



Investment Portfolio Optimization based on the Ant Lion Optimization Algorithm and comparing it with Conventional Approaches

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ABSTRACT

Nowadays, investors use different criteria for measuring risk. These criteria are selected depending on the behavior of investors in the capital market and the amount of knowledge and mastery of financial issues. Much has been said about the use of risk in stock portfolio analysis, and also, investors, apart from the principle of risk aversion, have always tried to optimize the relationship between risk and return on operations.

Therefore, in this research, the portfolio optimization was based on the approaches of the antlion optimization algorithm and the weed optimization algorithm. This research was conducted for the period of 1400 to 1401 in Tehran Stock Exchange. The statistical method used in this research is multivariate regression method and optimized algorithm. The results of this study showed that: choosing a portfolio using meta-innovative models is better than choosing a portfolio using traditional methods. The antlion optimization algorithm has the ability to select the optimal investment portfolio. Weed Making has the ability to select the optimal investment portfolio.

Keywords: investment portfolio optimization, ant lion optimization algorithm, conventional investment portfolio approaches



1. Introduction

For a long time, the selection of the optimal stock portfolio has been considered one of the topics that preoccupied the minds of investment experts. In other words, all investors are seeking to be able to make the best possible decision by observing the effective criteria in the investment decision and according to their personal preferences so that while minimizing the risk for a specific return, to some extent they take their preferences such as the degree of risk aversion into account. Investors who accept and apply the modern theory of the stock portfolio believe that they are not "market rivals". Hence, they keep various types of securities, so that their yield is equal to the average yield of the market. Since they do not have prediction ability, they attempt to keep a "diversified set" of securities, so that they can obtain their desired rate of return, which is close to the market rate of return (Wei Zhou et al., 2018).

The major challenge for every investor is determining the set of securities with maximum profitability. This issue is equivalent to choosing the optimal stock portfolio from the set of possible stock portfolios, which is called the stock portfolio selection problem. The mean-variance model presented by Markowitz is one of the models that is widely used in the stock portfolio selection problem. It should be noted that although this model can be solved theoretically with the mathematical programming method, but in practice there are problems in this field. Firstly, the variance criterion considering the real-world conditions and other risk criteria cannot be a suitable criterion for risk, and in addition, other risk criteria in other conditions and according to the preferences of investors in the real world have limitations such as the size of the stock portfolio. They add to their optimization model that such constraints form a quadratic-integer programming problem, which is much more difficult to solve than the original model (Fernandez et al., 2019).

Hence, the main objective of this research is to present a model for the investment portfolio optimization based on the ant lion optimization algorithm approach and compare it with conventional ones. Therefore, in this research, the stock portfolio optimization based on the ant lion model was investigated. Therefore, here, the investment portfolio optimization based on the ant lion optimization algorithm and its comparison with the conventional

approaches have been discussed. Then the outline of the theoretical foundations, literature review, research methodology description, the hypotheses and the theoretical foundations of the research is presented and then the results of the hypothesis test are explained, and in the final section, the conclusions and future perspectives are formulated.

2- Theoretical frameworks and literature review

In the past, investors inferred their experiences or others to achieve their goals. As financial management advanced, investors' choices became more scientific and they were able to achieve the optimal choice to some extent by applying different models and combining its results with their own experiences. According to modern financial theories, the factors that should be considered for choosing a suitable portfolio are very important. Rational investment decisions require attention to many criteria and various factors (liquidity, currency risk, transaction costs, different investment behaviors and financial market trends) at the same time. Difficulty in choosing an investment portfolio is one of the important topics of research in Finance. The main purpose of choosing an investment portfolio is to choose the best combination of assets and the highest expected return. Choosing an investment portfolio can be classified as decision under risk (17). Targeted analysis enables the decision-maker to consider all the factors related to the decision, so that he chooses the best final decision from a set of known options. The best method in the field of decision-making is with multiple criteria, which enables the decision-maker to combine several changes of restrictions and goals (Safari et al., 2016).

In fact, in this method, we seek to obtain a set of solutions that can optimize the maximum possible number of objective functions; this set of answers is called Pareto optimality. The problem of determining the Pareto curve in the space of mean and variance in asset portfolio optimization, in different cases, such as when the number of investable assets and limitations in the model market is small, or by referring to Markowitz simplification (instead of simultaneous optimization) The two opposing goals of minimum variance and maximization of returns, assuming the stability of one of them, the other optimal value can be solved by mathematical models. But when the

conditions and limitations of the real world are considered, the problem of asset portfolio optimization is simply and it cannot be solved using classical mathematical methods. Therefore, the importance of using new methods such as evolutionary algorithms in the optimization of the asset portfolio is one of the most important issues in recent times. In solving multi-objective optimization problems, many scientists and researchers have tried to use this algorithm to solve multi-objective problems, and so far, several versions of meta-heuristic algorithms have been presented for solving multi-objective problems. We will try to use this algorithm in the optimization of the investment portfolio. Hence, the purpose of this research is to optimize the investment portfolio based on the approach of the ant lion optimization algorithm and compare it with conventional approaches.

Therefore, in this research, the researcher seeks to answer a key question, what are the results of investment portfolio optimization based on ant lion algorithm? And compares its results with the conventional approach in terms of the functional criteria of capital portfolio selection, such as risk and return.

Literature review

Fernandez et al., (2019) presented an investment portfolio selection model with fuzzy returns. Two random optimization approaches and a fuzzy approach were used in their model. The model presented by them is a non-linear model. The presented model is first transformed into a deterministic equivalent model by the stochastic constraint approach. Then, the model that has fuzzy ideals is transformed into deterministic using common approaches.

Gulpinar and Rustem, (2019), in a paper entitled "A robust worst-case decision model for multi-period mean-variance portfolio optimization" extended the multi-period mean-variance optimization framework to worst-case design with competing return and multiple risk scenarios. presented an approach that includes a max-min algorithm and a multi-period mean-variance optimization framework for the random aspects of the scenario tree.

Najafi and Mushakhian (2015) in a paper entitled "Mean-Semi-Variance-Conditional Value-at-Risk Multi-Period Stochastic Investment Portfolio Optimization Model Considering Transaction Cost" using a combination of Genetic Algorithm and Particle

Optimization designed to solve the model and since the efficiency of the algorithms depends on the correct selection of the parameters, they used the Taguchi method to adjust the parameters of the algorithm.

Sun et al. (2016) in the paper entitled "Multi-period optimization of stock portfolio under potential risk" develop a minimax model for a multi-period stock portfolio selection problem. An analytical solution has been obtained and numerical simulations show the superiority of the multi-period model compared to the corresponding first period and also to the market index.

