Please find the process of leader strategies selection in the supplementary material file.

Upon solving the problem iteratively, a graph is constructed to depict the diminishing disparity between the best and worst strategies identified during each iteration. This graph provides valuable insights into the algorithm's progression and convergence towards optimal solutions. As the iterations proceed, the gap between the most favorable and less favorable strategies tends to narrow, indicating that the algorithm is effectively refining its search and honing in on more promising portfolio selections.

The graph showcasing the reduction in the performance gap is a powerful visual representation of the algorithm's effectiveness in systematically improving the quality of strategies over successive iterations. It highlights the algorithm's capacity to iteratively evolve and enhance the selected strategies, ultimately contributing to the identification of superior investment options within the portfolio.

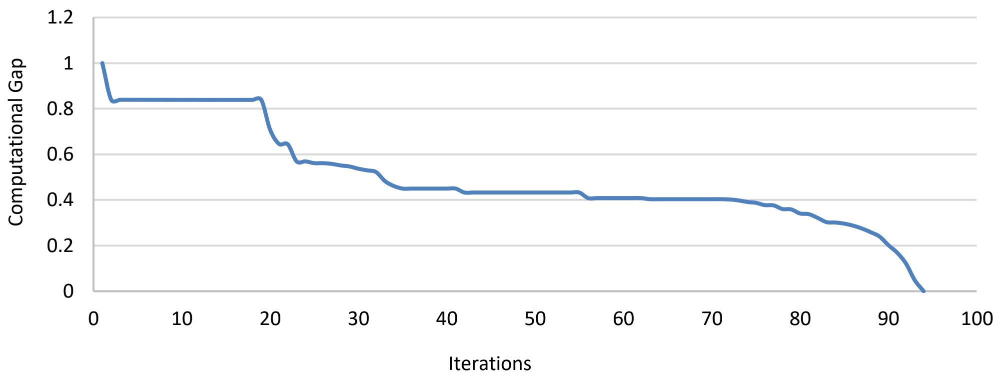

Figure 3 - Computational gap between the best and worst strategies discovered in each iteration

The depicted graph illustrates a significant observation regarding the reduction process within the calculation gap, specifically in the range between 60 and 40 percent. In this interval, the reduction process appears to slow down considerably, signifying the intricate and intricate nature of selecting various strategies. The intricate nature of strategy selection becomes evident as a substantial number of evaluations are required to discern optimal approaches within this particular range.

An interesting phenomenon unfolds during the initial iterations, as the disparity in solution quality is quite pronounced. The variance between the best and worst solutions is notably substantial, surpassing a factor of 100—indicating a stark contrast in quality. However, through successive iterations, this disparity diminishes progressively. In the concluding iterations, a convergence is observed, where the majority of strands converge towards a similar level of quality. Consequently, the gap between the best and worst solutions progressively approaches zero, affirming the algorithm's efficacy in identifying appropriate strategies.

It is evident that the algorithm's performance is influenced by factors such as the number of iterations, strand lengths, and the total number of strands. In this research endeavor, a configuration with 50 strands, 100 iterations, and strand lengths of 100 has been adopted. This configuration yields the final outcomes predicated on the superior leader strategy.

The culmination of this iterative process yields the following combinations for the stock portfolio, reflecting the optimal selections derived from the chosen leader strategy. This set of combinations encapsulates the culmination of the research's efforts, offering a portfolio structure that optimally balances investment quality, risk, and return within the context of Iran capital market.

Indeed, the visual representation illustrates a set of 18 distinct strategies, each contributing to a unique numerical value for the Sharpe function. Notably, the lowest value within this spectrum is attributed to the 12th strategy, denoting an impressively low value of 1.04388. It is worth mentioning that the omission of the other 32 strategies from the diagram serves the purpose of enhancing readability and facilitating effective comparisons.

Consequently, based on the comprehensive analysis and optimization conducted, the culmination of this research provides us with the optimal combination of investment in selected stocks. This culmination reflects a meticulous and strategic approach to stock portfolio selection, offering a tailored blend that aligns with the research objectives and the criteria encapsulated within the mathematical model. The optimal distribution of investment across the chosen stocks serves as a testament to the effectiveness of the proposed methodology and its capacity to deliver informed, data-driven investment decisions.

Absolutely, the rounding of percentages to two decimal places is a common practice and helps simplify the presentation of the investment allocation. It's entirely possible that companies with seemingly identical shares could have subtle differences in a fraction of a percentage point. While these minute variations may not be evident in the rounded percentages, they can still influence the overall portfolio structure and contribute to the optimization of the investment strategy. This attention to detail underscores the meticulous nature of your research and its implications for effective portfolio management.

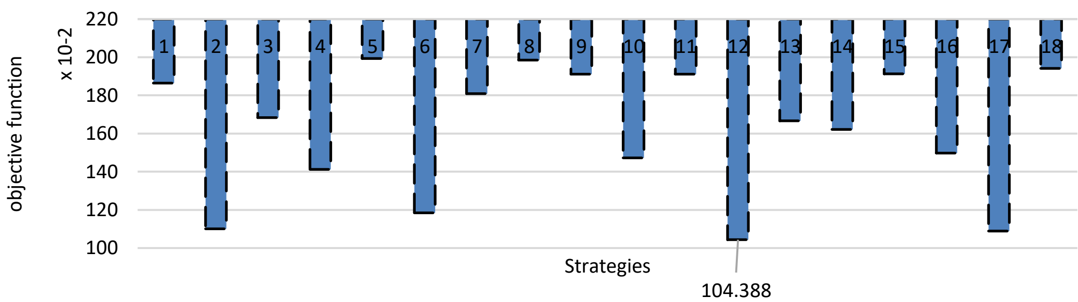

Figure 4- Optimal solutions of the problem in the two-level model

Name	Optimal value	Name	Optimal value	Name	Optimal value

Bzagros۱	0.08	1 Deshimi	0.07	1 Khakmak	0.07
1 Topkish	0.05	1 Deler	0.02	1 Daru	0.09
1 Hfares	0.09	1 Zebita	0.08	1 Davu	0.07
1 Khetrak	0.09	1 Zekesht	0.04	1 Sebhan	0.06
1 Sap	0.09	1 Sepaha	0.04	1 Fsabet	0.06

