



Predicting the Volatility Spillover in the Tehran Stock Exchange Market with Heston Switching Copula Model

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ABSTRACT

This article investigates the effects of global markets of gold, currency, metals, crude oil, and digital currency on the Tehran Stock Exchange and analyzes the spillover risk of these markets on the Tehran Stock Exchange. The research utilizes a combination of marginal models and copula models. The marginal models used include the stochastic volatility model, the Markov switching model, and the. The copula models include the normal, T Student, Clayton, Frank, and Gumbel model. The data used in this study are the daily values of the mentioned markets from December 2011 to January 2023. The results show that the aforementioned global markets have influence the Tehran Stock Exchange and the fluctuations occurring in these markets cause a reaction in the Tehran Stock Exchange index. The extent of this influence is also calculated using conditional Value at Risk (VaR). Considering the limitations of the stochastic volatility model and the Markov switching model, the Heston switching model is suggested as a marginal model, and the combination of this model with copula models can provide appropriate results.

Keywords: Heston switching copula model, Markov switching copula Model, Copula, Spillover.



1. Introduction

In 1929, a massive recession hit the American market. It was the first and largest crisis to ever occur in the modern financial markets of the United States. This crisis, considering the conditions of the financial markets during those years, mainly remained within the US financial markets, but the spread of the crisis in various markets in the country occurred to a significant extent. From the mid-1990s, financial crises in various markets around the world intensified. The difference between these financial crises and the crises of the previous years was their contagiousness to other markets in other parts of the world. Crises such as the 1994 Mexican financial crisis that affected Latin American markets, the 1997 Thailand crisis that affected Southeast Asia, and ultimately the 2007 US crisis that started from the housing market and quickly affected other markets in this country and other countries around the world, becoming a global economic crisis. (Ji et al., 2020)

These experiences have made researchers interested in examining the impact of markets on each other. This impact and risk transfer between markets occurs in two ways. In the first case, the markets do not compete with each other for capital but affect each other (technically they have parallel impacts). For example, one can refer to the impact of the global metals market on the Stock Exchange (Saghafi, 2018). The second model of market influence on each other is the competitive influence of markets on each other. In this case, markets compete with each other to attract capital (Forbes, 2012). The attractiveness of markets, the rate of profit, and the inherent risk in each market can entice capital to participate in these markets. For example, one can refer to the performance of the Stock Exchange and the digital currency market in attracting capital.

The impact of global markets on each other and Stock Exchanges has been of interest to researchers. Gold, due to its nature as a safe asset (Amir et al., 2018) plays a competitive role with the Stock Exchange. This role also intensifies during crises. Investors during crises such as the COVID-19 health crisis moved towards gold as a safe asset (Olausen et al., 2021). Another global market that interacts with the Stock Exchange is the crude oil market. Crude oil is considered the driving engine of the world, so it can be said that the economy all over the world is dependent on this market. Ran Ma (2019) stated that

oil price is an important element for the economy, changes and instability in oil prices can have a detrimental effect on stock prices in Stock Exchanges.

Among other markets that have a close relationship with the Stock Exchange is the currency market. Changes in the exchange rate affect the Stock Exchange and "economic firms, depending on whether they are inclined toward exports or imports.". The foreign exchange earnings of these firms affect their Stock Exchange. On the other hand, in Iran, due to the existing inflation, the dollar is considered a capital commodity and is used to cope with inflation, similar to the function that gold has in the capital market and is considered a safe capital. Research conducted by Tian et al., (2020) and also Utama and Pouryandani (2020) showed that the exchange rate has a significant effect on the Stock Exchange. In Iran's market, the exchange rate has both a trade link and a competitive link with the Stock Exchange.

The digital currency market is also in connection and competition with the Stock Exchange. The inclusion of digital currencies in individuals' asset portfolios can increase diversity and reduce risk (Kwon, 2020; Musialkowska, 2020). On the other hand, some researchers argue that digital currencies can serve as a safe asset in crisis-ridden countries or those with turbulent economies (Musialkowska, 2020). Another market that is connected to the Stock Exchange is the metals market. Given that approximately 67% of Tehran's Stock Exchange industries are dependent on the price of global commodities such as metals (Saghafi, 2018), examining the relationship between these two markets is significant. Since the outbreak of COVID-19, the contagiousness of risk in stock and commodity markets has also increased (Maitra and Dawar, 2019). In this research, we will examine the impact of the five mentioned markets on the Tehran stock market, the important point is that other markets also have impacts on this market, but according to previous research, these five markets have the most impact on the exchange markets. Of course, the special conditions of local stock exchanges and the impact of other markets on them are also important. In this study, we aim to investigate the impact of the aforementioned global markets on the Tehran Stock Exchange Index and to determine the extent of contagion from these markets to the Tehran Stock Exchange. The purpose of this article is to combine the marginal models of Switching

models with copula models and finally introduce the Heston Switching Copula model. Despite their many strengths, Markov Switching models suffer from weaknesses. For example, we can point to the weakness of these models in short time intervals. On the other hand, one of the basic assumptions of this model is the normal distribution of the input data, which is not possible in reality and can be accepted with a suitable approximation in large time intervals. It is also assumed in this model that the model outputs are independent from each other which this assumption is not correct. The Markov Switching model will perform poorly in cluster volatility. Many of these weaknesses will be overcome by the Heston Switching model. This model shows good performance in short time intervals. The problem of cluster volatilities can be solved in this model due to the existence of stochastic differential equations. On the other hand, this model does not require that the data follow a certain distribution (Ganti 2021). These features create high flexibility for Heston models, which can lead to better performance. So far, researchers have been interested in using the Markov switching model as a marginal model and combining it with various copula models and many researches have been done in this regard. In this research, we will use the Heston Switching model as the marginal model, considering the Markov Switching model's weaknesses and also to solve these weaknesses. Finally, we introduce the Heston Switching Copula model by combining the Heston Switching model and the copula models.

2. Literature review

Before the mid-1950s, there were few studies on the relationship between the Stock Exchange and other markets. From the 1960s onwards, researchers focused on optimizing stock portfolios and diversification, which led them to pay attention to the relationship between markets and made contagion issues more prominent. Studying the relationships and contagion between markets is important from several perspectives. For instance, examining this relationship is crucial for regulation and legislation (Brunnermeier et al., 2009; Kashyap et al., 2009), creating investment portfolios (Sahalia and Hurd, 2016), hedging risk (Balcilar et al., 2016), examining market behavioral patterns (Bartram, 2007), and the degree of risk

contagion (Zhang, 2015; Qiang Ji, 2020). Contagion models can be used for these purposes.

Over the years, regression models played a significant role in examining the impact of markets on each other. In 1959, copula models were introduced by Scalar. These models attracted researchers' attention due to the weaknesses of regression models. These models, due to their extensive flexibility, have been used in many studies about the relationship between markets. Among the researchers who worked on copula models, we can mention the studies of Costinot et al., (2000), Patton (2006), and Canela (2006) (Bargkar and Sohrabi, 2020). These researchers used copula models to examine the relationship and impact of markets on each other. Copula functions have become popular among researchers for two main reasons; first, they can be used to study the dependency structure between variables in a non-parametric way, and second, they can be the starting point for constructing multivariate distribution functions (Keshavarz Haddad and Heyrani, 2016).