Hassanlou (2017) in the paper "multi-period stock portfolio selection using chance constraint programming" has proposed a new multi-period model of stock portfolio selection with different borrowing rates and loans based on the concept of stochastic programming. In this proposed model to evaluate the uncertainty and random nature of stock portfolio selection, it is assumed that the rate of return and the rate of borrowing and lending are independently and normally distributed, and the genetic algorithm was presented to solve the non-linear programming.

Liagkouras and Metaxiotis (2018) in the paper titled "Mean-Variance Portfolio Optimization Model for Fuzzy Multi-period Stocks with Transaction Costs" propose a multi-faceted evolutionary algorithm (MOEA) to deal with fuzzy multi-period stock portfolio optimization problems with transaction costs.

Zhang and Li (2019) presented a valid multi-purpose mean-semi-entropy model with background risk for choosing a multi-period stock portfolio in the article "Mean-Semi-Entropy Credential Model for Multi-Period Stock Portfolio Selection with Background Risk". In addition, realistic restrictions such as liquidity, cardinality restrictions, transaction cost and purchase threshold have been considered.

Liu et al., 2019, in a paper entitled "Multi-period stock portfolio performance evaluation model based on possibility theory" considered a multi-period stock portfolio optimization problem and inspired by the Sharpe ratio or the ratio of "reward change to variability", reward ratio and low and potential fluctuations for choosing the stock portfolio by measuring the effects of negative and positive fluctuations in choosing the investment portfolio. Based on two new definitions, they present a fuzzy multi-period portfolio performance evaluation model. Then, using the weighted sum approach, the proposed

model is transformed into a single programming problem and a PSO algorithm with a mechanism of access to feasibility is designed for the solution.

Gupta et al., (2020) in the paper "optimistic and pessimistic fuzzy intuitive models of multi-period stock portfolio optimization" used intuitive fuzzy to deal with the uncertainty in the market and included the degree of membership, the degree of non-membership and the degree of uncertainty in their model. They consider and introduce the optimistic model and the pessimistic model and examine the results.

Faezy and Shahabi (2020) presented the zero and one mathematical model for selecting the optimal stock portfolio based on the gray model and solved it with the meta-innovative mixed frog jump algorithm. In this research, they used the price data of 100 companies. The results of their research indicate that the proposed method can be used not only in the problem of choosing the optimal portfolio but it can also be used in multi-criteria decision-making problems.

Fallah Shams et al (2013) investigated the performance of different risk criteria in selecting and optimizing the investment portfolio using the Ant algorithm. The results of the investigation showed that the model of the average value exposed to possible risk is able to show higher levels of return by minimizing the value exposed to possible risk. On the other hand, the time spent on the implementation of the average variance model was the lowest and the time spent on the average-value under the possible risk model was the highest. As a result, although CVaR provides better efficient boundaries, it is not a suitable criterion in terms of execution time, especially in high portfolio sizes. In many cases, variance is still used by many investors as a measure of risk due to the simplicity of its calculation.

Rezaei et al. (2017) used robust optimization in the problem of choosing an investment portfolio with capital loss subject to conditional risk. The results show the high efficiency of the model in the development of models under conditions of uncertainty. Also, the results show that if the level of conservatism increases, the value of the objective function will increase.

Asgari (2015), evaluated investment portfolio modeling for the optimal selection of the investment portfolio of companies admitted to the Tehran Stock

Exchange (Metaheuristic methods approach). The research modeling results show that based on the same accuracy, the memetic algorithm of symbiosis search has a good performance in specifying and estimating the mean-variance model compared to the two genetic algorithms and SOS methods, but it takes more time for optimization. Therefore, the most important policy recommendation of this study is that investors and financial analysts should try to minimize the investment risk by using the symbiosis search algorithm in the optimization and optimal modeling of the investment portfolio and in this way get the maximum return of the portfolio.

Rostgar et al.(2017) investigated the impact of investment horizon in optimizing the investment portfolio using wavelet and Copula-GARCH. The results show that the selected model performs better compared to the bootstrap model and the simple model without data analysis. Other results state, that investments with long-term horizon should pay attention to the data analyzed at a high level, with low frequency. Also, investing with a short-term horizon should consider low-level, high-frequency data.

Farid et al. (2019) investigated the application of the fuzzy logarithmic preferences planning approach in the selection of investment portfolio. In their research, in the first stage, through interviews with experts and the review of available documents, the main criteria for choosing the optimal stock portfolio are identified, and then the application of the fuzzy logarithmic preference planning approach is presented in how to determine the weight of these criteria. The results show that the criteria of profitability, efficiency and risk are respectively the most important criteria in choosing an investment portfolio.

Mohammadi (2019) evaluated the multi-criteria model for the optimal selection of investment portfolios based on Gordon's approach. The results showed that the index (market) has the most importance among all the existing indices. The index (income growth rate) has the second priority and the index (risk-free rate) has the third importance.

Amiri et al. (2019) investigated the simultaneous role of price and stock price awareness in choosing the optimal stock portfolio. The results show that these two variables behave differently in different environments, and if you pay attention to the timing and stock price awareness, you can create more optimal portfolios by analyzing the financial and non-

financial information of companies. Regardless of these criteria, the efficiency of the optimal stock portfolio is on average around 47%, and if considering the criteria of contemporaneity and stock price awareness, this efficiency increases to 86% and 75%, respectively.

Sina et al. (2018) optimized the investment portfolio with the extreme value theory approach in the Tehran Stock Exchange. The results of the research indicated that the formation of the optimal investment portfolio using extreme value theory is not much different from Markowitz's mean-variance model.

Khajezad et al. (2019) studied the prediction of the optimal stock portfolio using meta-heuristic algorithm and the Markov decision process. The results of the research show that the cultural meta-heuristic algorithm according to the Sharpe method has the ability to create an optimal stock portfolio using predicted data with the Markowitz method for risk-taking and risk-averse investors.

Ramoz et al. (2020) evaluated the optimal portfolio selection using a consensus planning model in the Tehran Stock Exchange. By examining the absolute value of the total difference of the profitability and safety indicators resulting from the optimization of the investment utility functions through direct and comparing it with the results of the optimization performed based on the consensual planning method, the research hypothesis was confirmed. Key words: Optimal portfolio, consensus planning model, consensus set, optimality of utility functions.

Haddadi et al. (2021) optimized the stock portfolio with MAD and CVaR criteria by comparing classical and meta-heuristic methods. The results show that the NSGA2 meta-heuristic method compared to the classical method in solving the portfolio optimization problem displayed more risk in two criteria, MAD and CVaR, and therefore it is a better method for solving portfolio optimization problems.