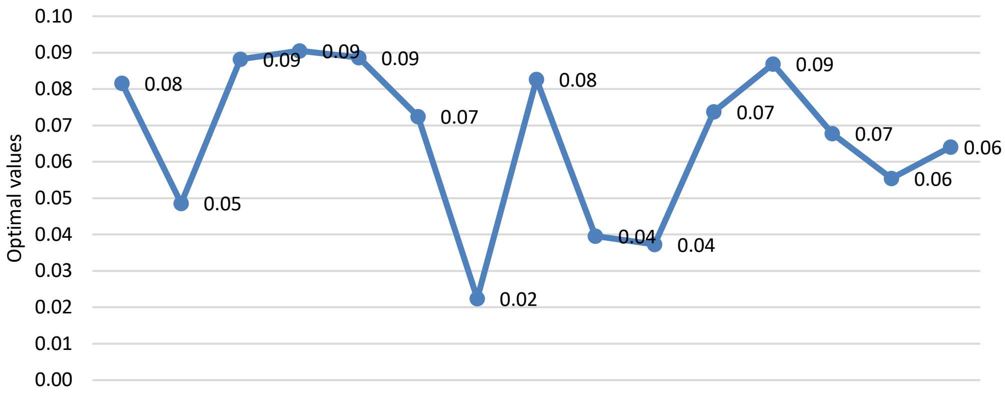

Figure 5- Percentage share of each company from the total available budget

Discussion and conclusion

This research reveals a groundbreaking approach to investment management, emphasizing the pivotal role of prudent decision-making in shaping portfolio outcomes. The optimization model, rooted in innovative algorithms, not only refines stock selection processes but also introduces a societal perspective to enhance investment quality. The strategic focus on determining the optimal number of stocks for investment, coupled with the quantitative assessment using the Sharpe ratio, propels decision-making to new heights.

A notable highlight is the algorithm's efficacy demonstrated in the Iran capital market, showcasing its relevance and effectiveness. The study's real-world applications extend beyond borders, promising insights for global markets. As technology evolves, the marriage of data analytics and ongoing algorithm refinement holds the key to further advancements in investment management. The comprehensive analysis and optimization methods presented offer a pathway to informed decisions, underlining the algorithm's effectiveness in addressing real-world complexities. This research paves the way for strategic portfolio management and capital growth in dynamic market landscapes.

In addition, this research has delved into a critical aspect of investment management, shedding light on intricate decision-making processes that impact portfolio optimization. Prudent investment decisions are of paramount importance, as they can either drive capital growth or lead to its decline. This study provides valuable insights applicable not only to Iran but also to a broader range of countries. At its core, the research has formulated an optimization model encompassing distinct objective functions reflecting the perspectives of two decision-making levels. The first level emphasizes the quality of investment and stock portfolio selection, going beyond conventional criteria to consider societal preferences. This innovative approach can also benefit companies engaged in portfolio management, offering a fresh perspective in their decision-making processes.

A significant focus has been placed on a key challenge in stock portfolio optimization—the determination of the number of selected stocks for investment. By making this a decision variable, the study enhances decision-making, ultimately leading to improved investment outcomes. The use of the Sharpe ratio as a metric for investment quality adds a quantitative dimension to decision-making, providing a valuable tool for evaluating investment choices and optimizing portfolios. An impressive aspect of this research is the development and application of an innovative iterative algorithm, solving complex problems and providing a comprehensive analysis of results. The case study within the Iran capital market demonstrates the algorithm's effectiveness and relevance. Looking forward, this research opens doors to numerous possibilities. Its model's applicability to other markets and regions holds promise for expansion. Integration of real-world data and ongoing algorithm refinement can enhance accuracy and practicality. As technology and data analytics evolve, there's potential for further advancements in investment management, and this research lays a strong foundation for contributing to this dynamic field. The section presenting numerical results thoroughly addresses the challenge of converting the multi-objective problem into a single-objective one, critical for the heuristic algorithm. Three methods—comprehensive criteria, ideal planning, and goal achievement—are rigorously compared, with the comprehensive criteria method emerging as the most effective. The two-level model's performance is meticulously evaluated through 30 numerical examples, revealing intriguing patterns based on varying the number of selected companies (N). For example, when N is set to 2, the focus is on Daru 1 and Sep 1 stocks, carrying notable investment risk. Expanding to N=3 maintains the prominence of Hafars 1, Daru 1, and Sep 1 stocks. Further increases in N lead to diversified portfolios and improved investment landscape. Addressing capital management complexities in the Iran capital market, the solution designates the highest share percentages to Hafars 1, Khatrak 1, and Sep 1 companies. These findings underscore the efficacy of the proposed algorithm in addressing real-world investment challenges in Iran capital market. The comprehensive analysis and optimization offer a path to informed investment decisions, enhancing portfolio management and capital growth potential.

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