Ning (2010) used copula functions to examine the relationship between the exchange rates of the US dollar and five other countries. Smith et al. (2012), by using copula functions, studied the data of European financial markets and global silver and gold markets. Dibmen et al. (2013) examined the mutual effects of European countries' Stock Exchanges with the help of copula functions. The risk contagion across different markets has been extensively studied by utilizing copula models. For instance, Zhang and colleagues (2014) were able to solve the challenge of quantifying contagion. In their paper, they estimated the value at risk and expected shortfall for indices of international markets in the United States, Japan, China, Germany, France, the United Kingdom, Brazil, Russia, Italy, and India from 2006 to 2013 using copula models and a GARCH approach. The models of Dibmen and Zhang did not allow for the examination of different regimes. In markets that face regime shifts, we require models that can account for these changes. Markov switching models, and their integration with copula models, can facilitate consideration of regime changes in different markets (Tansuchat et al., 2017). From that period onward, the use of Markov Switching Copula models gained popularity (Bargkar and Sohrabi, 2020).

Selmi et al., (2019) investigated the impacts of digital currency and gold markets using a Markov switching copula model and considered the possibility

of substituting them in an investment portfolio. Tiwari et al., (2020) examined the contagion risk between the crude oil market and the economies of the G7 countries, also utilizing a Markov switching copula model to estimate contagion. Mwamba and Mwambi (2021) examined the relationship between the BRICS markets and the price of crude oil. Their model of choice was the Markov switching copula, which they used to investigate contagion through the value-at-risk index. A strong point of their research was the examination of the COVID-19 pandemic period. Their model successfully displays the regime change conditions resulting from this event.

On the other hand, we also introduce an index to measure the level of contagion. The conditional value at risk can be a suitable index for investigating contagion (Liu, Ji, and Fan, 2017; Reboredo and Ugolini, 2015). To investigate contagion between markets such as stock exchanges (Yao and Li, 2023), commodity markets and interest rates (Chang, 2023), and the impact of the COVID-19 disease on various markets using a Markov switching model (Bouteska, 2023), we can benefit from the conditional value-at-risk index. Combining this index with switching copula models can yield suitable results.

Most research in this field in recent years has utilized the Markov switching copula model and investigated contagion using this model. However, other models like Heston switching, which exhibits an appropriate performance in regime changes and the volatility within each regime, have not been investigated. Combining this model with a copula function can create a practical and suitable model. In most previous studies, only the value at risk was used to quantify contagion, but in this study, in addition to using this index, the conditional value at risk will also be used.

Methodology

This research examines the combination of marginal models, such as the stochastic volatility model, Markov switching model, and Heston switching model, with Copula models. The present research is included in the basic research category in terms of introducing the Heston switching Copula model, but it is also included in the category of applied research due to the application of this research results to create diverse portfolios. Also, this research is placed in the correlation research category according to the research

purpose to investigate the global market's impact on the Tehran stock market. On the other hand, the present study is quantitative research according to the type of data. The basic hypothesis of this research is that the Heston switching Copula model performance will be more suitable than the Markov switching Copula model in predicting transitivity. For this purpose, we reviewed the data obtained from global markets as well as the Tehran Stock Exchange index from December 2011 to January 2023. Finally, global markets transitivity to this market is checked according to Copula switching models.

Copula functions enable us to combine marginal distributions that have different distributions and create a multivariate function. These functions were introduced by Scalar in 1959. Copula functions are a powerful tool for investigating dependency structures. In general, Copula functions are a flexible mathematical technique that can convert marginal cumulative probability functions into a multivariate probability function. This feature leads to the great flexibility of these functions. Copula functions are based on non-linear relationships and dependence between variables and link joint distribution and marginal functions. These functions can use marginal probability functions with any distribution and generate multivariate distribution. More importantly, the Copula function is capable of explaining the variables' correlation changes in different parts of the joint probability distribution. This feature is not observed in other simulation methods of stochastic variables.

Different types of Copulas can be generally divided into parametric and non-parametric categories. Elliptical and Archimedean are among these categories. These models join the marginal distributions to each other and create a multivariate distribution. Both categories have the same structure except for their sequence dependency.

Suppose we have two stochastic variables X_1, X_2 and F represents the joint distribution function for variables X_1, X_2 with marginal distribution functions F_1, F_2 . will be a Copula function C as follows

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \quad (1)$$

where C is the Copula parameter, $F_i(x_i)$ represent marginal functions. In this equation, $F_i(x_i)$ can be different distributions. In this research, there are three

distributions for these functions. Consider the marginal distribution to be a stochastic volatility model, this model is derived from Ball and Torous (1999) They sought to present their stochastic volatility model under a simple extension of discrete-time diffusion models and finally presented the following equation:

$$dr_t = (a + br_t)dt + \sigma_t r_t^\gamma dB_t \quad (2)$$

Where γ represents diffusion, σ is volatility and a and b represent constant parameters. Also, r_t is the instantaneous rate and dB_t represent a standard Brownian process. Suppose we are faced with only two distributions and the time series $x_t = (x_{1t}, x_{2t}), t = 1, 2, \dots$ is a two-dimensional vector, and the two marginal functions are stochastic volatility models. Then our Copula function will be as follows:

$$SV(X_t|a_i, b_i, \sigma_{it}) = C(F_1(X_{1t}|a_1, b_1, \sigma_{1t}), F_2(X_{2t}|a_2, b_2, \sigma_{2t})) \quad (3)$$

Where C , Copula parameter, $F_i(X_{it})$ represents marginal functions. In this equation, $F_i(X_{it})$ has a stochastic volatility distribution. Equation 3 can be written as follows (Filho, 2012):

$$sv(X_{it}) = c_i(F_1(X_{1t}), F_2(X_{2t})) \prod_{i=1}^2 f_i(x_i) \quad (4)$$

Where f_i represents marginal distribution density function and c_i is the copula density function of the twice pair. This function value is:

$$c_i(F_1(X_{1t}), F_2(X_{2t})) = \frac{\partial^n C(F_1(X_{1t}), F_2(X_{2t}))}{\partial(F_1(X_{1t})) \partial(F_2(X_{2t}))} \quad (5)$$

Now, suppose that our marginal functions follow switching distributions such as Markov switching and Heston switching distributions. Ganot's study (1958) is the first study on switching discussion which deals with regression along with switching. Markov switching models allow the economy to be in a regime with any number of constraints and at any point in time. The regime has a great influence on the dynamic behavior of the series. In Markov switching models, the regime can occur at any point. This is an important point in the discussion of determining the parameter that transfers probability from regime i to j at time t is defined as $P_{ij} = Pr(S_t = i | S_{t-1} = j)$.

$$P = Pr(S_t = 1 | S_{t-1} = 1) \quad (6)$$

$$q = Pr(S_t = 2 | S_{t-1} = 2)$$

A Markov switching model, as represented in Equation (7) allows the volatility to switch between regimes. That is, it allows σ to change and volatile between two regimes

$$\Delta r_t = a_i + b_i r_{t-1} + \sigma_i r_{t-1}^\gamma \varepsilon_t \quad (7)$$

where $i \in \{1, 2\}$ is a regime input at time t . Also, γ represents the diffusion parameter, σ is the volatility parameter, and a & b represent constant parameters in each regime. Here, ε_t represents a white noise process which is independent of r_t . All ε_t are independent and have the same distribution with mean zero and variance σ^2 . According to the above specifications, the Markov switching model has many strengths, including high flexibility against volatility, considering jumps that lead to regime change, and also considering different regimes, but it also suffers from limitations. This model usually does not perform well in short intervals and when we face cluster fluctuations or non-normality of the data. Considering the limitations of the Markov switching model, we introduce the Heston switching model here. Some of the above limitations are solved in this model.