Heydari et al. (2021) conducted a research on the stock portfolio optimization based on robust probabilistic planning model using genetic algorithms and mixed frog jump. The obtained results show that there is no significant difference between the two algorithms in choosing the investment portfolio, but the combined approach of TOPSIS and entropy weighting selects the genetic algorithm as the superior algorithm.

Mehrani et al. (2019) Comparison of the accuracy of black hole algorithms and gravitational research and the hybrid method in portfolio optimization, portfolio optimization in Tehran securities exchange using the black hole algorithm and the Gravitational Research algorithm and an algorithm named Hybrid Algorithm which combines the two algorithms above to cover the weaknesses of these two algorithms. Finally compare the results with the Markowitz model and choose the optimal algorithm. attempts to optimize the portfolio using a black hole algorithm. In all the years of research, the hybrid method introduced in this research has obtained the nearest solution to the exact solution, which is the same Markowitz. In order to optimize the portfolio, black hole meta-heuristic algorithms, Gravitational Research and hybrid algorithm (hybrid) can be used instead of the Markowitz algorithm with higher accuracy and speed. The results of the present case study and other studies show that black hole algorithms, Gravitational Research, and hybrid algorithms are very quick in solving portfolio optimization problems

Mehrani et al. (2019) Portfolio optimization using black hole meta-heuristic algorithm. in this research is optimization portfolio by using black hole algorithm in Tehran security exchange and comparing with Markowitz. The results of research show that we conclude by using black hole algorithmic that all years the black hole method has obtained results the same as Markowitz's results and can be the suitable pattern for optimization portfolio.

4- Research hypotheses:

Selecting an investment portfolio using meta-heuristic models is better than choosing an investment portfolio using conventional methods.

The ant lion optimization algorithm has the potential to choose the optimal investment portfolio.

5- Research method:

According to the scientific classification in terms of the purpose of this research, it is of the type of applied research and since the subject of this research is the optimization of the investment portfolio based on the approach of the ant lion optimization algorithm and its comparison with the conventional approaches, evidence from the Tehran Stock Exchange. The power of this research is classified as descriptive research.

From a theoretical point of view, it is a part of proof research, and from the point of view of reasoning, it is also a deductive-inductive part. Also, the research methodology is post-event, which means that the research is based on past information.

The current research is quantitative in terms of data type, applied in terms of result and descriptive and exploratory in terms of purpose. Given the fact that in this research, the past performance of the companies is examined, or in other word, the historical information of the companies is used, so the research is considered a post-event in terms of the research plan. Since research method is field-based and deals with real data, in order to provide the information of the companies accepted in the Tehran Stock Exchange, which according to the variables of the research, from different sources, Rahavard Novin software and Databases of the Stock Exchange Organization are used. Aggregating data in Excel columns, we calculate research variables and analyze them using R2021b matlab software.

The statistical population of this research is all the companies listed in the Tehran Stock Exchange between 2021 and 2022. The data of this research is from the two-year price of stock companies (symbols traded in Tehran Stock Exchange) from the beginning of 2021 to the end of 2022. The purpose of choosing this sample is to investigate the efficiency of the meta-heuristic algorithm of ant lion optimization in portfolio investment optimization.

1-5- Research model and variables:

the portfolio calculation method using the Markowitz model:

In the mean-variance model designed by Markowitz, it shows the average expected return and the variance is the risk of the portfolio.

$$\begin{aligned} Minz &= \delta_p^2 \\ St: \bar{r}_p &= \sum_{j=1}^n w_j \cdot \bar{r}_j \\ \sum_{j=1}^n w_j &= 1 \\ w_j &> 0 \end{aligned}$$

The portfolio return variance is calculated according to the following equation:

$$\delta_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j cov(\bar{r}_i, \bar{r}_j)$$

Ri represents the real yield of stock i on day t and is calculated from the following equation:

$$R_i = \frac{(P_{t+1} - P_t) + D + M + N}{P_t}$$

Pt+1 = price on the day after t

Pt = price on day t

D = cash interest

M=right of first refusal

N= dividend profit

An efficient portfolio means the optimal combination of assets in such a way that the risk of the portfolio is minimized for a given rate of return. In fact, two important components in making investment decisions are the amount of risk and the yield of capital assets. Rational investors consider the return to be favorable and are risk averse. In addition, they act rationally in decision-making, which maximizes their desired efficiency. Therefore, investors' utility is a function of expected return and risk, which are the two factors that are the basic parameters of investment decisions. In other words, in the asset portfolio optimization problem, we are looking for a portfolio that produces less standard deviation (risk) and more expected value. This problem can be modeled in the form of an operation research model, in the form of the following equation:

$$\begin{aligned} Min Risk \sigma_i^2 &= \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \\ Max Return R_p &= \sum_{i=1}^n w_i E(R_i) \end{aligned} \tag{1}$$

$$s. t. \sum_{i=1}^n w_i = 1$$

where w_i is the weight of asset i, or in other words, the percentage of funds that can be invested in this asset, Ri is its yield, and Rp is the yield of the entire asset portfolio. σ_i^2 also represents the variance of the i-th asset and σ_{ij} represents the covariance between the i-th and j-th assets. Also, sometimes it is necessary to add another restriction to the model in order to prevent borrowed sales, which is the positive weight of each of the assets in the portfolio. The last limitation in this model is that the sum of the assets' weights is equal to one.

Model 1 is a model that has been used in most papers and similar researches in this field as a basic model that, with different mathematical or meta-heuristic methods, different combinations of wis that

optimize the two criteria of minimum risk and maximization of returns It is estimated. Also, since the presence of fewer assets in the portfolio is expected to reduce the cost of buying and selling shares and also make portfolio management easier and better, in this research the optimization problem has three objectives (that is, maximizing the return of the stock portfolio, minimizing its risk and the third objective function, i.e. minimizing the number of assets or shares) has been studied. Based on this, investors accept a small amount of risk and approximately the same amount of return, and choose a portfolio that has fewer assets. Therefore, the innovation considered in this research is to consider the third criterion or goal under the title of minimizing the number of assets or shares in the stock portfolio ($Min\ count(i|w_i \ll 0)$) in the form of model 2.

$$\begin{aligned}
 Min\ Risk\ \sigma_i^2 &= \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \\
 Max\ Return\ R_p &= \sum_{i=1}^n w_i E(R_i) \quad (\Upsilon)
 \end{aligned}$$

$$Min\ count(i|w_i \ll 0)$$

$$s. t. \sum_{i=1}^n w_i = 1$$

As it can be seen from the optimization model, this model is a multi-objective optimization problem in which the decision-maker has a set of conflicting objectives (here, maximizing the return of the portfolio, minimizing the corresponding risk and the minimum number of stocks in the portfolio). In this type of problems, unlike single-objective optimization problems and due to the existence of several conflicting objectives, instead of only one solution, a set of solutions is obtained, which is called Pareto (non-dominated) solutions. In other words, it is almost impossible to find an optimal solution that simultaneously optimizes all functions in case of conflicting objectives. However, it is possible to find a set of solutions that provide the best interaction between the objectives so that they do not improve unless they cause other objectives to deteriorate.

