Modeling and simulation of non-linear fractional-order chaotic system of supply chain and financial model

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ABSTRACT

The purpose of this paper is to investigate modeling and optimal control of nonlinear fractional order chaotic system of supply chain and financial system. Modeling approach of differential equations with fractional derivatives was used. Mathematical model related to the supply chain was built with the help of fractional calculators, and then it was shown that the presented fractional order model has chaos and needs optimal control. The presented method was carried out with the help of an applicable and dynamic model in the form of a simulation using genetic algorithm and particle swarm optimization algorithm.The results of the control applied to the model can control the supply chain system. When the controller is applied from the beginning; the Results of genetic algorithm method are excellent. Results obtained for the particle swarm optimization method show that this method has also been very successful and have results very close to the genetic algorithm method.

Keywords: supply chain, financial system, optimal control, chaotic system, particle swarm optimization.

1. Introduction

In its simplest form, supply chain is the activities that organizations need to provide goods or services to consumers. The main focus of the supply chain is on the core activities of the organization that are used to transform raw materials or components into final products or services. In a traditional manufacturing environment, supplier liaison activities will generally be supported by procurement; also, operations management, throughout the supply chain through logistics, plays a fundamental role in the movement of incoming materials and outgoing goods to ensure that the final product reaches the consumer [1]. A supply chain can be in the form of a product-service-based supply chain, where services come together to provide an overall service to the customer in return for the final product, an example of which can be transportation of goods, employees and customers. When the supply chain is connected to our consumers and suppliers, we begin to create a supply chain network, where we can understand the flow of materials and information in a much more complex way [2]. Supply chain professionals need knowledge of managing supply chain functions such as transportation, warehousing, inventory management, and production planning. In the past, supply chain professionals emphasized logistics skills such as knowledge of shipping routes, familiarity with warehousing equipment and distribution center locations and footprints, and an accurate understanding of freight rates and fuel costs. Recently, supply chain management has expanded to inter-company logistics support and management of global supply chains. Supply chain professionals need to understand business continuity principles and strategies [3]. It is clear that business managers must be aware of the efficient flow of products from the point of origin to the point of consumption. For example, in the diagram of the food supply chain, the two-way flow of information between different parts of the chain clearly defines the amount of demand and also the problems of the chain. Supply chain managers need this data to make appropriate decisions about what to buy as raw materials, what products to manufacture, and what goods to deliver to the customer. Other currents are equally important. For example, suppliers pay a lot of attention to financial flows, because they want to receive the cost of the products and services they have provided very soon. Of course, sometimes defective and damaged products need to be sent back in the supply chain for return, repair, recycling, etc. Because of all these processes, every company needs supply chain management to improve and manage the flow of products, information, money, etc. This means the formation of a wide range of jobs in the supply chain management sector [3,4]. When a company's products are in the sales phase, they must go through some steps. At this stage, the supply chain is used. The activities that are carried out in the field of supply chain include natural resources and raw materials that are involved in the sale of a product in the supply to the customer. In large organizations, the supply chain of products may have the ability to return and re-enter the supply chain at any point where it can be recycled. Supply chain management refers to the processes that supply raw materials or organizational components that the company needs during the production of a product and the services of that product to customers. In simpler terms, it can be said that the purpose of supply chain management is to improve the performance of the organization's supply chain. In simpler terms, to answer the question of what is supply chain management, it should be said that timely and accurate supply chain information to manufacturers enables manufacturers and retailers to produce and supply only products that can be sold. Supply chain management to create and use in organizations has categorized strategies known as supply chain strategy [5]. Fisher was the inventor and initiator of supply chain management. In order to choose a strategy, he says that companies need to differentiate between new products and basic products to improve the company's performance, and we know that new and innovative products that have been produced need a reactive supply chain strategy [6]. The most important advantage of supply chain management is its effect on completing orders and responding to customer needs. In other words, this strategy provides an easy way to determine whether the organization has the right supply chain to meet customers' needs. This is the basis of competition. The second advantage of supply chain management is the alignment of supply chain activities with business activities. This orientation completes the mentioned activities. The last advantage of the integration of strategies is to improve the responsiveness of the supply chain to the business environment and change the basis of competition. This advantage ensures the evolution of the supply chain along with the change in market demand and the adaptation of the chain to the changing needs of customers [7]. Now, it is possible to ensure the alignment of supply chain strategy and business if all three aspects of supply chain management goals and business, supply chain processes, management tools and the supply chain focus area of senior management are coordinated. Today, companies are trying to respond to customers' demands due to the economic characteristics of modern business on a global scale and complex supply chain operations. Many companies around the world have come to the conclusion that providing services alongside production is more profitable than producing products alone. They seek to maximize their total benefits in the value chain up to the end consumer level. These businesses are involved in downstream activities and have moved towards providing services to the final consumer in order to engage in valuable economic activities that are created throughout the production cycle. value chain approach; That is, the supply chain includes all the activities required to provide a product or service to the final customer [11]. With this attitude, production and shipping activities are also added to the supply chain. Therefore, in the new world where there is fierce competition for the production of goods, producers have come closer to consumers. In order to gain the trust of the consumer, in addition to the product, manufacturers also offer a variety of services to minimize the cost of using the product and maintaining it. The supply chain is the complete process of providing goods and services for consumption. finalizer and supply chain management; It means managing the flow of goods and services, information and money to increase profitability. Uncertainty in global demand is a major challenge that is exacerbated by disruptions. Although the strategic importance of integration operations with suppliers and customers in supply chains is widely accepted, many questions about how best to identify supply chain strategies still remain unanswered. It is more important to communicate with suppliers or with customers or both. Similarly, little is known about the relationship between supplier and customer integration and overall performance improvement. Performance evaluation is essential in order to use the right combination of all resources in the chain in the best way to provide products and services. According to the above, the aim of this research is to answer the question of how to mathematically model the nonlinear fractional order chaotic system of the supply chain and the optimal control of this model with the help of evolutionary algorithms [12].In the crisis of 2020, businesses around the world realized the importance of the supply chain. When the corona epidemic caused the supply of primary resources to be disrupted and producers were unable to provide essential materials. In those circumstances, both companies and consumers realized the value of flexible and scalable supply chains. Companies are now taking a closer look at global supply chains and related technologies to take steps to stabilize their businesses. On the other hand, research shows that in the future of the capillary broadcasting industry and the supply chain of software and information technologies, they play an important role in the stability of various industries [17]. One of the concerns, at the most basic level, is that until we can describe what we do and demonstrate our success to stakeholders, we cannot gain recognition and support. But by defining the supply chain as the functions and activities that start from the procurement of raw materials and other components to the production of goods or services, we can outline how the goods will reach our consumers. When we talk about supply chain management, we are actually dealing with product development, material procurement, production of quality goods and product logistics. The supply chain can be the beating heart of an industry and has a tremendous impact on people's lives in different scales. Although supply chain management has experienced a lot of progress in the past few decades, many big brands and companies are facing many problems in different parts of the supply chain [18]. Essentially, supply chain management is focused on customer needs and can be summed up as: providing the right product at the right scale for a certain amount of cost. Therefore, it is very important to adjust the process as well as its correct timing. It may seem simple at first, but there are many challenges in this way. Customers have different standards and manufacturers must always adapt to customer needs. Providing unique services to each customer can make supply chain management face many challenges. The companies that excel in this field have actually been able to use their ingenuity and new technologies to find ways to respond to these different