The Heston model is one of the most famous and widely used stochastic volatility models introduced by Heston in 1993. In this model, the asset price follows the Brownian motion. In the following years, this model was revised and strengthened by Cox, Ingersoll, and Ross. Now, consider $S = (S_t)$ to be a stochastic process in our probability space. This space can exist in any of the markets under our research. Consider that $v = (v_t)$ is another stochastic process that models the instantaneous variance of S . It is important to note that all the parameters of the stochastic volatility process v and the correlation coefficient between each market index and its instantaneous variance v are dependent on X . In simpler words, each regime's parameters depend on that regime. Consider Heston's baseline model:

$$dS_t = \mu S_t + S_t \sqrt{v_t} dZ$$

$$dv_t = k(\theta - v_t)dt + \sigma \sqrt{v_t} dB$$

where $\theta, k, \sigma > 0$ are our constant parameters. Z & B are two independent Brownian processes. Now we have:

$$\frac{dS_t}{S_t} = \mu + \sqrt{v_t}dZ \quad (8)$$

We also have:

$$\mu = rdt$$

where r is the risk-free interest rate, so we have:

$$\frac{dS_t}{S_t} = rdt + \sqrt{v_t}dZ \quad (9)$$

The transfer function between the two regimes follows the Poisson distribution. The transfer possibilities in this model are as follows:

$$P = Pr(S_t = 1 | S_{t-1} = 1)$$

$$q = Pr(S_t = 2 | S_{t-1} = 2)$$

For each regime we will have:

$$dv_i = k_i(\theta_i - v_i)dt + \sigma_i\sqrt{v_i}dB_i \quad (10)$$

$$dZ_i dB_i = \rho_i dt \quad (11)$$

$$dZ_i dZ_j = dB_i dB_j = 0 \quad (12)$$

In the above equations, ρ_i is the correlation between two Brownian processes, θ is the long-term variance of volatility, k is the rate of return to the average volatility, and r is the interest rate. Now consider that we have only two marginal switching distributions and the time series $x_t = (x_{1t}, x_{2t}), t = 1, 2, \dots$ is a two-dimensional vector. In this case, the switching copula model is defined as follows:

$$RS(X_t | S_t) = C_t^{S_t}(F_1(x_{1t} | S_{1t}), F_2(x_{2t} | S_{2t})) \quad (13)$$

In $C_t^{S_t}$, copula process, $F_i(x_{it})$ represents marginal functions. In this equation, $F_i(x_{it})$ can have Markov switching or Heston switching distributions. Patton (2006) presented an equation for copula time-varying functions. These equations were derived from scalar equations. Consider that C can be considered as a simple ARMA function. Then we will have:

$$C_t = H(\alpha + \beta C_{t-1} + \gamma \Psi_t) \quad (14)$$

Where, $H(\cdot)$ is an exponential function, α, β , and γ are constant parameters and Ψ_t represents a pressure variable. In this equation, there is a possibility of regression to the mean. Also, the pressure variable Ψ_t is different for elliptical copula or Archimedean copula. Now, consider a copula switching regime function. Then Patton's equation will be as follows:

$$C_t^{S_t} = H(a^{S_t} + \beta C_{t-1}^{S_t} + \gamma \Psi_t) \quad (15)$$

Unlike the first case, in this equation, the copula will behave differently in each regime. Therefore, each regime, may represent a different copula than the elliptical or Archimedean copula based on the pressure variable. Therefore, we will not necessarily have a copula for the entire data. In both Markov switching and Heston switching models, we have a probability transfer matrix in which the transfer probability p_{ij} is calculated as follows:

$$p_{ij} = Pr(S_t = j | S_{t-1} = i) \quad (16)$$

Finally, it can be concluded that the regimes in each of the marginal distributions are selected in pairs based on the determined index after estimating the parameter in the switching copula. This index can be the Akaike index, which works based on the amount of missing information. The maximum likelihood estimation (MLE) method can be used to estimate these model parameters. So, we will have:

$$\log Rx(X_t | S_t)$$

$$= \sum_{t=1}^2 \log(C_t^{S_t}(F_1(X_{1t} | a_{1t}, b_{1t}, \sigma_{1t}), F_2(X_{2t} | a_{2t}, b_{2t}, \sigma_{2t}))$$

$$\times \prod_{i=1}^2 f_{it}(X_{it} | a_{it}, b_{it}, \sigma_{it})) \quad (17)$$

The parameters estimation will be complicated by (17), but the two-step maximum likelihood estimation method can be used since this equation is a separable equation. Therefore, Eq (17) can be rewritten as follows:

$$\log Rx(X_t | S_t) = \sum_{t=1}^2 \log(C_t^{S_t}(F_1(X_{1t} | \vartheta_{1t}), F_2(X_{2t} | \vartheta_{2t}))$$

$$\times \prod_{i=1}^2 f_{it}(X_{it} | \vartheta_{it})) =$$

$$\sum_{t=1}^2 \log f_{1t}(X_{1t}|\vartheta_1) + \sum_{t=1}^2 \log f_{2t}(X_{2t}|\vartheta_2) + \sum_{i=1}^2 \sum_{t=1}^2 \log C_t^{s_i}(U_{it}|\vartheta_{it}) \quad (18)$$

Where $U_i = F_i(X_i)$, ϑ_i represents the marginal model parameters such as a_i, b_i, σ_{it} . You can start from the first and the second functions, the maximum likelihood functions of marginal functions, then go to the next step and estimate the third function, the copula function. Due to the existence of two regimes in Eq (18), we are expected to face two different copulas in calculating the third part of the equation. To estimate each parameter, you can estimate the parameter in a special way in each regime. Finally, it can be stated that in the first step, each marginal model parameter should be estimated based on the maximum likelihood estimation method. Then the copula parameter should be estimated in the second step and each regime.

Results

In this article, we will examine 6 markets which these markets are Tehran Stock Exchange, the crude oil market, the currency market (USD-Rial exchange rate), the global steel market, the global gold market, and the digital currency market (Bitcoin). After examining these markets and fitting the models of Stochastic volatility model, Markov switching, and Heston switching to these markets, we will examine

the contagion of fluctuations in the Tehran Stock Exchange from other markets. The data for this research spans from December 2011 to January 2023. Since the Tehran Stock Exchange is closed on Thursdays and Fridays, the data have been selected on common working days, and this point has been taken into account in the analysis of these markets with the Tehran Stock Exchange. We analyze the data and provide a statistical description of each market

Now, after the statistical analysis of the mentioned markets, we will analyze the survivability of these markets. The lack of stability in the above markets can cause problems such as false regression. Dickey-Fuller and Phillips-Perron tests can be used to check the significance. These two tests are very similar to each other and their results are usually the same. Therefore, we will use the Dickey-Fuller test to check the significance. we can conclude that the existence of unit root is not rejected in all the above markets. to eliminate the root of unity, the logarithm of the differential series can be used.