In 1950, Harry Markowitz presented his portfolio model, which became the basis for modern theories of portfolio management. For the first time, he quantitatively showed why and how asset portfolio

diversification can reduce the risk of an investor's portfolio (investment portfolio). According to Markowitz, since investors are not sure about the future, they should diversify their investments to reduce risk. Asset portfolio diversification means choosing the best combination of financial assets in a way that maximizes the return of the investment portfolio and minimizes its risk as much as possible. Risk aversion of all investors is the main assumption of this model. In the Markowitz model, the average asset portfolio shows the expected return.

And the variance represents the risk of the asset portfolio. In fact, Markowitz presented the asset portfolio optimization problem as a quadratic planning model with the aim of minimizing the variance of the asset portfolio with the condition that the expected return is equal to a constant value. Therefore, this model is also called mean-variance. An asset portfolio that has sufficient diversity is called an efficient asset portfolio. An efficient portfolio means the optimal combination of assets in such a way that the risk of the portfolio is minimized for a given rate of return. Investors can determine the efficient asset portfolio by specifying the expected rate of return of the portfolio and minimizing its risk at this level of return. In Markowitz's theory, we look for a portfolio that produces lower risk and higher returns. Therefore, by minimizing the risk (variance) of the asset portfolio with the assumption (or limitation) of the constant rate of its return, it is possible to model the Markowitz theory. Also, sometimes it is necessary to add another limitation to the Markowitz model; this limitation is the positivity of the weight. Each of the assets in the portfolio is due to the fact that the weights can be negative It is in the form of variance minimization, hence the Markowitz model is a non-linear optimization model (quadratic) that can be modeled in the form of an operation research model in the form of equation 3:

$$\begin{aligned}
 Min\ Var(R_p) &= \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \\
 s. t. \sum_{i=1}^n x_i E(R_i) &= R_{Expected} \quad (\Upsilon) \\
 s. t. \sum_{i=1}^n w_i &= 1
 \end{aligned}$$

Note that by solving the above optimization model, only one efficient point on the efficient boundary

(corresponding to $R_{Expeted}$) is produced, and as a result, to determine the efficient boundary, the above model must be equal to the number of points on this boundary, per $R_{Expeted}$ to solve different

In a simple two-stage model, based on the model proposed by Hu, Xu and Zhang (2015), which will be called HXZ (2016) from now on, this theory can be defined in the form of relation number 4:

$$E_t[r_{it+1}^s] = \frac{E_t[\Pi_{it+1}/A_{it+1}] + 1}{1 + a(I_{it}/A_{it})}$$

While in this relation A is the book value of assets for company i on date t , I is the level of investment and $a > 0$ is a fixed parameter in the asset cost adjustment function. According to the quadratic adjustment of costs, the company's final costs for adding an additional unit of assets at time t are equal to $1 + a(I_{it} / A_{it})$. The mentioned final cost is actually the sum of the final costs of the invested asset unit and the final cost adjustments.

The numerator, $[\Pi_{it+1}/A_{it+1}] + 1$, is the expected return for this additional investment in the time period $t + 1$, which includes the expected return and the final value of the asset unit. Therefore $(E_t[\Pi_{it+1}] + 1) / [1 + a(I_{it} / A_{it})]$ is the expected investment return of a final unit of assets. Without considering the financial leverage and in fact assuming no use of external financing and borrowing, the cost of capital is equal to the expected return E_t , and in this case the company's investment level is optimized if the expected investment return is equal to Expected stock returns per unit of final investment. This is the balance that Zhang (2015) refers to as the capital assets evaluation model or CAPM.

Based on what is explained in the explanation of the evaluation pattern of investment returns based on the Q theory and with the assumption of multi-period return of capital, the evaluation pattern will be defined as described in relation number 5 and as follows, which contains additional variables compared to the previous relation. :

$$E_t[r_{it+1}^s] = \frac{E_t[\Pi_{it+1}/A_{it+1}] + (a/2)E_t[(I_{it+1}/A_{it+1})^2] + (1 + aE_t[I_{it+1}/A_{it+1}])}{1 + a(I_{it}/A_{it})}$$

In this regard, the component $(a/2) E_t [(I_{it+1} / A_{it+1})^2]$ is the expected savings in the adjusted costs at the time point $t + 1$, which is from an additional unit of investment in Time period t is obtained and $1 + a E_t [I_{it+1} / A_{it+1}]$ is the continuous value of an investment unit in time period t . and $(1 + a E_t [I_{it+1} / A_{it+1}]) / [(1 + a (I_{it} / A_{it}))]$ is almost equal to $E_t[I_{it+1} / I_{it}]$ intuition]. , a lower investment growth rate is expected at date t with higher investment at date $t+1$ (i.e., high expected investment growth), cost savings and the continuous value of the firm's return on investment for the required revision Stop the market. This investment return component is not available from the two-period model.

Stock price fluctuations, profitability, investment growth and stock returns:

Fama and McBeth (1973) regression has been used to estimate the relationship between stock price fluctuations, profitability fluctuations, investment growth fluctuations and stock return fluctuations. In this regard, the fluctuations in stock returns based on the annual standard deviation of stock risk as a dependent variable, the annual standard deviation of the difference between the lowest and the highest stock price in each month, the deviation of the company's profitability standard based on the return on equity over the past three years, the deviation The criteria for the growth of capital expenditures during the last three years have been defined as independent variables.

In order to predict stock price fluctuations, a logarithmic function will be used. Konrad et al. (2014) provided evidence that stocks that are likely to have a price bubble will experience low returns and price declines in the following periods. Zhang and Kang (2016) found that stocks that are most likely to experience future price declines experience little profit in the capital market afterwards. The findings of this research show that the possibility of falling prices is a characteristic that indicates the existence of excessive prices. Since there is currently a possibility of the stock price increasing too much, until the stock price reaches the highest possible price in the stock market, there is a possibility of this increase.