needs with minimal cost and time. Due to continuous changes. Market, crisis management is an important parameter in the survival and success of a group. This shows itself in the supply chain sector more than in other sectors. In crisis management, unwanted changes can come from different sources. For example, changes in consumer demand countries' political plans, global resources, diseases, and other human and natural factors that can cause the chain to undergo unwanted changes. do Floods and road closures, ship hijackings, or labor strikes in any corner of the world may cause major problems in supply chain operations, and crisis management must be able to manage them well. The primary and basic solution in this direction is to rely on a prepared crisis management plan [19].You should plan a crisis management plan based on the type of services, capabilities, financial and human resources, as well as the characteristics of your company. Based on this plan and in case of unforeseen changes, the collection can complete the cycle with minimal loss. It is better to plan this program in collaboration with logistics management and special software for such situations. The supply cycle is a living organism that needs care. Since human interactions are one of the basic foundations of this cycle, lack of clear communication can cause various problems for the chain. Always build a healthy and harmonious relationship with suppliers and other partners. With this work, you will be able to provide high-quality and high-standard products to your customers in a certain time and progress. The work is an important weight in the supply chain. Not using skilled labor and not guiding them can cause small and large disturbances in this chain. Meanwhile, today, finding interested and motivated people and having qualified personnel has become a challenge. Personnel employed in this field must have a correct understanding of their duties and responsibilities in the cycle. You can overcome many problems caused by workforce and personnel by creating an organizational culture, hiring based on specific standards and also promoting the level of successful employees [28 , 29]. Procuring raw materials may be easy, but delivering them on time is another challenge. Unforeseen delays outside the organization can change the timing of other parts of the chain. One of the key solutions in this regard is to store goods. Through an efficient warehousing system, you can replace what's in stock and move on schedule, even with material delay issues. By doing this, you will prevent domino delays in the supply chain and minimize losses [30]. The concept of chaos is one of the new and fundamental concepts of modern science that offers us a new insight into the real world. Today, with the discovery of chaos in the behavior of many phenomena in the surrounding world, many researchers have drawn attention to this concept. On the other hand, fractional calculations, which provide an opportunity to better model phenomena, have led to the development of chaotic systems [39]. Recognizing the presence or absence of turbulence in a dynamic system is an important issue that extends from the irregularities of heart fluctuations to the problem of the stability of the solar system. Among these methods, we can mention the estimation of the largest component of the Lyapunov system and the visual method of observing the behavior of the phases of the system [31,32,33]. The first real experiment in turbulence was performed by a meteorologist named Edward Lawrence. In 1960, while working on a weather forecasting problem, he accidentally came up with nonlinear differential equations that are very sensitive to their initial conditions, and a slight change in the initial conditions of such systems causes many changes in future behavior. These systems will. This phenomenon is known as the butterfly effect in chaos theory [34,35]. A well-known example of such a phenomenon is the weather [40]. The nonlinear dynamics of the atmosphere makes it impossible to make long-term predictions due to the lack of all the initial conditions (lack or inaccuracy of measuring temperature, humidity, etc. in all parts of the earth's surface (at the beginning of calculations) [36,37,38]. It says that flying a butterfly in another country can cause a storm in your country or not. Some systems that use or are heavily influenced by chaotic behaviors are highly advanced, expensive, and critical. However, the uses and methods of using chaotic systems are important areas of research. Evolutionary algorithms are a subset of evolutionary computations and are placed in the branch of artificial intelligence and include search algorithms in which the search process starts from several points in the solution space. Evolutionary algorithms are fundamentally different from other optimization methods and traditional search. Scalable algorithms do not search for a single point, but examine a population of points in parallel. Scalable algorithms do not need implicit information and other complementary knowledge; only the objective function and the relevant competence influence the search directions. Adaptive algorithms use probabilistic changing rules and not fixed ones. The use of scalable algorithms is generally very straightforward because there are no restrictions on the definition of the objective function. Adaptive algorithms obtain a large number of acceptable answers and the final choice is up to the user; therefore, in cases where the problem in question does not include a single answer, for example, a family of Pareto-optimal answers, similar to what exists in multi-objective optimization and scheduling problems. Evolutionary algorithms are inherently efficient at identifying these multiple responses simultaneously [49,52,53,56]. Evolutionary algorithms include: genetic algorithm, bee colony algorithm, ant group optimization method, evolutionary strategy, colonial competition algorithm. Evolutionary algorithms use elementary mechanisms and operations to solve the problem and reach the appropriate solution to the problem during a series of iterations. These algorithms often start from a population containing random solutions and try to improve the set of solutions. At the beginning of the work, a number of community members are randomly guessed, then the objective function is calculated for each of these members and the first generation will be created. If none of the optimization termination criteria are met, a new generation will begin. Members are selected according to their ability to produce babies. These people are considered as parents and produce recombination babies. Then all the babies are genetically changed with a certain amount of probability, that is, the same mutation. Now, the level of competence (fitness) of babies is determined and they replace their parents in society and create a new generation. This cycle is repeated until one of the optimization end criteria is obtained [50,51,57]. The theory of optimal control has the goal of maximizing efficiency or minimizing costs in the operation of physical, social and economic systems. Optimal control problems are mathematically challenging and practically have a certain mathematical order and structure [41,42,43]. Optimal control problems can be divided into definite and random branches. These problems have many applications in various sciences such as engineering, economics and financial mathematics [44,45,46]. In practice, many optimal control problems are subject to constraints on control and state variables. The goal in the problem of optimal control is to find the control law in one of two ways of open-loop or closed-loop control for a given system in a way that maximizes or minimizes a certain performance index [47,48]. The reasons for using the genetic algorithm method can be mentioned below. The nature of this algorithm's random search in the problem space is considered to be a parallel search. Because each of the random chromosomes generated by the algorithm is considered a new starting point for searching a part of the problem state space and the search is done in all of them simultaneously. Due to the wideness and dispersion of the points that are searched, it obtains favorable results in problems that have a large search space. It is considered a type of targeted random search and it will reach different answers from different paths. In addition, it does not face any restrictions in the path of searching and selecting random answers. Due to the competition (conflict) of answers and choosing the best among the population, it will reach the global optimal point with a high probability. Its implementation is simple and does not require complex problem solving procedures. Also, the reasons for using the particle swarm optimization algorithm can be explained as follows. The particle swarm optimization algorithm has a memory in such a way that the knowledge of good solutions is kept by all particles. In other words, in the particle swarm optimization algorithm, each particle benefits from its past information, while there is no such behavior and feature in other evolutionary algorithms, for example, there is no such memory in the genetic algorithm. The previous knowledge of the problem is lost once the population changes. In the particle swarm optimization algorithm, each member of the society changes its position according to the personal experiences and the experiences of the whole society. The social sharing of information among the members of a uniform society leads to evolutionary advantages, and this hypothesis is the basis of the optimal algorithm. It is considered the creation of particle swarm and its development, and as a result, there is beneficial cooperation between particles and particles in the group share their information with each other. In the particle swarm optimization algorithm, the members of the population communicate with each other and solve the problem by exchanging information. It has a high convergence speed. The collective movement of particles is an optimization technique where each particle tries to move to the direction where the best personal and group experiments took place. The particle swarm optimization algorithm shows more flexibility compared to other optimization strategies by using a large number of swarming particles against the local optimal problem. Many articles have examined fractional order equations, but there are very few articles that examine fractional order financial models and especially non-linear ones, and genetic algorithms and particle swarm optimization have not been used to numerically solve these models and have not been compared. . For this reason, this article is innovative and non-repetitive and has no similar. Therefore, this article has been published. Control issue Optimal has been reviewed in many books and articles [54,55]. In order to study the control of chaotic systems, we express the genetic algorithm and the particle swarm algorithm, and in the following we describe the financial system and theoretical formulation, and we express the optimal control of the proposed mathematical model.