According to the Dickey Fuller test, it can be concluded that the null hypothesis, which indicates the existence of a unit root, is not rejected and there will be a unit root. In this test we are looking to check the model stationary. Differential series logarithm can be used to eliminate the unit root. Now we will examine autocorrelation at different levels of data. Then we will have:

Table 1: Statistical description of markets

	Num	Mean	Median	Min.	Max.	Std. Dev.	Skew.	Kurt.	Jarque-Bera
Tehran Exchange	۲۳۸۱	476538.7	95929.50	38602.60	2078512.	584002.6	1.031752	2.326844	467.3892
Gold	۳۲۹۳	1477.687	1395.950	1051.740	2061.220	251.4223	0.313687	1.688009	290.1844
Dollar	۳۱۶۷	96958.66	37590.00	13350.00	416150.0	95500.92	1.199776	3.022625	759.8634
Oil	۳۳۳۳	69.78408	65.07000	12.82000	124.1400	23.22490	0.156187	1.850327	197.1089
Steel	۲۴۸۴	755.5781	650.0000	364.0000	1940.000	322.9080	1.750579	5.623865	1981.277
Bit	۳۰۳۰	11050.27	4971.905	68.43100	67566.83	15577.78	1.748379	5.052382	2417.990

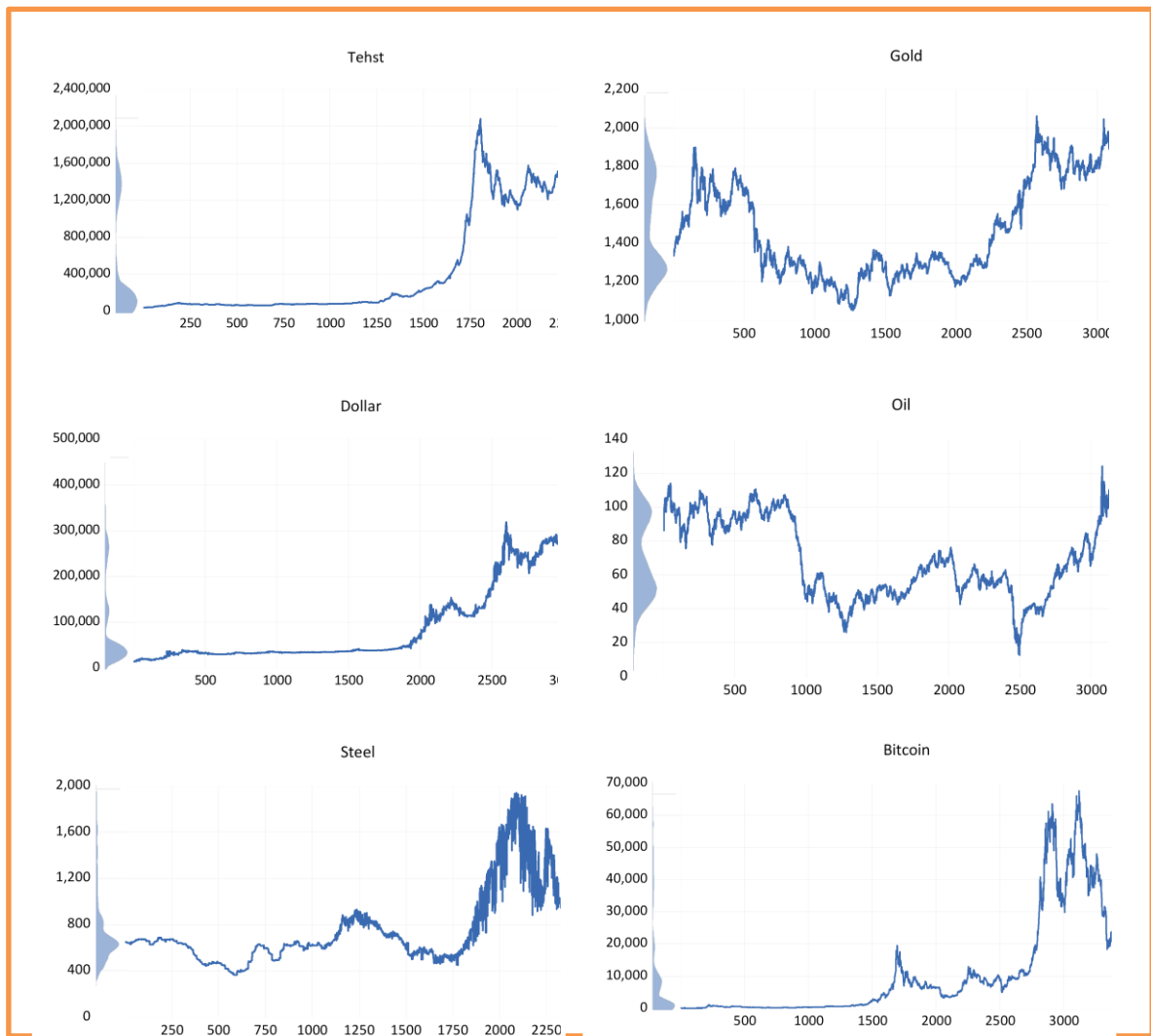


Diagram 1: Distribution diagram of markets

Table 2: The result of unit root tests

Dickey Fuller Test		t-Statistic	Prob.	Augmented Dickey-Fuller test statistic
Tehran Exchange	1% level	-3.432926	0.9552	-0.024493
	5% level	-2.862564	0.9552	-0.024493
	10% level	-2.567360	0.9552	-0.024493
Oil	1% level	-3.432125	0.3319	-1.901499
	5% level	-2.862209	0.3319	-1.901499
	10% level	-2.567170	0.3319	-1.901499
Bitcoin	1% level	-3.432016	0.5481	-1.471732
	5% level	-2.862161	0.5481	-1.471732
	10% level	-2.567144	0.5481	-1.471732
Steel	1% level	-3.432800	0.5154	-1.535836

Dickey Fuller Test		t-Statistic	Prob.	Augmented Dickey-Fuller test statistic
	5% level	-2.862508	0.5154	-1.535836
	10% level	-2.567331	0.5154	-1.535836
Dollar	1% level	-3.432232	1.0000	3.430338
	5% level	-2.862257	1.0000	3.430338
	10% level	-2.567196	1.0000	3.430338
Gold	1% level	-3.432148	0.5963	-1.374572
	5% level	-2.862220	0.5963	-1.374572
	10% level	-2.567176	0.5963	-1.374572

Table 3: The result of the autocorrelation test in Tehran Stock Exchange

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
***	***	1	0.362	0.362	313.09	0.000
*		2	0.110	-0.024	342.14	0.000
*	*	3	0.174	0.163	414.12	0.000
*		4	0.118	0.004	447.43	0.000
*		5	0.091	0.050	467.31	0.000
*		6	0.103	0.040	492.42	0.000
*		7	0.109	0.052	520.87	0.000
		8	0.037	-0.041	524.14	0.000
		9	0.042	0.027	528.44	0.000
*		10	0.081	0.037	543.94	0.000
		11	0.068	0.023	555.12	0.000
*	*	12	0.112	0.081	584.98	0.000
*		13	0.109	0.027	613.32	0.000
*		14	0.094	0.037	634.32	0.000
		15	0.051	-0.023	640.55	0.000
		16	0.070	0.037	652.22	0.000
		17	0.047	-0.026	657.55	0.000
		18	-0.024	-0.060	658.90	0.000
		19	-0.015	-0.021	659.43	0.000
		20	0.058	0.061	667.47	0.000
		21	0.053	0.016	674.31	0.000
		22	0.050	0.032	680.25	0.000
		23	0.046	-0.002	685.28	0.000

table 3. So, it can be concluded that small jumps and big jumps follow small and big jumps, respectively which fully confirms the hypothesis of autocorrelation. Now, according to the above evidence, we will perform the ARCH test and have:

The null assumption in this test is that we do not have an arch. According to the table, this assumption is rejected and we are facing the arch phenomenon and variance heterogeneity. We will examine and estimate the parameters of two switching models in this group

of data after determining the GARCH model coefficients.