In this research, a generalized logit model will be used in order to predict the probability of price collapse and bubble for individual exchanges, following the proposed model of Zhang and Kang (2016). While Konrad et al. (2014) have used a binary logit model to predict the probability of the return of

price increases and additional returns. Zhang and Kang (2016) used a generalized logit model in which the dependent variable is a multi-state variable in which the states: extreme negative return (collapse), extreme positive return, price bubble and non-extreme return are defined.

In this research, the binary model was used to evaluate the two situations of collapse or price bubble, and the probability of occurrence of accidental situations was evaluated separately. This model could not detect the skewness, and as a result, it was possible that the probability of occurrence of random events and the price bubble were correlated with each other and the future fluctuations. The generalized Logit model was designed to solve these problems. Specifically, a price crash or a large negative return event with a small probability is defined as the entry return in the next 12 months. In this research, it is assumed that for each company, the probability of collapse and price bubble will have the following distribution as described in relation (6):

$$\Pr_t(Y_{i,t,t+12} = -1) = \frac{\exp(\alpha_{-1} + \beta_{-1}X_{i,t})}{1 + \exp(\alpha_{-1} + \beta_{-1}X_{i,t}) + \exp(\alpha_1 + \beta_1X_{i,t})}$$

$$\Pr_t(Y_{i,t,t+12} = 1) = \frac{\exp(\alpha_1 + \beta_1X_{i,t})}{1 + \exp(\alpha_{-1} + \beta_{-1}X_{i,t}) + \exp(\alpha_1 + \beta_1X_{i,t})}$$

While in this equation, where $Y_{i,t,t+12}$ is a three-state variable that is equal to -1 if the input of company i is less than 25% during the months $t+1$ to $t+12$, if the same efficiency is greater than 25% is equal to 1 or zero. The term $X_{i,t}$ represents a vector of explanatory variables measured at the end of month t . In this research, according to Konrad et al. (2014), Jang and Kang (2016) and Jang (2018), explanatory variables have been chosen. These explanatory variables include: past returns (RET12), total volatility (TVOL), total skewness (TSKEW), company size (SIZE), financial turnover of retail stock exchanges (TURN), company age (AGE), current assets (are CRRNT), sales growth (SALESG), and past return (RET12) is defined as the return on stock inputs over the past 12 months. Total Volatility (TVOL) and Total Skewness (TSKEW) are calculated as standard deviation and inconsistency of daily returns over the last three months, respectively. Company size (SIZE) is measured as input from the logarithm of company

value. Net turnover (TURN) is defined as the average six-month stock return of the previous 18-month average stock return. Age of the company (AGE) is defined as the number of years since the establishment of the company in the data set. Current assets (CRRNT) are obtained by dividing current assets by total assets. Finally, sales growth (SALESG) is defined based on the change in net sales compared to the past in terms of percentage. In some researches such as Konrad et al. (2014) and Jang and Kang (2016) tangible assets have been used to predict extreme returns, in this research current assets have been used. Because current assets may be more effective than long-term tangible assets compared to long-term returns compared to future years.

Lagging structures of annual accounting data such as current assets and sales have also been used to ensure that all information used as predictors at the beginning of the 12-month period for investors in measuring extreme returns. The future is used. In addition, the pseudo coefficient of determination will be used as a criterion for measuring the explanatory power of the estimated relationship. This standard has been used based on the model proposed by Nagelkark. In this research, as it was said, a three-state variable has been used in predicting anomalies in stock returns, including crashes, price bubbles, and other cases. Classification of stock portfolios and the possibility of collapse and bubble:

In this section, it has been assumed that the probability of random events, crash and bubble is significantly a function of cross-sectional stock returns. In particular, the securities are sorted based on each of the predicted possibilities of random changes, collapse and price bubbles, and the yield of each group of observations that takes place in the next month is observed. In order to avoid forecasting error, the yield of each month is re-estimated using the available historical data, in the period under review, using a developed logarithmic relationship. Then, the observations related to random situations in month t with a set of parameters estimated from the window that ends in month $t-12$ are examined to ensure that other predictions before the time period that Future falls in stock prices are measured, not observed.

In other words, the indirect expectations in month t of an accidental event in the future during the next 12 months as the probability of falling calculated on the basis of the explanatory variables of the time period

and other variables are determined and the estimated parameters using the data for the explanatory variables up to Month $t-12$ has been estimated and therefore, the data for the dependent variable up to month t ignores the predicted probabilities before the beginning of the period. In this research, the probability of a stock price crash is estimated and called it (CRASHP) and the probability of a price bubble is also estimated and it is called (JACKPOTP). Based on each of the predicted possibilities, the investment return at the end of each month is determined and the return of each stock portfolio is calculated in month $t + 2$. In this research, monthly estimates were used to prevent possible short-term effects. The cross-sectional efficiency estimation model is defined in the form of equation 7, which is a non-linear model and the Jang and Kang (2016) model was used in its calculation:

$$r_{i,d} - r_{f,d} = \alpha_i + \beta_i(r_{m,d} - r_{f,d}) + \gamma_i(r_{m,d} - r_{f,d})^2 + \varepsilon_{i,d}$$

where the dependent variable is the stock risk compared to the market return and the explanatory variable is the excess return of the shares compared to the risk-free return, which is linear and quadratic in the estimation relationship. In evaluating cross-sectional stock returns, the model used in researches such as: Ang et al. (2006), Bali, Kakiki and Whitelove (2011), Strambagh, Yu and Yuan (2012) and finally Konrad (2014) were followed and the parameters A position such as decimals has also been used to describe the yield dispersion.

The results of cross-sectional efficiency estimation cannot be expressed with the three-factor model of Fama and French (1993) and to increase the explanatory power, the proposed model of Zhang and Kahn (2016) has been used, although the difference between the two models can be estimated. For comparison, the cumulative return is taken from a similar investment strategy in the market portfolio, defined as the value-added portfolio of all listed stocks. Based on this criterion, the cross-sectional difference in yield between the lowest and highest falling circles is studied and considered as the probability of falling stock price. Portfolio calculation using the ant lion optimization algorithm

Ant lion is a member of net-winged insects. These types of insects use a strange method and technique to hunt and feed ants. The algorithm presented in this project is inspired by the interaction between the ant lion and the ant in hunting. Like the meta-heuristic algorithm of genetics, this algorithm is also based on the population, so a set of candidate answers is collected at each stage when the algorithm is executed. The ant lion optimization algorithm produces the initial population of ants (information) in the first step, which creates a set of solutions to solve the problem. In the second step, the correctness of the values given to the ant position (information) is checked.