1.2. Riemann-Liouville Integral and Caputo fractional derivative

Suppose that $n > 0$ and fare continuous segments on the interval (α, ∞) and are integrable on any finite sub-interval(α, ∞). Then, the fractional Riemann-Liouville Integral *f* for *t*>*a* of order *n* is defined as [29]

$$ {}_aD_t^{-n}f(t) = \frac{1}{\Gamma(n)}\int_\alpha^t (t-T)^{n-1} f(T)dT, \quad (1)$$

which can also be displayed with the symbols I_a^n orJ_a^n . In addition, if f is continuous on $[a,t]$, then $\lim_{n\to\alpha} D_t^{-n} f(t) = f(t)$. Furthermore, the following equation can be true:

$$ {}_aD_t^{\cdot} f(t) f(t). \quad (2)$$

When $n - m \in N$, the definition of (1-1) is compatible$-m$ with -fold integral as follows:

$$ {}_aD_t^{-m}f(t) = \int_\alpha^t dT_1 \int_\alpha^{T_1} dT_2 \ldots \int_\alpha^{T_{m-1}} f(T_m)dT_m = \frac{1}{(m-1)!}\int_\alpha^t (t-T)^{m-1} f(T)dT, \quad m \in N. \quad (3)$$

Regarding $m \geq 0$ and $v > -1$, the integral from the defined real order in Equation (1) has the following properties [30]:

$I. {}_\alpha D_t^{-n}(t-\alpha)^v = \frac{\Gamma(v+1)}{\Gamma(n+v+1)}(t-\alpha)^{n+v}$,

$II. {}_\alpha D_t^{-n}k = \frac{k}{\Gamma(n+1)}(t-\alpha)^n$,

If $f(t)$ for $t \geq a$ is continuous, then:

$III. {}_\alpha D_t^{-n}({}_\alpha D_t^{-m} f(t)) = {}_\alpha D_t^{-m}({}_\alpha D_t^{-n} f(t)) = {}_\alpha D_t^{-n-m} f(t)$.

Caputo defined a derivative operator in 1976 that differs from previous derivatives in terms of characteristics. The symbol of this operator is as ${}_aD_*^n$ and is defined as [28]:

$$ {}_aD_*^n f(t) = \frac{1}{\Gamma(m-n)}\int_\alpha^t (t-T)^{m-n-1} f^{(m)}(T)dT \quad (m-1 < n \leq m) \quad (4)$$
$$ = \alpha D_t^{-(m-n)} f^{(m)}(t),$$

On the conditions that $n \to m$ are exercised on the f function, then the Caputo derivative transforms to the m^th order derivative of the f(t) function. Suppose that $0 \leq m-1 < n < m$ and function f(t) have m+1 continuous bounded derivative in the interval$[a,t]$, then by partial integration for each $t > a$ per $m = 1,2,\ldots$, we have [30]:

$$\lim_{n\to m} {}_\alpha D_*^n f(t) = \lim_{n\to m}\left(\frac{f^{(m)}(\alpha)(t-\alpha)^{m-n}}{\Gamma(m-n+1)} \int_\alpha^t (t-T)^{m-n} f^{(m+1)}(T)dT\right)$$
$$ = f^{(m)}(\alpha) + \int_\alpha^t f^{(m+1)}(T)dT = f^{(m)}(t). \quad (5)$$

1.3. Chaotic fractional-order systems

The parameters and conditions for which the fractional-order system could have chaotic behavior are both investigated in this article [9,13]. In this section, two relevant theorem for fractional-order systems are stated [8,10,14]. The theorem is about proportional fractional-order systems.

Theorem 1. *Consider the autonomous system*

$$\frac{d^\alpha x}{dt^\alpha} = Ax, \quad x(0) = x_0.$$

Suppose $0 < \alpha < 1$ *and* $x \in \mathbb{R}^{n\times n}$ *, then, matrix A* $\in \mathbb{R}^{n\times n}$ *is asymptotically stable if and only if* $|arg(\lambda)| > \frac{\alpha\pi}{2}$ *is valid. In this equation,* λ *is the eigenvalue of matrixA In addition, this matrix is stable if and only if* $|arg(\lambda)| \geq \frac{\alpha\pi}{2}$.

The equilibrium point in fractional-order systems is calculated by the following system of ODEs [15,16]:

$$\frac{d^{\alpha}x}{dt^{\alpha}} = f(x), \quad (7)$$

$$f(x) = 0, \quad (8)$$

for $0 < \alpha < 1$ *and* $x \in \mathbb{R}^{n\times n}$*. The equilibrium point achieved by the solution of the system is asymptotically stable if the calculated eigenvalue* λ *related to the Jacobian matrix* $J = \frac{df}{dt}$ *satisfies the following equation in equilibrium point [8,9]:*

$$|arg(\lambda)| > \frac{\alpha\pi}{2} \quad . \quad (9)$$

Proof: See [8,10] for the proof.

Theorem2. *The n-dimensional dynamic fractional-order system could be specified as follows [17]:*

$$\frac{d^{\alpha_1}x_1}{dt^{\alpha_1}} = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n,$$
$$\frac{d^{\alpha_2}x_2}{dt^{\alpha_2}} = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n,$$
$$\vdots$$
$$\frac{d^{\alpha_n}x_n}{dt^{\alpha_n}} = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n. \quad (10)$$

In the system above, α_i*'s are between 0 and 1. It is assumed that M is the least common multiple of* u_i *that is expressed as* $\alpha = \frac{v_i}{u_i}$*. Here* $(u_i, v_i) = 1$ *and* $u_i, v_i \in \mathbb{Z}^+$ *for* $i = 1,2,3,\dots,n$*. Finally* $\Delta(\lambda)$ *Is described as below [18]:*

$$\Delta(\lambda) = \begin{pmatrix} \lambda^{M\alpha_1} - a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & \lambda^{M\alpha_2} - a_{22} & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \dots & \lambda^{M\alpha_n} - a_{nn} \end{pmatrix}. \quad (11)$$

The system response described in (10) is asymptotically stable if all roots (λ) *of equation* $det(\Delta(\lambda) = 0)$ *satisfy the condition:*

$$|arg(\lambda)| > \frac{\alpha\pi}{2}.$$

Denoting the matrix $\Delta(s)$ *is the characteristic matrix, and* $det(\Delta(s))$ *is the polynomial characteristic of the system (10).*

Proof: See [8,10] for the proof.

Definition 1. Consider the fractional-order system [20]:

$$\frac{d^{\alpha_i}x_i}{dt^{\alpha_i}} = f_i(x_1, x_2, x_3, \dots, x_1), \quad i = 1,2,3,\dots,n, \quad (12)$$

Where all α_i coefficients have values between 0 and 1.