After defining the statistical description of each of the markets as well as defining the limitations of the data, we will examine the marginal models in each of the above markets. As mentioned, to standardize and homogenize the data in all six markets, we use the logarithm of the differential series. In each of the above six markets, we fit the three investigated models, the Heston switching model, the Markov switching model, and a sample of We fit Stochastic

volatility models to each of the six markets and specify their parameters, including the transfer matrix in the switching models. The results are shown in Table 6

After fitting the above models and estimating the parameters of each of the above models in the data space of the specified markets, it is necessary to check

the above models and determine the most suitable model for fitting in each of the above markets. To check the optimal model in each market, we use three indices MAE, RMSE, and R^2 . Our primary index will be to measure the coefficient of determination, and then the MAE and RMSE indices will be examined.

Table 4: Arch test result of Tehran Stock Exchange

	F-statistic	Prob.
Arch Test	115.0594	0.0000

Table 5: Estimation of Conditional Variance Equation

Tehran Stock Exchange	Conditional Variance Equation
	$c = 0.002645$ $RESID(-1)^2 = 0.184354$ $GARCH(-1) = 0.824031$ <hr/> $S.E. of regression = 0.467503$ $GARCH = C(1) + C(2) * RESID(-1)^2 + C(3) * GARCH(-1)$

Table 6: Estimated parameters of the models

	Heston Switching Model			Markov Switching Model			Stochastic Volatility Model
	Regime 1	Regime 2	Trans. Matrix	Regime 1	Regime 2	Trans. Matrix	
Tehran Exchange	$k = 0.0246$ $\theta = 0.3279$ $v = 0.0006$	$k = -0.0001$ $\theta = 1.0751$ $v = 0.0005$	$\begin{bmatrix} 0.4169 & 0.5831 \\ 0.5852 & 0.4148 \end{bmatrix}$	$\alpha = 0.0018$ $\beta = -0.0583$ $\sigma = 0.6245$	$\alpha = 0.0060$ $\beta = -0.0555$ $\sigma = 0.6118$	$\begin{bmatrix} 0.5720 & 0.4280 \\ 0.4117 & 0.5883 \end{bmatrix}$	$\alpha = 0.0066$ $\beta = -0.1937$ $\sigma = 0.3814$
Gold	$k = 0.0018$ $\theta = 0.3242$ $v = 0.0048$	$k = 0.0028$ $\theta = 0.3213$ $v = 0.0034$	$\begin{bmatrix} 0.3160 & 0.6840 \\ 0.6717 & 0.3283 \end{bmatrix}$	$\alpha = -0.0034$ $\beta = -0.0918$ $\sigma = 0.8429$	$\alpha = 0.0049$ $\beta = -0.1075$ $\sigma = 0.8557$	$\begin{bmatrix} 0.5115 & 0.4885 \\ 0.4897 & 0.5103 \end{bmatrix}$	$\alpha = 0.0008$ $\beta = -0.1968$ $\sigma = 0.3608$
Dollar	$k = 0.0061$ $\theta = 6.9591$ $v = 0.0166$	$k = 0.0058$ $\theta = 5.3734$ $v = 0.0093$	$\begin{bmatrix} 0.3035 & 0.6965 \\ 0.6449 & 0.3551 \end{bmatrix}$	$\alpha = -0.0131$ $\beta = -0.1661$ $\sigma = 1.7486$	$\alpha = 0.0243$ $\beta = -0.1401$ $\sigma = 1.5522$	$\begin{bmatrix} 0.4503 & 0.5497 \\ 0.5533 & 0.4467 \end{bmatrix}$	$\alpha = 0.0209$ $\beta = -0.4429$ $\sigma = 1.5318$
Oil	$k = 0.0061$ $\theta = 2.9831$ $v = 0.0017$	$k = 0.0069$ $\theta = 1.7039$ $v = 0.0417$	$\begin{bmatrix} 0.3148 & 0.6852 \\ 0.6612 & 0.3388 \end{bmatrix}$	$\alpha = -0.0098$ $\beta = -0.1168$ $\sigma = 0.9883$	$\alpha = 0.0115$ $\beta = -0.1168$ $\sigma = 1.0193$	$\begin{bmatrix} 0.4785 & 0.5215 \\ 0.5110 & 0.4890 \end{bmatrix}$	$\alpha = -0.0008$ $\beta = -0.2327$ $\sigma = 1.0783$
Steel	$k = 0.0030$ $\theta = 30.9943$ $v = 0.0037$	$k = 0.0059$ $\theta = 20.5422$ $v = 0.0038$	$\begin{bmatrix} 0.2320 & 0.7680 \\ 0.4849 & 0.5151 \end{bmatrix}$	$\alpha = -0.0525$ $\beta = -0.2007$ $\sigma = 2.0887$	$\alpha = 0.0308$ $\beta = -0.1574$ $\sigma = 2.3854$	$\begin{bmatrix} 0.3916 & 0.6084 \\ 0.4479 & 0.5521 \end{bmatrix}$	$\alpha = 0.0023$ $\beta = -0.7530$ $\sigma = 3.1747$
Bitcoin	$k = 0.0354$ $\theta = 8.1260$ $v = 0.1146$	$k = 0.025$ $\theta = 7.3236$ $v = 0.1027$	$\begin{bmatrix} 0.3187 & 0.6813 \\ 0.6740 & 0.3260 \end{bmatrix}$	$\alpha = -0.0097$ $\beta = -0.0925$ $\sigma = 0.8976$	$\alpha = 0.0210$ $\beta = -0.0917$ $\sigma = 0.8698$	$\begin{bmatrix} 0.4963 & 0.5037 \\ 0.5051 & 0.4949 \end{bmatrix}$	$\alpha = 0.0119$ $\beta = -0.1945$ $\sigma = 1.6268$

Table 7: Comparison of models

		Stochastic volatility Model	Markov Switching Model	Heston Switching Model
Tehran Exchange	MAE	0.7918	0.7637	0.7309
	RMSE	0.4848	0.4784	0.4897
	R^2	0.3747	0.5000	0.9976
Gold	MAE	0.4790	0.2771	0.2767
	RMSE	0.5071	0.1307	0.4227
	R^2	0.0127	0.9907	0.8599

		Stochastic volatility Model	Markov Switching Model	Heston Switching Model
Dollar	MAE	۱,۳۹۰۱	۰,۷۴۸۱	۰,۶۸۴۰
	RMSE	۱۲۱,۷۰۷۰	۱۰۰,۳۱۰۱	۸۸,۲۰۷۹
	R ²	۰,۰۹۷۰	۰,۶۹۰۰	0.899
Oil	MAE	۱,۳۴۰۱	۰,۶۹۳۰	۰,۶۹۲۰
	RMSE	۱۰۹,۲۰۳۹	۶۷,۴۸۹۰	۶۷,۴۲۹۱
	R ²	۰,۴۷۷۷	۰,۹۹۶۰	۰,۹۹۷۲
Steel	MAE	۲,۰۲۴۱	۱,۴۰۴۳	۱,۴۲۹۷
	RMSE	۲۰۱,۳۴۷۰	۱۰۹,۸۷۳۶	۳,۱۸۷۴
	R ²	۰,۶۷۰۰	۰,۹۸۰۴	۰,۸۶
Bitcoin	MAE	۲,۱۹۹۰	۱,۱۶۹۶	۱,۱۴۹۰
	RMSE	۱۷۶,۶۴۰۷	۱۰۸,۸۳۴۴	۱۰۶,۹۳۱۱
	R ²	۰,۴۷۴	۰,۹۶۶	۰,۸۹۹

Our initial expectation is that switching models perform better than Stochastic volatility models. This is due to the nature of switching models, taking into account regime changes and extreme jumps. This hypothesis is proven in practice. Considering the R^2 index along with the MAE and RMSE indices, it can be concluded that the Heston switching model exhibits better performance in the markets of the Tehran Stock Exchange, the global gold market, the dollar exchange market, and the global steel market. However, in markets of global crude oil and bitcoin, the Markov switching model will perform better. In the next part, according to the estimation of the marginal values in this research, the values of structural dependence between the performance of the Tehran Stock Exchange and other markets, the approach of conditional copula functions are specified in Table 8. In this study, functions with a better ability to explain the dependence structure among financial markets are determined based on Akaike's criterion.