In the third step of the optimization algorithm of ant lion, which is considered the most important step, the calculation of the fitness function of the ant (information) is performed. An ant's merit function is used to show how optimal this solution is. This algorithm takes advantage of the reactions of ants that are trapped by ant lion.

If we want to briefly explain how the ant lion algorithm works in one sentence; It can be said that the ALO algorithm imitates the interaction between ants and their behavior in the trap. To model such an interaction, ants must move through the search space. Ants are allowed to hunt them and improve by using traps. Since ants move completely randomly when searching for food in nature; A random walk equation is chosen to model the movement of ants as follows.

$$X(t) = [0, \text{cumsum}(2r(t_1) - 1), \text{cumsum}(2r(t_2) - 1), \dots, \text{cumsum}(2r(t_n) - 1)]$$

In the equation above, when Cumsum calculates the cumulative sum; n will be the maximum number of iterations and also in this case, t represents the random walk step and $r(t)$ is a random function and is defined as follows, where t represents the random walk step and rand , is a random number that is generated by a uniform distribution in the interval $[0,1]$.

$$r(t) = \begin{cases} 1 & \text{if } \text{rand} > 0.5 \\ 0 & \text{if } \text{rand} \leq 0.5 \end{cases}$$

6- Results:

Estimation of patterns based on return (PTH) and asset growth (AG):

In this section of the findings, the predictions of return and investment growth have been discussed. It

is expected that growth stocks with high returns will have lower returns in the future and stocks with low returns will have better returns and growth in the future, and at the same time, the ant lion optimization algorithm approach is better able to predict returns compared to conventional approaches. The future and growth of investment has. In this regard, he started to sort stock portfolios based on investment yield and

growth, and according to two factors: 1) adjusted yield and 2) investment growth, 25 portfolios or investment portfolios were formed and based on this classification 25 types have been estimated. In the following table, firstly, the explanation of the efficiency of these 25 portfolios and then the estimation of the parameters of the multi-factor models and the model based on the Q factor are mentioned:

Table 1: stock returns in portfolios based on profitability (PTH) and asset growth (AG)

average excess return ($r_{it}-r_{ft}$)										
AG5	AG4	AG3	AG2	AG1	AG5	AG4	AG3	AG2	AG1	گروه
0/05 (0/81)	0/48 (1/09)	0/67 (2/65)	0/59 (1/98)	0/48 (1/35)	0/05 (0/35)	0/61 (1/96)	0/68 (2/64)	0/58 (1/35)	0/43 (1/66)	PTH1
0/47 (0/84)	0/69 (1/64)	0/79 (2/38)	0/82 (2/29)	0/89 (3/28)	0/35 (0/78)	0/68 (2/34)	0/98 (4/38)	1/08 (5/35)	1/25 (3/38)	PTH2
0/37 (2/02)	0/67 (2/66)	1/02 (2/65)	1/14 (2/98)	1/28 (3/36)	2/34 (1/99)	0/64 (2/05)	0/76 (3/17)	0/82 (3/37)	0/86 (4/48)	PTH3
0/67 (2/08)	0/76 (2/21)	0/89 (2/25)	1/12 (3/05)	1/24 (3/38)	0/47 (2/28)	0/58 (3/22)	0/72 (4/05)	0/87 (4/33)	0/99 (4/58)	PHT4
0/65 (2/02)	0/79 (2/25)	0/96 (3/31)	1/05 (3/61)	1/25 (4/58)	0/48 (2/02)	0/67 (2/29)	0/67 (3/22)	0/87 (3/65)	0/98 (3/99)	PHT5
Adjusted return factors (alphas) in different models based on 25 portfolios										
Average portfolio					total portfolio					
T statistic		Estimated factor			T statistic		Estimated factor		factor	
(5/61)		1/89			(3/58)		1/38		One factor	
(2/21)		0/68			(2/01)		0/81		CAPM	
(4/41)		1/61			(3/12)		1/25		Three factor	
(3/88)		1/35			(4/48)		0/99		carhartt	
(3/16)		1/05			(4/05)		0/67		Five factor	
(6/16)		0/78			(5/11)		0/41		Q	
Factor loadings in the model based on Q factor based on 25 portfolios										
Average portfolio					total portfolio					
T statistic		estimated			T statistic		estimated		q factor	
(4/45)		0/87			(3/66)		0/89		ROE	
(4/68)		1.36			(4/48)		1/35		L_A	
(3/33)		-0/24			(1/38)		-0/25		ME	
(3/39)		-0/46			(2/25)		-0/48		MKT	

The results obtained in the regression estimations and statistical significance tests of the estimated coefficients as summarized in section B of the above table show that:

- 1) The Student's t-statistic in examining all 25 investment portfolios, according to the model based on the Q factor, is equal to 5.11 and is significant at the 5% test level. While in single-factor models, CAPM, three factors of Fama and French (1995), 4 factors of Carhartt (1997) and 5 factors of Fama and French (2015) are respectively equal to: 3.58, 2.01, 3.12, 4.48 and 4.05 were calculated, which were significant in all cases at the 5% confidence level, but the significance of the Q

pattern was more significant due to the significance level corresponding to Student's t-test tending to zero. In other words, in the regression estimation for the model based on the Q factor, it is possible to predict future returns and investment growth based on 25 classified investment portfolios based on returns and investment growth.

- 2) Student's t-statistic in the average investment portfolio analysis, according to the model based on the Q factor, is equal to 6.16 and is significant at the 5% level of the test. While in single-factor models, CAPM, three factors of Fama and French (1995), 4 factors of Carhartt (1997) and 5 factors of Fama and French

(2015) are respectively equal to: 5.61, 2.21, 4.41, 3.88 and 3.16 were calculated that in all cases it was significant at the 5% confidence level, but the significance of the Q model was more significant due to the tendency of the significance level corresponding to Student's t-test to be zero. In other words, in the regression estimation for the model based on the Q factor, it is possible to predict future returns and investment growth based on the average of 25 classified investment portfolios based on return and investment growth.

Comparing the estimation of investment portfolio optimization based on the ant lion optimization algorithm approach and comparing it with conventional approaches:

First, we design the neural network using the toolbox. We also introduce the input data of the network, which are the components of the investment portfolio with risk minimization and efficiency maximization, to the network and specify it for it.

Now, using this toolbox, the network should be trained. The training of the network is done using the ant lion optimization algorithm.