The equilibrium point of the system (12) is acquired with solution of the following equations [19,21]:

$$f_i(x_1, x_2, x_3, \dots, x_1) = 0, \quad i = 1,2,3,\dots,n. \quad (13)$$

It is assumed that $x_1^* = (x_1^*, x_2^*, x_3^*, \dots, x_n^*)$ is the equilibrium point of the system(12) meaning $f_i(x_1^*, x_2^*, x_3^*, \dots, x_n^*) = 0$. Considering the values for *i*, the equation below is defined to evaluate the stability of equilibrium point [22]:

$$\varepsilon_i = x_i - x_i^*, \quad i = 1,2,3,\dots,n. \quad (14)$$

As the Caputo differentiation by a constant value is zero, we would conclude:

$$\frac{d^{\alpha_i}\varepsilon_i}{dt^{\alpha_i}} = f_i(x_1^* + \varepsilon_1, x_2^* + \varepsilon_2, \dots, x_n^* + \varepsilon_i), \quad i = 1,2,3,\dots,n. \quad (15)$$

If the second partial differentiation of function f_i around the equilibrium point x^* exists in the *n*-dimensional space of $\mathbb{R}^n$, the right-hand side of equation (15) could be rewritten as [22]:

$$f_i(x_1^* + \varepsilon_1, x_2^* + \varepsilon_2, \dots, x_n^* + \varepsilon_i) = f_i(x_1^*, x_2^*, \dots, x_n^*) + \left[\frac{\partial f_i}{\partial x_1}\Big|_{x^*} \frac{\partial f_i}{\partial x_2}\Big|_{x^*} \dots \frac{\partial f_i}{\partial x_n}\Big|_{x^*}\right]\varepsilon + \overline{f_i}(\varepsilon). \quad (16)$$

In the equation above, $\varepsilon = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T$, and $\overline{f_i}(\varepsilon)$ consist of the higher-order terms of Taylor expansion that is neglected. In addition, it is assumed that we have $f_i(x_1^*, x_2^*, \dots, x_n^*) = 0$, for $i = 1,2,3,\dots,n$. As a result, we could conclude:

$$f_i(x_1^* + \varepsilon_1, x_2^* + \varepsilon_2, \dots, x_n^* + \varepsilon_i) \approx \left[\frac{\partial f_i}{\partial x_1}\Big|_{x^*} \frac{\partial f_i}{\partial x_2}\Big|_{x^*} \dots \frac{\partial f_i}{\partial x_n}\Big|_{x^*}\right]\varepsilon + \overline{f_i}(\varepsilon). \quad (17)$$

Furthermore, we could assume the following equation:

$$\begin{pmatrix} \frac{d^{\alpha_1}x_1}{dt^{\alpha_1}} \\ \frac{d^{\alpha_2}x_2}{dt^{\alpha_2}} \\ \vdots \\ \frac{d^{\alpha_n}x_n}{dt^{\alpha_n}} \end{pmatrix} = J\varepsilon, \quad (18)$$

Where we have $f = [f_1, f_2, \dots, f_n]^T$ and $J = \frac{\partial f}{\partial x}\Big|_{x^*}$.

It is assumed that M is the least common multiple of α_i that is defined as $\alpha_i = \frac{v_i}{u_i}, (u_i, v_i) = 1$, and $u_i, v_i \in \mathbb{Z}^+$ for $i = 1,2,3,\dots,n$. According to Theorem (1.3.1), if $|arg(\lambda)| > \frac{\alpha\pi}{2}$ for all λ calculated by the equation below, the equilibrium point $x = x^*$ of the system (12) is asymptotically stable [10,17]:

$$det(diag([\lambda^{M\alpha_1} \;\; \lambda^{M\alpha_2} \;\; \dots \;\; \lambda^{M\alpha_n}]) - J) = 0, \tag{19}$$

It should be noted that $diag([m_1 \;\; m_2 \;\; \cdots \;\; m_n])$ is represented by a square $n \times n$ matrix as below:

$$diag([m_1 \;\; m_2 \;\; \dots \;\; m_n]) = \begin{pmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_n \end{pmatrix}. \tag{20}$$

1.4. The required conditions for the presence of chaos in fractional-order system

The saddle point is an equilibrium point in a three-dimensional integer-order system with at least one eigenvalue at the stable region (the left-hand part of the imaginary axis) and at least one eigenvalue in the unstable area (the right-hand part the imaginary axis) [23,24]. This saddle point is called saddle point of kind one if one of the eigenvalues is unstable and the others are stable, and if one eigenvalue is stable while two others are unstable [15,23], the saddle point is of kind two. The chaotic behavior in a chaotic system is demonstrated around a saddle point of kind two. The chaotic behavior could also be observed around a saddle point of the second kind in a three dimensional fractional-order system, just as the three-dimensional integer order one [25]. It is considered that the chaotic three-dimensional system of the form $\dot{x} = f(x)$ have chaotic attractors. It is also assumed that Ω is a set of equilibrium points of the system surrounded by a twisting [15]. On the other hand, both system $D^\alpha x = f(x)$ with defined $D^\alpha \equiv \left(\frac{d^{\alpha_1}}{dt^{\alpha_1}}, \frac{d^{\alpha_2}}{dt^{\alpha_2}}, \frac{d^{\alpha_3}}{dt^{\alpha_3}}\right)$ and $\dot{x} = f(x)$ have equal equilibrium points [15,26]. Therefore, the required condition for a fractional-order system of $D^\alpha x = f(x)$ to have chaotic attractor is stated as the following equation [26]:

$$\left(\frac{\pi}{2M}\right) - min|arg(\lambda_i)| \geq 0, \tag{21}$$

where λ_i are the roots of the equation below:

$$det([\lambda^{M\alpha_1} \;\; \lambda^{M\alpha_2} \;\; \dots \;\; \lambda^{M\alpha_n}]) - \left.\frac{\partial f}{\partial x}\right|_{x-x^*} = 0, \quad \forall x^* \in \Omega. \tag{22}$$

The system's behavior around this point cannot tend to a chaotic attractor if the system has a stable equilibrium point, and the initial conditions related to the system do not lie inside the attracting region [17,27]. In other words, the system cannot have a chaotic behavior for any initial condition, and some of the initial conditions do not actually represent chaotic behavior. In general, there is not a specific mathematical relation to the present attracting region. The condition of being chaotic for the fractional-order system of (12) could be stated as follows (by assumption $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. See [8,22] for more details):

$$\alpha \geq \frac{2}{\pi} min|arg(\lambda_i)|, \tag{23}$$

where λ_i are the eigenvalues of the Jacobin matrix defined by $\left.\frac{\partial f}{\partial x}\right|_{x-x^*} = 0$ for every $x^* \in \Omega$. The relation (23) states the necessary condition for chaos to occur in a fractional-order system [8,10]. This relation could be used to acquire the minimum order of the system for which the chaotic behavior could occur.

2. Research method and Modeling

In this chapter, we studied the optimal control problem of the system of supply chain and financial system of fractional-order. We were ready to solve the specified fractional-order model by the particle swarm optimization and genetic algorithms.

2.1. Supply chain fractional-order model and financial system fractional-order model

Supply chain the fractional-order model is given by:

$$x' = a_1 y - (a_2 + 1)x,$$
$$y' = a_3 x - z, \tag{24}$$
$$z' = xy + (a_4 - 1)z$$

where x(t) is the amount of asymmetric demand by the retailer in the current period, y(t) represents the

amount that distributors can asymmetric supply in the current period, and z(t) is the amount produced in the period It shows the current depending on the order. In the above relationships, $x'(t)$ is the derivative of the variable x(t) in relation to t, $y'(t)$is the derivative of the variable y(t) in relation to t, $z'(t)$ is the derivative of the variable z(t) in relation to t.
To find the lowest fractional order for the system to be in the chaotic region, we put

$$\alpha \geq \frac{2}{\pi} min|\arg(\lambda_i)|, \quad (25)$$

Where for the parameters [2.0.2,0.09,3], the order of the system is considered as [1, 0.99, 0.99]. The reason for choosing these parameters is that by choosing these parameters and orders, relation (25) will be established. The model presented in relation (24) is of correct order, but the purpose of this article is to investigate and control the non-linear fractional order model. It was shown that if equation (24) is written as equation (26), then the model will be of fractional order and will also have chaotic behavior, which must be controlled in the model. The variable " u " is added to the model with specific coefficients so that optimal control can be performed for the model.we show the chaotic system related to the supply chain control system with the following system of fractional differential equations

$$x'(t) = a_1 y - (a_2 + 1)x - u,$$
$$D_t^{0.99} y(t) = a_3 x - z - 0.5u, \quad (26)$$
$$D_t^{0.99} z(t) = xy + (a_4 - 1)z - 0.5u,$$

The parameters used in the model (26) are defined in the table (1).