The BIC evaluation criterion, like the AIC criterion, represents the amount of information that is lost by the model, and as a result, the smaller the value of the BIC evaluation criterion, the more suitable the model is compared to other models. The estimation results of the switching copula model shown in Table 4 show that the structural dependence between the Tehran Stock Exchange and the gold markets in the first regime with a dependence size of 1.3555 and in the second regime with a dependence size of 3.4852 by Clayton's copula shows the best performance. give The structural dependence between the Tehran Stock Exchange and the free foreign exchange market is also

best explained by Clayton's copula. This relationship is best explained in the crude oil market in the first regime by Frank's copula and in the second regime by Gamble. The connection between the Tehran Stock Exchange and the metal markets And Bitcoin also shows the best performance by the Gamble copula. concerning the Tehran Stock Exchange and other markets, in each of the regimes of the Tehran Stock Exchange, the structural relationship with other markets has been determined and the best copula function has been determined.

Using Kendall's rank correlation coefficient test and Spearman's rank correlation coefficient test were used, the significance of the correlation between research variables was determined.

According to the results of Kendall's and Spearman's rank tests, the existence of a correlation between the pair of variables of the Tehran Stock Exchange index with the five introduced markets is established. Now, after determining the results of the correlation between the Tehran Stock Exchange and the markets of gold, currency, crude oil, metals, and bitcoin, we need to determine the extent of contagion of the Tehran Stock Exchange from these markets. The specified results are consistent with previous research, including the research of Saghafi (2018) regarding the relationship between the Stock Exchange and metals. Now, after determining the results of the correlation between the markets, the existence of causality between the positive and negative risk of each market with the systematic risk of another market is examined. For this purpose, a value at conditional risk of 5% is considered a negative risk, and a value at risk

of 95% is considered a positive risk. Other risk levels can also be used, for example, the risk level of 97.5% and 2.5%. However, the choice of these levels is the

responsibility of the researcher, which naturally varies based on the individual's research strategy (Saghafi,2018).

Table 8: Comparison of copulas in different regimes

copula		Tehran exchange -Gold		Tehran exchange - Dollar		Tehran exchange -Oil		Tehran exchange - Steel		Tehran exchange Bitcoin	
		Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
Normal Copula	Copula parameter	۰,۷۱۱۳	۰,۹۰۳۳	۰,۸۱۳۱	۰,۶۶۶۶	-0.0490	۰,۱۸۴۴	۰,۰۰۸۴	۰,۷۷۱۳	۰,۸۴۲۱	۰,۷۲۷۳
	Max likelihood	۱۹۷,۳۱۸۶	۳۰۴,۶۲۳۱	۰۴۸,۰۰۷۴	۱۰۱,۳۱۶۳	۰,۸۳۴۸	۸,۲۰۶۱	۸۶,۹۷۶۹	۲۱۰,۱۳۷۹	۰۴۷,۱۰۴۲	۲۱۸,۹۶۳۰
	AIC	۳۹۶,۶۳۷۱	۷۱۱,۲۴۶۲	1.0990e+03	۳۰۴,۶۳۲۶	۳,۶۶۹۷	۱۸,۴۱۲۲	۱۷۰,۹۰۳۸	۴۲۲,۲۷۰۸	1.0962e+03	۴۳۹,۹۲۷۱
	BIC	۴۰۰,۸۶۷۶	۷۱۰,۲۱۲۴	1.1039e+03	۳۰۸,۷۰۹۰	۸,۰۴۸۱	۲۲,۳۲۰۷	۱۸۰,۱۸۲۳	۴۶۶,۳۱۶۰	1.1010e+03	۴۴۴,۲۰۳۷
T Student copula	Copula parameter	۰,۸۱۹۰	۰,۹۰۰۴	۰,۹۳۰۲	۰,۷۸۷۲	۰,۰۸۸۸	۰,۳۳۶۲	۰,۰۹۸۶	۰,۸۱۷۰	۰,۹۲۲۸	۰,۸۰۱۲
	Max likelihood	۲۴۳,۴۱۹۲	۳۰۷,۲۴۴۱	۷۷۳,۰۶۱۶	۱۹۱,۸۸۷۴	۱۶,۸۰۶۶	۳۲,۰۶۶۰	۱۰۹,۰۰۲۶	۲۷۸,۷۳۳۶	۷۳۴,۰۶۴۶	۲۹۴,۶۰۶۰
	AIC	۴۹۰,۸۳۸۴	۷۱۸,۴۸۸۲	1.5511e+03	۳۸۷,۷۷۴۹	۳۷,۷۱۳۳	۶۹,۱۲۸۱	۲۲۲,۰۰۰۲	۰۶۱,۴۶۷۲	1.4721e+03	۰۹۳,۲۱۳۰
	BIC	۴۹۹,۲۹۹۴	۷۲۶,۴۲۰۰	1.5609e+03	۳۹۶,۰۲۸۶	۴۶,۴۷۰۱	۷۶,۹۰۰۱	۲۳۰,۴۶۲۲	۰۶۹,۰۴۷۷	1.4816e+03	۶۰۱,۷۶۶۳
Clayton Copula	Copula parameter	۱,۳۰۰۰	۳,۴۸۰۲	۲,۲۳۹۸	۰,۹۷۰۶	۰,۱۹۱۳	۰,۰۷۴۹	۱,۰۲۳۳	۳,۶۶۸۰	۰,۰۶۰۲	۴,۸۱۰۶
	Max likelihood	۱۲۶,۱۶۴۰	۳۸۲,۳۰۱۸	۳۹۹,۴۳۳۲	۷۴,۱۰۷۴	۱۰,۲۹۰۴	۲۷,۹۴۰۰	۹۰,۹۰۴۲	۳۱۶,۸۱۹۰	۷۹۳,۳۹۶۷	۴۶۴,۴۶۶۴
	AIC	۲۰۴,۳۲۷۹	۰۶۶,۶۰۳۶	۸۰۰,۸۶۶۴	۱۰۰,۲۱۴۸	۲۲,۰۸۰۷	۰۷,۸۸۰۹	۱۹۳,۹۰۸۰	۶۳۰,۶۳۸۹	1.5888e+03	۹۳۰,۹۳۲۸
	BIC	۲۰۸,۰۰۸۴	۰۷۰,۰۶۹۸	۸۰۰,۷۴۸۸	۱۰۴,۳۴۱۷	۲۶,۹۰۹۱	۶۱,۷۹۴۴	۱۹۸,۱۳۷۰	۶۳۹,۶۷۹۲	1.5935e+03	۹۳۰,۲۰۹۴
Gumbel Copula	Copula parameter	۲,۰۷۳۸	۳,۱۶۰۸	۲,۸۷۸۰	۲,۰۳۱۱	۱,۰۴۰۳	۱,۰۹۹۷	۱,۴۴۰۸	۱,۹۸۰۰	۲,۶۳۹۰	۱,۸۳۸۸
	Max likelihood	۱۹۲,۱۱۰۲	۳۳۲,۰۸۴۹	۰۹۰,۰۶۳۸	۱۷۱,۸۷۴۹	۲,۸۷۱۸	۴,۷۷۹۲	۷۱,۰۴۹۸	۱۴۳,۲۴۷۰	۴۶۰,۶۹۲۹	۱۳۶,۱۹۷۲
	AIC	۳۸۶,۲۳۰۰	۶۶۶,۱۶۹۷	1.1821e+03	۳۴۰,۷۴۹۸	۷,۷۴۳۶	۱۱,۰۰۸۴	۱۴۴,۰۹۹۶	۲۸۸,۴۹۴۹	۹۲۳,۳۸۰۷	۲۷۴,۳۹۴۳
	BIC	۳۹۰,۴۶۰۹	۶۷۰,۱۳۰۹	1.1870e+03	۳۴۹,۸۷۶۷	۱۲,۱۲۲۰	۱۰,۴۷۱۹	۱۴۸,۳۲۸۱	۲۹۲,۰۳۰۲	۹۲۸,۱۳۱۰	۲۷۸,۶۷۱۰
Frank Copula	Copula parameter	۱۱,۲۲۲۷	۱۱,۸۰۰۷	۲۲,۸۶۶۲	۹,۴۰۸۰	۰,۱۶۹۷	۱,۷۷۲۰	۴,۱۰۹۰	۱۲,۷۶۸۷	۱۷,۰۹۹۶	۱۲,۷۶۳۱
	Max likelihood	۳۳۲,۶۸۱۶	۲۹۶,۹۱۹۱	1.1602e+03	۲۰۷,۱۴۸۹	۰,۱۸۳۹	۱۲,۳۰۳۴	۹۸,۴۰۹۲	۳۱۰,۴۰۴۰	۸۶۷,۶۰۹۸	۴۱۲,۰۱۴۲
	AIC	۶۶۷,۳۶۳۱	۰۹۰,۸۳۸۲	2.3224e+03	۰۱۶,۲۹۷۸	۲,۳۶۷۸	۲۶,۷۰۶۷	۱۹۸,۹۱۸۴	۶۹۲,۸۰۰۰	1.7373e+03	۸۲۶,۰۲۸۴
	BIC	۶۷۱,۰۹۳۶	۰۹۹,۸۰۴۳	2.3272e+03	۰۲۰,۴۲۴۶	۶,۷۴۶۳	۳۰,۶۲۰۲	۲۰۳,۱۴۶۹	۶۹۶,۸۴۸۳	1.7421e+03	۸۳۰,۳۰۰۱