The neural network model includes the second stage

1- Train

2- Testing weights and error calculation

Based on the ant lion optimization model, we define 70% x (investment portfolio optimization components) and y (investment risk) in the input and output layers. The network defines some weights by using training

and using artificial intelligence, and in the next step, it tests these weights with the remaining 30% of x's (the components of the investment portfolio optimization) and obtains the investment risk and tests to determine how close the predicted value is to reality and calculate the error.

The network has done the training until it reaches the optimal point of training, (the optimal point of training is the level of training where the network was able to estimate the best weights using training and learning), when we test the weights with 30% of the remaining x's The predicted values have the least difference with the reality.

In order to prevent the network from being overtrained, a number of repetitions are gradually taught and then it is tested with the test data to find at least a few errors on the screen (the base is the mean squared error MSE) and the least of these is the lowest and the optimal training of the network is selected based on them.

After training in different times, the best training is selected. Here, 21 times the training has been done, and in the 15th order, the learning was better than the next times. So set 15 is selected as network learning.

The optimal amount of training in learning is given in the form of a diagram. The goodness of their fit has also been determined and the input and target data have been matched.

The results of neural network training are network weights that sit on the communication lines between neurons and other components, both input and output.

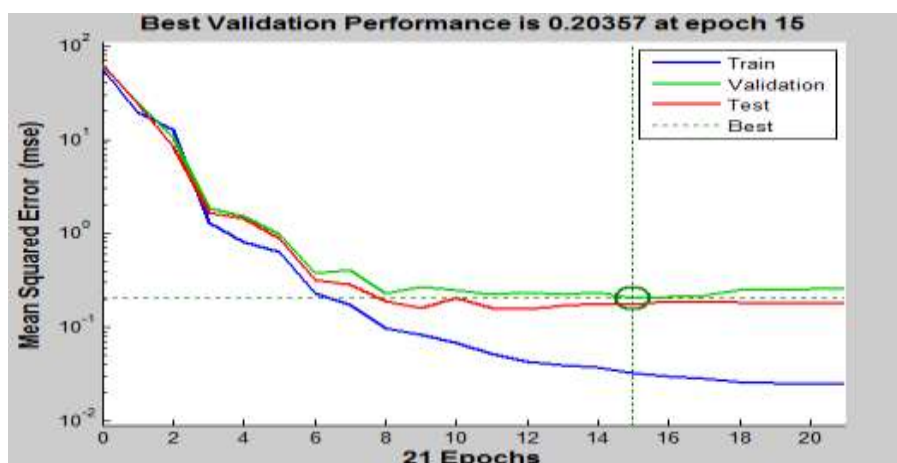


Figure 2-4: neural network training diagram of the ant lion optimization algorithm

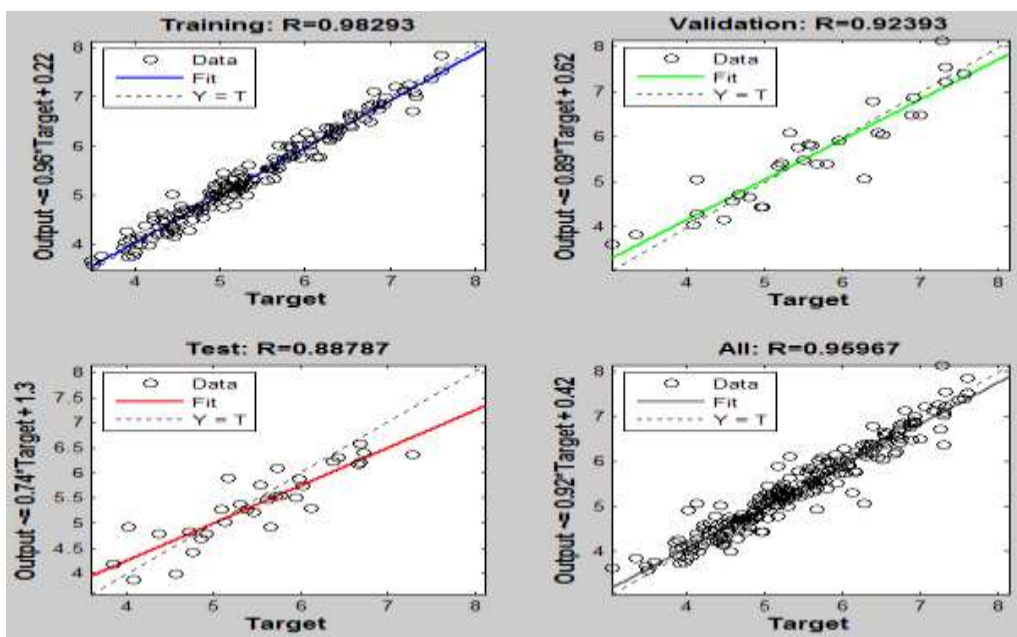


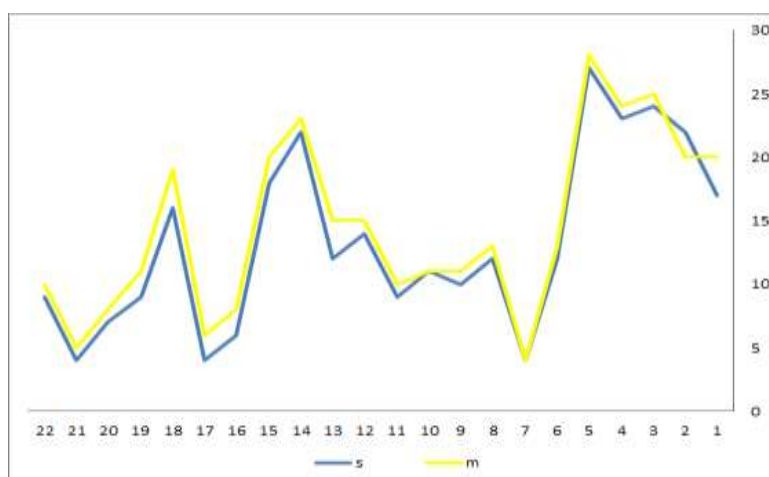
Figure 3-4: neural network training diagram

The optimal amount of training in learning is given in the form of a diagram. The goodness of their fit has also been determined and the input and target data have been matched.

The results of neural network training are network weights that sit on the communication lines between neurons and other components, both input and output. The weights between the input layer and the hidden layer are shown in the table below. According to the

table of weights, it is possible to predict investment risk using the ant lion optimization algorithm.