Table1. The parameters presented in the model (26)

parameter	Parameter definition	Value
a_1	The degree of satisfaction of customer demand from the retailer	2
a_2	Distributors' inventory levels	0.2
a_3	Manufacturer's safety reserve factor	0.09
a_4	The amount of information distortion of products demanded by the retailer	3

Also in this article reported a model composed of three differential equations to describe the running of financial system [21]:

$$x' = a_1(x + y),$$
$$y' = -y - a_2 xz, \quad (27)$$
$$z' = a_3 + a_1 xy,$$

Where the variable x, y, z indicates the interest rate, the investment demand, the price index. The a_1, a_2, are constants.

In the above relationships, $x'(t)$ is the derivative of the variable x(t) in relation to t, $y'(t)$is the derivative of the variable y(t) in relation to t, $z'(t)$ is the derivative of the variable z(t) in relation to t.

In this dissertation, we aim to control the fractional-order financial model. Therefore, we consider a chaotic model with fractional-order derivatives based on the stability theorem related to fractional-order systems. Because modeling a system with fractional derivatives can show the system behavior better than ordinary derivatives. To find the lowest fractional-order for the system to be in the chaotic region, we put:

$$\alpha \geq \frac{2}{\pi} min|\arg(\lambda_i)|, \quad (28)$$

where for the parameters [12,40.2], the order of the system is considered as [1, 0.99, 0.99]. The reason for choosing these parameters is that by choosing these parameters and orders, relation (28) will be established. The model presented in relation (27) is of correct order, but the purpose of this article is to investigate and control the non-linear fractional order model. It was shown that if equation (27) is written as equation (29), then the model will be of fractional order and will also have chaotic behavior, which must be controlled in the model. The variable " u " is added to the model with specific coefficients so that optimal control can be performed for the model. we show the chaotic system related to the financial with the differential equation of fractional-order as follows.

$$x'(t) = a_1(x + y) - u,$$
$$D_t^{0.93} y(t) = -y - a_2 xz - 0.75u, \quad (29)$$
$$D_t^{0.93} z(t) = a_3 + a_1 xy - 0.75u .$$

The parameters used in the model (29) are defined in the table (2).

Table1. The parameters presented in the model (29)

parameter	Parameter definition	Value
a_1	saving	12
a_2	Investment cost	4
a_3	Elasticity of product demand	0.2

2.2. Optimal control of supply chain and financial system

First it is necessary to determine the purpose of the control for optimal control of the supply chain control fractional-order system model. It is necessary to define a standard mathematical function based on the specified goal. It is feasible to represent the function by the following relation;

$$j = \int_0^{t_f}(x^2 + u^2)dt \,. \qquad (30)$$

Now, the supply chain control fractional-order model is regarded by considering the control variable as the following relation:

$$\begin{aligned} &x'(t) = 2y - 1.2x - u, \\ &D_t^{0.99}y(t) = 0.09x - z - 0.5u, \qquad (31) \\ &D_t^{0.99}z(t) = xy - 2z - 0.5u, \end{aligned}$$

Also the financial system control fractional-order model is regarded by considering the control variable as the following relation:

$$\begin{aligned} &x'(t) = 12(x + y) - u, \\ &D_t^{0.93}y(t) = -y - 4xz - 0.75u, \qquad (32) \\ &D_t^{0.93}z(t) = 0.02 + 12xy - 0.75u \,. \end{aligned}$$

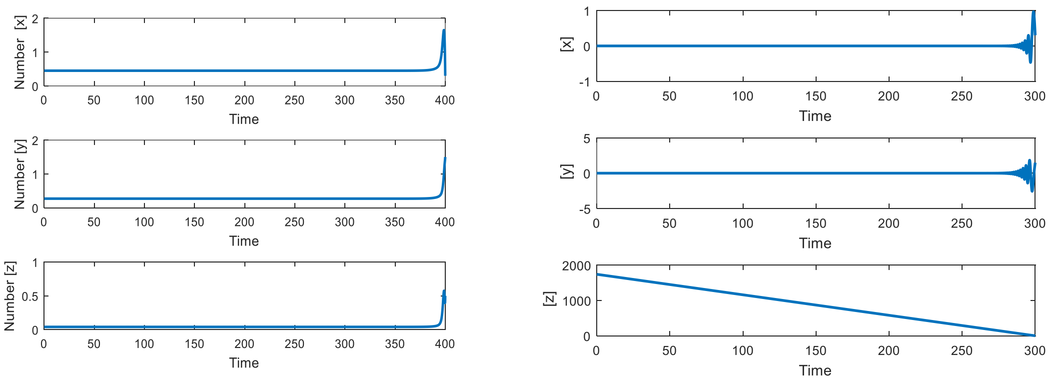
In this article, we used the particle swarm optimization algorithm and genetic algorithm methods to solve the problem.

3. Computational Experiments and Results

The results of each method are presented in the sequel.

3.1. Without control

In uncontrolled mode, in Fig. 1, the following results for three-mode variables are obtained that are not desirable:

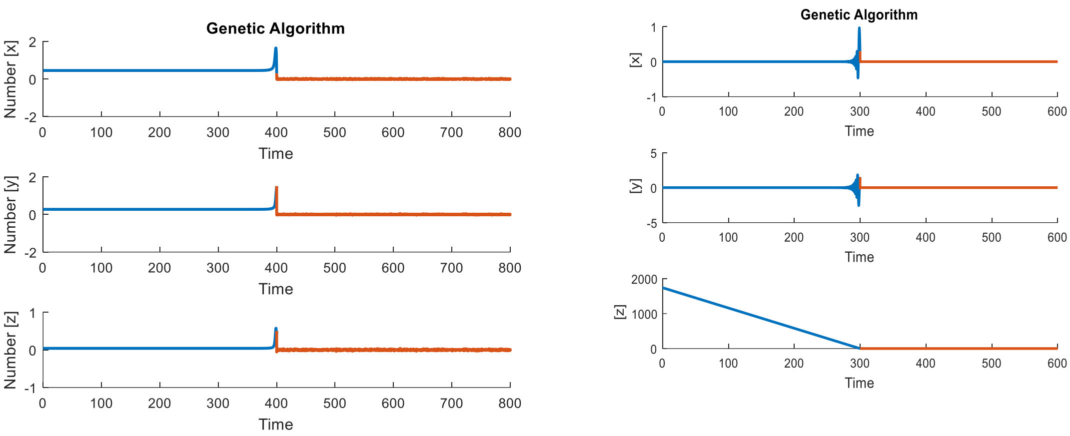


a) supply chain model **b) financial system model**

Fig 1. Results for three-mode variables

3.2. Results of genetic algorithm method

First, we consider the time of implementing the control input of 400 and 300 seconds and thus the following results are obtained. It is clear that the results are excellent as soon as the control input is applied (in Fig. 2, blue lines are for the uncontrolled method and red are for the controlled ones):