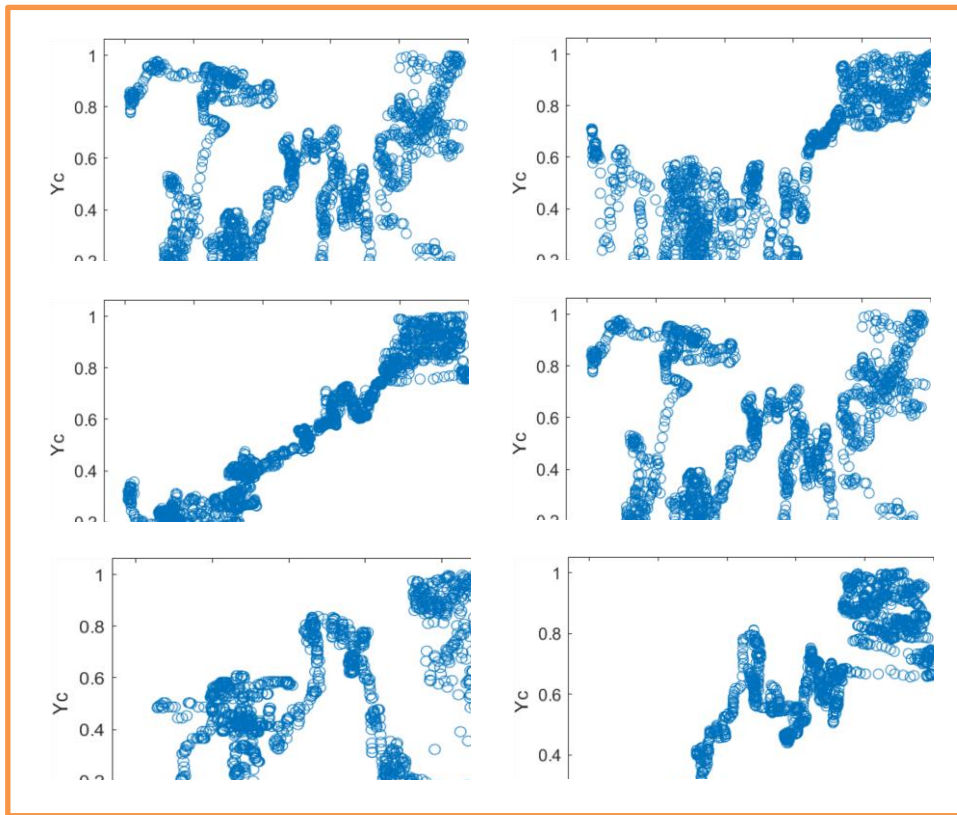


Chart 2: Performance of the best Copula in different markets

	Tehran exchange - Gold	Tehran exchange - Dollar	Tehran exchange - Oil	Tehran exchange - Steel	Tehran exchange - Bitcoin
correlation coefficient	۰,۷۷۴۹	۰,۹۴۴۰	۰,۱۱۳۰	۰,۶۰۰۶	۰,۷۷۹۳

Table 10: Risk spillover from different markets to the Stock Exchange

Risk spillover	Negative domain	Positive domain
Risk spillover from the Dollar exchange market on the Tehran Stock Exchange	۰,۰۱۳۶	۰,۲۶۱۰
Risk spillover from the global gold market on the Tehran Stock Exchange	۰,۰۰۲۵	۰,۴۱۰۰
Risk spillover from the global crude oil market on the Tehran Stock Exchange	۰,۰۱۱۵	۰,۲۰۸۲
Risk spillover from the steel exchange market on the Tehran Stock Exchange	۰,۰۱۹۱	۰,۴۰۸۸
Risk spillover from the bitcoin market on the Tehran Stock Exchange	۰,۰۱۱۹	۰,۲۰۰۰

The term positive risk here means fluctuation towards the upper range of the distribution. Fluctuation alone is considered a bad product due to creating uncertainty, but moving towards the upper range of the distribution means a price increase, and if there is no borrowing sale, it is a good product, and if there is a borrowing sale, it can be considered a bad product. In some financial instruments that trade on volatility, the volatility itself can be considered a good product or a

bad product according to the specific structure of that financial instrument. As can be seen in Table 6, global markets have an impact on the Tehran Stock Exchange, and this impact is such that according to the positive and negative fluctuations of these markets, the Tehran Stock Exchange also reacts. This reaction can be more about currency, digital currency, and gold markets than other markets.