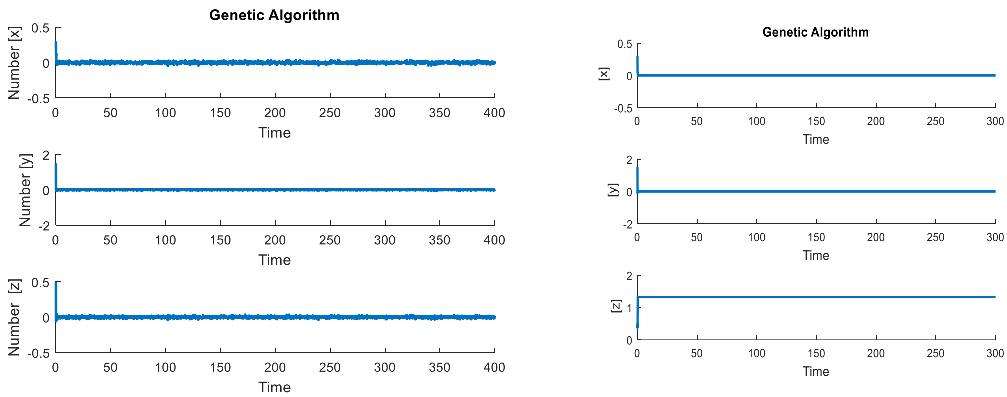


a) supply chain model

b) financial system model

Fig 2. Blue lines are for the uncontrolled method and red are for the controlled ones

Again, in Fig. 3, we examine the results when the controller is in use from the beginning. It is easy to see that the answers are excellent from the beginning.

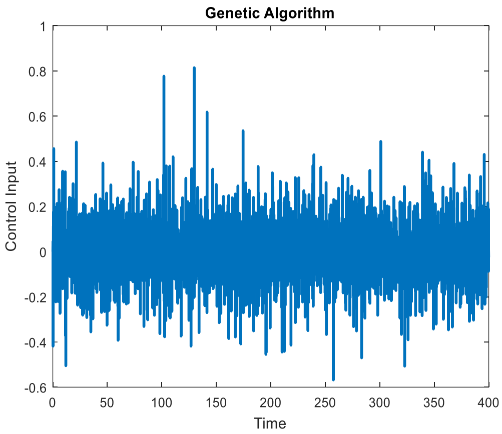


b) financial system model

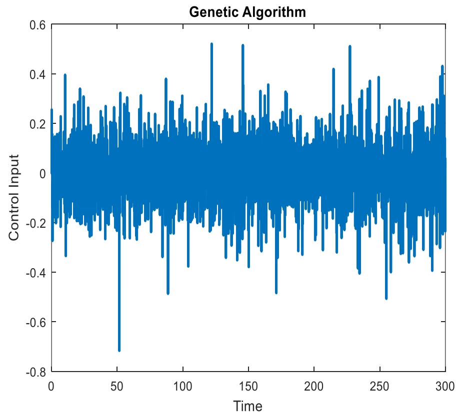
a) supply chain model

Fig 3. The results when the controller is in use from the beginning

In Fig. 4, changes in control input are as follows:



a) supply chain model



b) financial system model

Fig 4. Changes in control input

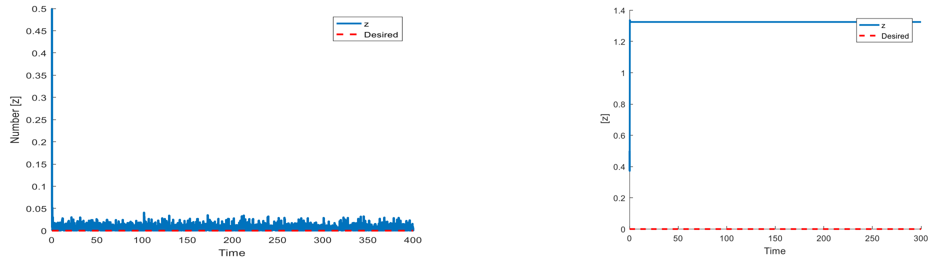
We saved an excel file that contains the numeric values of the model variables and the control input (in full control mode). The following picture is only part of the Table 3.

Table 3.the numeric values of the model variables and the control input in supply chain model

Time	X	Y	Z	U	Time	X	Y	Z	U
0	0.3000	1.5000	0.5000	0	205.0000	0.0089	0.0080	0.0070	0.0590
5.0000	0.0083	0.0059	0.0028	0.0094	210.0000	0.0068	0.0063	0.0053	0.0048
10.0000	0.0035	0.0012	0.0019	0.0282	215.0000	0.0014	0.0014	0.0011	-0.0501
15.0000	0.0020	4.5300e-04	0.0037	0.0018	220.0000	0.0040	0.0045	0.0051	-0.0102
20.0000	0.0056	0.0064	0.0069	0.0069	225.0000	0.0109	0.0094	0.0073	-0.0378
25.0000	0.0116	0.0085	0.0042	0.0322	230.0000	0.0128	0.0130	0.0127	0.0201
30.0000	0.0055	0.0058	0.0058	0.0889	235.0000	0.0162	0.0147	0.0123	-0.0623
35.0000	0.0170	0.0133	0.0082	0.0734	240.0000	0.0046	9.7376e-04	0.0033	0.0300
40.0000	0.0103	0.0093	0.0076	0.0115	245.0000	0.0039	0.0038	0.0037	-0.0851
45.0000	4.9172e-04	0.0051	0.0107	0.0967	250.0000	8.9033e-04	0.0015	0.0045	-0.0077
50.0000	0.0149	0.0154	0.0155	0.0666	255.0000	0.0045	0.0032	0.0016	-0.0300
55.0000	0.0037	0.0046	0.0055	0.0336	260.0000	0.0142	0.0116	0.0079	0.0353
60.0000	0.0250	0.0208	0.0148	-0.0098	265.0000	0.0074	0.0075	0.0069	-0.1338
65.0000	0.0111	0.0100	0.0087	-0.0208	270.0000	0.0152	0.0159	0.0158	-0.0899
70.0000	0.0110	0.0084	0.0048	-0.0381	275.0000	0.0044	0.0039	0.0032	0.0526
75.0000	0.0042	0.0033	0.0017	-0.0011	280.0000	0.0020	0.0026	0.0029	0.0412
80.0000	0.0135	0.0088	0.0026	-5.5921e-04	285.0000	0.0080	0.0118	0.0159	-0.2127
85.0000	0.0037	0.0034	0.0027	0.0742	290.0000	0.0110	0.0094	0.0069	-0.0061
90.0000	0.0045	0.0053	0.0063	-0.0628	295.0000	0.0069	0.0057	0.0041	-0.1247
95.0000	0.0145	0.0133	0.0110	0.1345	300.0000	0.0030	0.0025	0.0016	-0.0253
100.0000	3.8009e-04	5.1590e-04	0.0014	0.0315	305.0000	0.0130	0.0122	0.0106	-0.1740
105.0000	0.0121	0.0104	0.0080	-0.0123	310.0000	2.4725e-04	1.2453e-04	3.7591e-04	2.4374e-04
110.0000	0.0036	0.0022	3.7817e-04	0.0252	315.0000	0.0119	0.0108	0.0093	-0.0174
115.0000	0.0023	0.0013	3.2067e-04	-0.0177	320.0000	0.0137	0.0096	0.0039	0.0763
120.0000	6.7363e-04	0.0022	0.0038	0.0541	325.0000	9.9656e-05	6.0372e-04	0.0015	0.0457
125.0000	0.0198	0.0162	0.0112	-0.0390	330.0000	0.0095	0.0079	0.0052	-0.0694
130.0000	0.0073	0.0024	0.0151	0.0916	335.0000	0.0081	0.0065	0.0042	0.0548
135.0000	0.0063	0.0065	0.0064	0.0479	340.0000	0.0079	0.0059	0.0034	0.0092
140.0000	0.0052	0.0079	0.0106	0.2413	345.0000	0.0085	0.0082	0.0073	-0.0983
145.0000	0.0120	0.0129	0.0135	-0.0701	350.0000	0.0146	0.0119	0.0080	0.0998
150.0000	0.0162	0.0169	0.0166	0.2972	355.0000	0.0011	1.0440e-04	0.0017	0.0033
155.0000	0.0116	0.0092	0.0058	0.0031	360.0000	0.0102	0.0099	0.0090	0.0849

Time	X	Y	Z	U	Time	X	Y	Z	U
160.0000	0.0081	0.0039	0.0014	-2.9531e-04	365.0000	1.1956e-04	0.0013	0.0032	-0.0145
165.0000	0.0049	0.0044	0.0035	0.0311	370.0000	0.0060	0.0061	0.0059	-0.0736
170.0000	0.0206	0.0181	0.0141	0.0753	375.0000	0.0061	0.0053	0.0040	0.0421
175.0000	0.0130	0.0115	0.0094	-0.0296	380.0000	0.0042	0.0041	0.0038	-0.0048
180.0000	0.0016	0.0021	0.0029	0.0173	385.0000	0.0012	0.0015	0.0018	-0.0063
185.0000	0.0047	0.0040	0.0029	0.0050	390.0000	0.0138	0.0118	0.0089	0.0226
190.0000	0.0072	0.0064	0.0050	0.0955	395.0000	0.0019	0.0021	0.0024	0.0472
195.0000	0.0107	0.0114	0.0113	0.0717	400.0000	0.0020	0.0018	0.0016	-0.0250
200.0000	0.0084	0.0074	0.0059	-0.0171	-	-	-	-	-

For this reason, in Fig.5 we draw a diagram for the approximation and error of the zero reference signals:



a) supply chain model **b) financial system model**

Fig 5. Diagram for the approximation and error of the zero reference signal

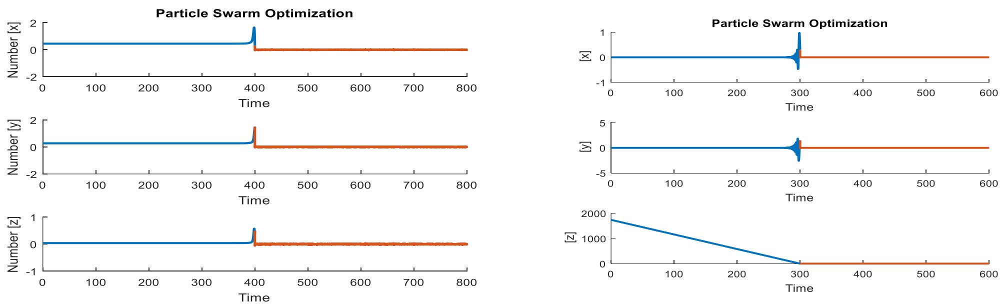
The MSE and RMSE specifications for error are on the MATLAB command page. We observe that their values are small. Consequently, the simulation is effective. It can be seen in Table 4.

Table 4.The MSE and RMSE specifications for error

model	MSE	RMSE
supply chain	9.4872e-05	0.009740
financial system	8.7541 e-05	0.007310

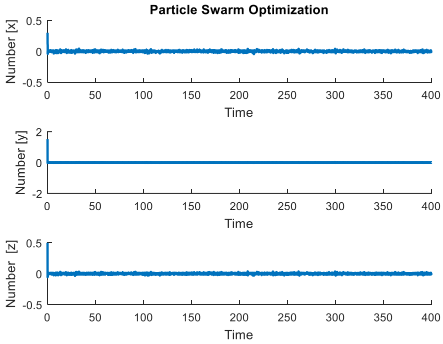
3.3. Results of particle swarm optimization algorithm

We also repeated all the above steps for this method and observed that it is very successful. Moreover, in Figs. 6, 7, 8 and 9 its results are very close to the genetic algorithm method.

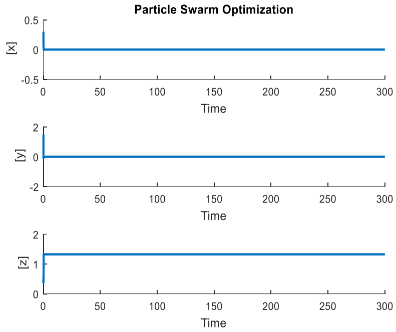


a) supply chain model **b) financial system model**

Fig 6. Blue lines are for the uncontrolled method and red are for the controlled ones

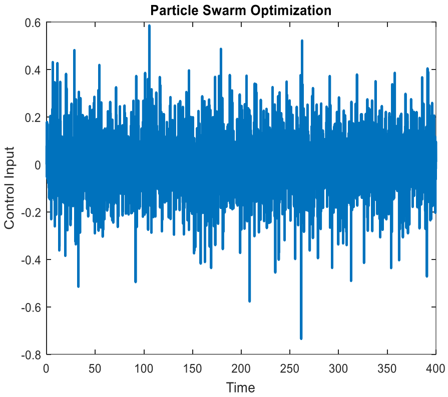


a) supply chain model

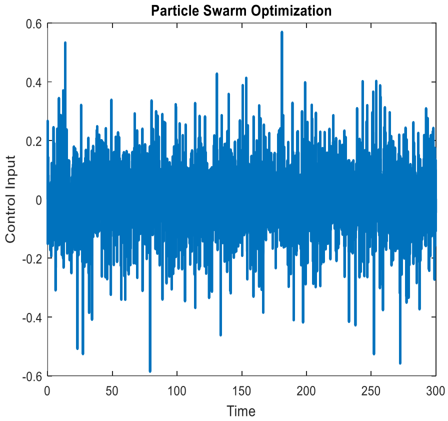


b) financial system model

Fig 7. The results when the controller is in use from the beginning

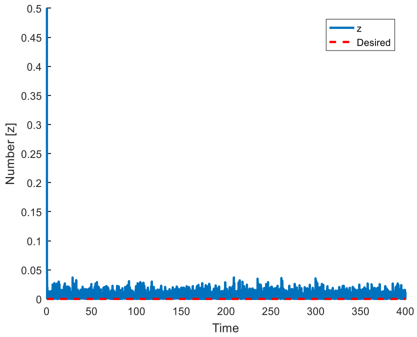


a) supply chain model

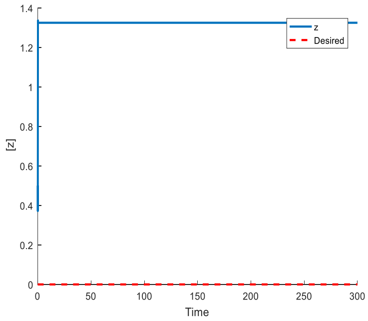


b) financial system model

Fig 8. Changes in control input



a) supply chain model



b) financial system model

Fig 9. Diagram for the approximation and error of the zero reference signal

The MSE and RMSE specifications for error are on the MATLAB command page. We observe in Table 5 that their values are small. Consequently, the simulation is effective.

Table 5.The MSE and RMSE specifications for error

model	MSE	RMSE
supply chain	9.6512e-05	0.009824
financial system	8.6632 e-05	0.006584

The numerical values of model variables and control input (in full control mode) can be seen in Table 6.