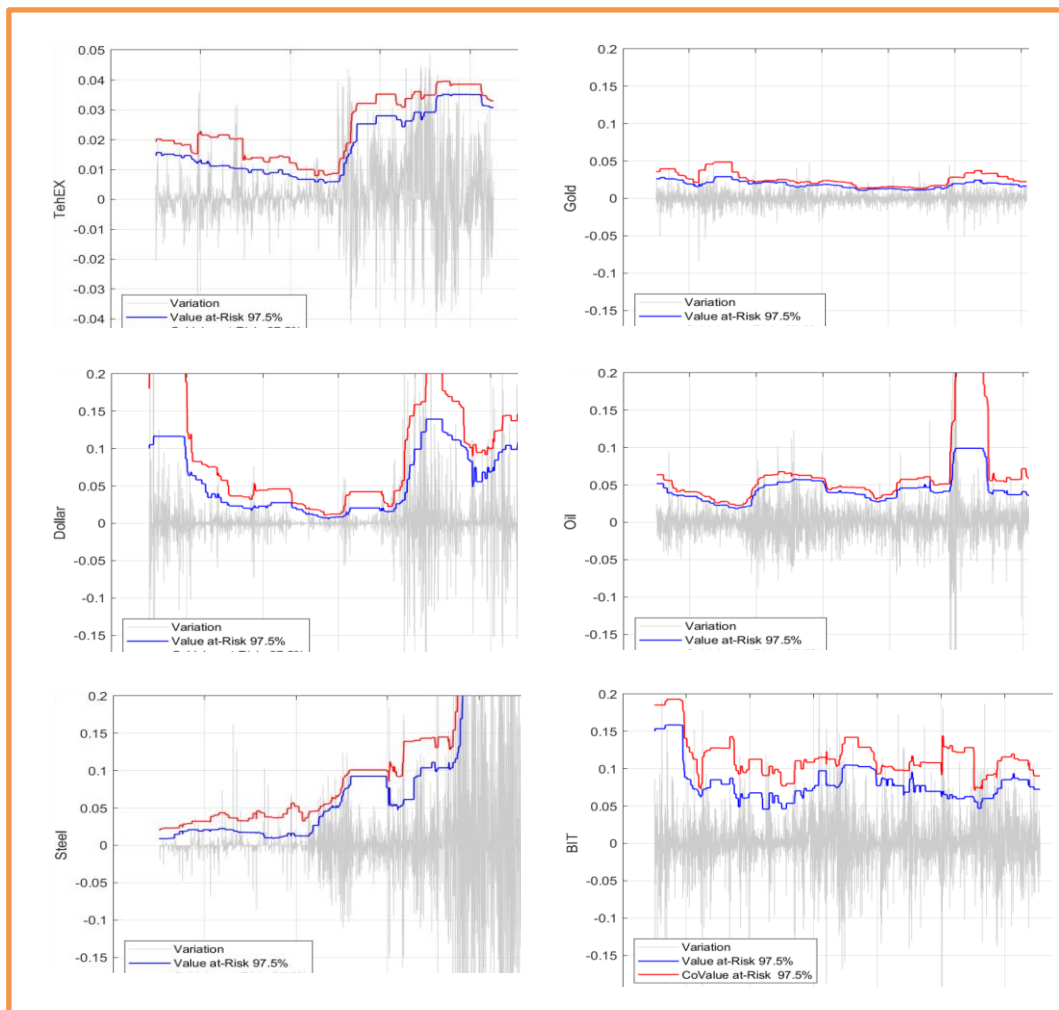


Chart 3: Value at risk in different markets

5 - Conclusion

The main goal of this research is to investigate the impact of global markets on the Tehran Stock Exchange. The markets whose impact on the Tehran Stock Exchange has been investigated include the gold market, the currency market (dollar), the digital currency market (Bitcoin), the global crude oil market, and the global steel market. The daily data of these markets have been collected between December 2011 and January 2023. To investigate the impact of these markets on the Tehran Stock Exchange, a combination of copula models and marginal models including Heston switching, Markov switching, and Stochastic volatility models have been used. The first step is to determine the most suitable distribution for marginal models, which is determined by comparing the above three models in each market. Considering the performance of switching models against extreme fluctuations and regime changes, we expect that these models will perform better than the Stochastic volatility model.

It seems that in these markets, the performance of switching models, including Heston switching and Markov switching, is better than the performance of the Stochastic volatility model. In the Tehran Stock Exchange markets, the dollar market, and the world market of crude oil, the Heston switching model shows a better performance. However, in the gold world markets, steel, and the Bitcoin markets, the Markov switching model shows a better performance. The reason for using the Heston switching model is to remove the limitations that we face in the Markov switching model, and in practice, we see that it shows better performance in several examples of the markets that we are facing.

After specifying the optimal marginal functions, it is necessary to select the best copula model. Therefore, we examined five copula categories and finally, based on the Akaike criterion, the best copula was selected in each of the regimes. The estimation results of the switching copula model show that the structural dependence between the Tehran Stock Exchange and the gold markets, as well as the currency market, is best performed by the Clayton copula. This relationship in the crude oil market is best explained by Frank's copula in the first regime and by Gamble in the second regime. The connection between the Tehran Stock Exchange market and the metal and bitcoin markets is also displayed by the Gamble copula.

After determining the optimal copula, it is necessary to determine the contagion in the positive and negative range with the help of the conditional risk value. Finally, it can be concluded that the Tehran Stock Exchange is most affected by the currency market. Considering the nature of the Iranian market and the direct impact of the dollar rate on the existing inflation in the society, this result is consistent with the empirical evidence. In the turbulent market of Iran, the dollar market has a competitive effect as well as a parallel effect on the Stock Exchange. When it comes to Iran's economy, the dollar acts as a safe asset. On the other hand, the increase in the exchange rate, in addition to affecting the existing inflation, also affects the cost of products. The gold and bitcoin markets will act as safe assets in the current situation and will have a competitive relationship with the Tehran Stock Exchange. The global steel market also has a great impact on the Tehran Stock Exchange, considering that about 67% of Tehran Stock Exchange industries are dependent on the price of global commodities such as metals.

Over time and with the development of markets, the transfer of information between markets has increased significantly, and this transfer of information can cause markets to influence each other. Freeing capital movement, improving the level of technology, and increasing the number of financial instruments including derivatives in the financial markets have made the financial markets more sensitive and responsive to the information and movement of other markets. In this space, the impact of economic, political, social, etc. factors is also important in addition to the relationship of markets with each other. Using switching models is also according to the above conditions.

In Iran's volatile economy, global markets have both parallel and competitive effects on this economy. Gold market as a safe capital, currency and digital currency markets are also considered as safe capital due to the turbulences of the Iranian market and compete with the Tehran Stock Exchange to attract funds. Sometimes incorrect economic policies have also caused this flow to double.

One of the most important causes of economic crises in the Tehran Stock Exchange as well as the country's markets, apart from the government's intervention and directive policies in these markets, is the currency exchange crisis. The lack of an organized

market for foreign exchange transactions has created an atmosphere of speculation in this market, and failure to organize this market can have serious consequences. It should be noted that the government's intervention in this market and organizing it should be done very carefully. Considering the political and economic records of the governments in the country, creating such a mechanism without being careful about its possible results may lead to the aggravation of the currency crisis. Considering the primary and secondary markets for currency can be a good alternative for the current situation. The movement of the government and macroeconomic policies of the country from central planning and directive guidance of the market towards market governance can achieve good results in the field of economy. The experience of East Asian countries in this regard is a confirmation of this claim

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