Table 6.the numeric values of the model variables and the control input in supply chain model

Time	X	Y	Z	U	Time	X	Y	Z	U
0	0.3000	1.5000	0.5000	0	205.0000	0.0029	0.0044	0.0063	0.0185
5.0000	5.4599e-04	0.0019	0.0036	0.0425	210.0000	0.0052	0.0061	0.0073	-0.0086
10.0000	0.0175	0.0185	0.0194	-0.2109	215.0000	0.0341	0.0333	0.0303	0.3382
15.0000	0.0017	3.4975e-04	0.0016	-0.0078	220.0000	0.0147	0.0102	0.0042	-0.0026
20.0000	0.0275	0.0280	0.0267	0.3812	225.0000	0.0061	0.0045	0.0024	-0.0438
25.0000	0.0022	0.0016	5.0362e-04	0.0320	230.0000	0.0162	0.0138	0.0105	0.0219
30.0000	0.0274	0.0280	0.0267	-0.3366	235.0000	0.0338	0.0310	0.0262	-0.0070
35.0000	0.0113	0.0090	0.0061	-0.0136	240.0000	8.7544e-04	2.8593e-05	0.0011	0.0466
40.0000	4.7595e-04	0.0023	0.0048	-0.0339	245.0000	0.0064	0.0049	0.0033	-0.0426
45.0000	0.0013	1.1525e-04	0.0017	-0.0285	250.0000	0.0075	0.0053	0.0018	0.0508
50.0000	0.0061	0.0048	0.0034	0.0115	255.0000	0.0077	0.0085	0.0093	-0.0618
55.0000	0.0038	0.0029	0.0015	0.0196	260.0000	0.0147	0.0156	0.0155	-0.1488
60.0000	0.0074	0.0066	0.0054	0.0200	265.0000	0.0016	0.0016	0.0015	0.0229
65.0000	2.6203e-05	8.1465e-06	4.8160e-06	0.0264	270.0000	0.0022	0.0027	0.0030	-0.0251
70.0000	0.0066	0.0055	0.0038	0.0277	275.0000	0.0013	0.0024	0.0035	0.0234
75.0000	0.0028	0.0019	5.1425e-04	0.0156	280.0000	0.0095	0.0102	0.0102	0.1692
80.0000	0.0178	0.0126	0.0059	-0.0226	285.0000	0.0073	0.0072	0.0069	-0.0126
85.0000	0.0154	0.0140	0.0117	0.0132	290.0000	3.5336e-04	8.8801e-04	0.0015	-0.0052
90.0000	0.0152	0.0147	0.0135	-0.0893	295.0000	7.8043e-04	0.0031	0.0059	-0.0180
95.0000	0.0119	0.0105	0.0082	-0.1411	300.0000	0.0127	0.0089	0.0043	-0.0015
100.0000	0.0021	0.0021	0.0019	-0.0341	305.0000	0.0040	0.0040	0.0036	-0.0035
105.0000	0.0109	0.0074	0.0029	0.0128	310.0000	0.0095	0.0089	0.0079	-0.0062
110.0000	0.0093	0.0100	0.0104	0.0608	315.0000	0.0032	0.0016	5.3946e-04	-0.0700
115.0000	0.0052	0.0040	0.0023	0.0393	320.0000	0.0062	0.0075	0.0089	-0.1212
120.0000	0.0037	0.0041	0.0044	-0.0112	325.0000	0.0019	7.8394e-04	6.9136e-04	-0.0285
125.0000	0.0090	0.0082	0.0070	-0.0570	330.0000	0.0269	0.0256	0.0224	-0.2794
130.0000	0.0057	0.0041	0.0019	-0.0020	335.0000	0.0026	0.0043	0.0061	-0.0654
135.0000	0.0045	0.0053	0.0059	-0.0414	340.0000	0.0010	0.0019	0.0030	-0.0537
140.0000	0.0162	0.0160	0.0146	0.2422	345.0000	9.3081e-04	6.7358e-04	0.0026	-0.0147
145.0000	0.0109	0.0156	0.0209	0.1353	350.0000	0.0058	0.0045	0.0028	0.0333
150.0000	0.0044	3.3794e-04	0.0046	0.0422	355.0000	0.0062	0.0038	3.3311e-04	0.0303
155.0000	0.0058	0.0062	0.0066	0.0210	360.0000	1.4614e-05	0.0019	0.0041	0.0487
160.0000	0.0148	0.0146	0.0135	0.0486	365.0000	0.0057	0.0053	0.0047	0.0155
165.0000	0.0167	0.0159	0.0142	0.0317	370.0000	0.0078	0.0062	0.0042	0.0874
170.0000	5.3622e-04	3.1249e-04	9.3720e-05	-0.0684	375.0000	0.0028	0.0018	5.8991e-04	0.0699
175.0000	0.0055	0.0034	6.7465e-04	-0.0503	380.0000	0.0055	0.0031	2.0803e-04	0.0590
180.0000	0.0032	0.0037	0.0043	-0.0458	385.0000	0.0031	0.0042	0.0054	0.0930
185.0000	9.5840e-04	0.0014	0.0021	-0.0466	390.0000	0.0104	0.0099	0.0089	0.0578
190.0000	0.0136	0.0110	0.0073	-0.0238	395.0000	0.0057	0.0060	0.0059	0.0929
195.0000	0.0117	0.0096	0.0066	0.0690	400.0000	0.0082	0.0065	0.0040	0.0104
200.0000	0.0018	0.0014	7.1512e-04	-0.0136	205.0000	0.0029	0.0044	0.0063	0.0185

4. Discussion and Conclusion

Chaos is a characteristic that can show itself in non-linear systems under certain conditions. The optimal control law was expressed as a mathematical equation using particle swarm optimization algorithm and genetic algorithm. For this reason, using this method in relation to other methods is both simpler and more practical because its application has the necessary conditions and optimization. Regarding the optimal control of the supply chain in Iran, few measures have been taken. However, no specific control rules have been established. If the control signal contains a mathematical equation, it can be used for practical applications. A specific equation for the control signal was obtained through the algorithm method, and this is one of the advantages of using this method. If supply chain modeling is done with higher accuracy, using this control method can be suitable for reducing supply chain. Therefore, there is a need for a more accurate model of the supply chain to operationalize the results and make them more effective. Having a mathematical relationship with control signals can be used for practical applications. The optimal control of the supply chain and financial system were carried out by the particle swarm optimization algorithm and the genetic algorithm. Using this method is more suitable than other methods. Because using this method has a necessary and sufficient condition to be optimal. Also, in the calculated control law, there is a definite mathematical relationship between the control variables, and this is very suitable for applications. As we expect, the results show that evolutionary algorithm methods, including particle swarm optimization algorithm and genetic algorithm, are better, more accurate and faster than other methods. In the present research, the simulation was done by MATLAB. At first, the chaotic deficit order of the supply chain was obtained. By shaping the structure of the controller, there will be a need to use a tool to solve the problem. In this case, particle swarm optimization and genetic algorithms were used to solve the problem. Clearly, once the control input is applied, the results are excellent. When the controller is first used, you check the results. It's easy to see that the answers are excellent from the start. It was observed that the error values of MSE and RMSE are small. As a result, the simulation is effective. Also, we repeated all the above steps for the particle swarm optimization algorithm and observed that it is very successful. The following suggestions are presented as strategies for future research, It is suggested to the investors to use IT specialists in addition to independent auditors and accountants and financial specialists for the intelligent review of the supply chain and financial system when reviewing the financial statements and making decisions about investing in the company. Considering the concerns of inflationary conditions and sanctions governing the economic situation, it is recommended to use evolutionary simulation algorithms and artificial intelligence in addition to other application software to reduce the time and cost of checking the financial hub statement, increase accuracy and high reliability. Accounting should be used continuously and optimally.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work report in this paper.

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