The graph above shows the prediction of the network based on the input data and comparing them with the actual values. The red graph is the prediction with neural network and the blue graph is the values of the conventional approach of stock portfolio formation. Now, in the figure below, the results of the ant lion optimization algorithm and their comparison with the conventional approach are presented



In this diagram, the yellow color corresponds to the ant lion optimization algorithm and the blue color corresponds to the conventional approaches.

As can be seen in the diagram above, the error of the ant lion optimization algorithm and its jumps are less compared to the conventional approach, so it can be said that due to the less deviations of the neural network in the ant lion optimization algorithm, its estimation power can be higher. He knew about the conventional approach.

According to the topics raised and the hypotheses raised in this research, it can be seen that the combination of artificial neural network and investment portfolio optimization components are capable of estimating the investment portfolio optimization, and according to the RMSE of the presented model using the network. In this research, nerves have the power to optimize the investment portfolio, but more precisely, the results show that:

The ant lion optimization algorithm has a higher ability to optimize the investment portfolio than the conventional approach.

7- Discussion and conclusion:

As stated earlier, the main goal of this research is to optimize the investment portfolio based on the ant lion optimization algorithm and compare it with conventional approaches. To achieve this demand, we first created stock portfolios and calculated the risk and return of the created portfolios. In the data analysis section, each hypothesis was tested and the relationship between the factors of each hypothesis and the optimization of the investment portfolio was analyzed.

According to the analysis, we observed that the selection of the optimal investment portfolio using the ant lion algorithm has the ability to optimize the investment portfolio using the components of maximizing returns and minimizing risk, and at the same time, based on the results, The ant lion optimization algorithm has a high ability (more than 93%) to optimize the investment portfolio with emphasis on the components of maximizing efficiency and minimizing risk.

One of the important issues in capital markets is the issue of choosing the optimal investment portfolio, and in this regard, many investigations and studies have been conducted by investors regarding the amount of risk and its return. It is usually assumed that

investors do not like risk and avoid it, and they always seek to invest in assets that have the highest return and the lowest risk. In other words, investors view investment return as a favorable factor and view the return variance (risk) as an inappropriate element. The model presented in this research is by considering both these objective functions and by considering the conditions governing the elements of the portfolio. As a result, in order to achieve such a goal, it is necessary to propose mathematical planning according to the financial information and conditions governing investment in the real world. In order to acquire securities and any investment, different types of expenses are created. In the meantime, the most important cost factor is the purchase cost, but there may be other cost factors such as fixed costs. The purpose of this research is to consider the uncertainty in the models related to stock portfolio optimization to bring the model closer to the real world. The issue of interest here has the objective of minimizing the ratio of risk to return of each share.

In the past, investors used their experiences or others to achieve their goals. With the progress of financial management, investors' choices became more scientific and they were able to make the optimal choice to some extent by using different models and combining its results with their own experiences. According to modern financial theories, the factors that should be considered for choosing a suitable portfolio are very important. Rational investment decisions require attention to many criteria and different factors (liquidity, currency risk, transaction costs, different investment behaviors and financial market trends) at the same time. Difficulty in choosing a stock portfolio is one of the important research topics in financial affairs. The main purpose of choosing a stock portfolio is to choose the best combination of assets and the highest expected return; while providing an acceptable level of risk.

On the other hand, finding a suitable method for prioritizing stocks with the aim of creating a suitable vision for the investor regarding the future macro situation and long-term investment horizon is of interest in this research, and as a result, using the time horizon method, prioritizing stocks and finally the main model has been solved using meta-heuristic algorithms. The proposed algorithm in this research is the ant lion optimization algorithm. Finally, for a better analysis of the process, the results obtained from

the ant lion optimization algorithm have been compared with the results of conventional approaches. The results obtained from the test of research hypotheses showed that:

- 1) Selecting an investment portfolio using meta-heuristic models is better than selecting an investment portfolio using conventional methods.
- 2) The ant lion optimization algorithm has the ability to choose the optimal investment portfolio.

The results obtained from the research of Faizi and Shahabi 2020, Liu et al. 2019, Liagoras and Metaxiotis 2018, Hassanlou 2017, Najafi and Moshkhian 2015, Gupta et al. 2013, Modares et al. 2013, and Heydari et al. 1400 are in line with the results of this research.

An efficient portfolio means an optimal combination of securities in such a way that the risk of that portfolio is minimized in exchange for a certain return. The Markowitz model showed that the most important factors in choosing the optimal stock portfolio are the two factors of return and risk. The results of this research show that the optimization methods based on the defined objective function sought to select the stock portfolio that has the highest return and the lowest risk. According to the Markowitz model in the problem of choosing the optimal financial portfolio, due to the quadratic nature and the upper and lower limits of investment in each asset and according to the exact solutions available in mathematical programming, there are always problems in obtaining the optimal answer for it. has had Therefore, in this research, an attempt has been made to evaluate the ability and application of the ant lion optimization algorithm as a smart method in solving investment portfolio optimization problems with an integer limit as well as the limit related to the upper and lower limits of the investment ratio in each Assets should be shown using different definitions of risk, i.e. mean variance, semi-variance and mean absolute deviation. On the other hand, the use of meta-heuristic algorithms enables the investor to apply his desires and preferences towards risk and return in the final model. The aim of the current research is to optimize the investment portfolio based on the approaches of the ant lion optimization algorithm and compare it with conventional approaches.

The results obtained from the test of research hypotheses showed that the components of

maximization of efficiency and minimization of risk using the ant lion optimization algorithm have a higher power than the conventional approach to optimize the investment portfolio.

References

- Amiri, Maitham, Hosseini, Seyyed Mojtabi. (2019). The role of price simultaneity and stock price awareness in choosing the optimal stock portfolio. *Financial Management Strategy*, 7(3), 25-60.
- Haddadi, Mohammad Reza, Nadami, Yunus, Tafi, Fatemeh. (2021). Stock portfolio optimization with MAD and CVaR criteria by comparing classical and meta-heuristic methods. *Financial Engineering and Securities Management*, 12(47), 514-533.
- Heydari, Mohammad Saeed, Walidi, Javad, Ebrahimi, Seyed Babak. (2021). Stock portfolio optimization based on robust probabilistic programming model using genetic algorithms and mixed frog jump. *Financial Engineering and Securities Management*, 12(47), 564-586.
- Mehrani, K., Mirshahvalad, A., & Abbasi, E. (2019). Comparison of the accuracy of black hole algorithms and gravitational research and the hybrid method in portfolio optimization. *International Journal of Finance & Managerial Accounting*, 4(14), 111-126.
- Mehrani, K., Mirshahvalad, A., & Abbasi, E. (2019). Portfolio optimization using black hole meta-heuristic algorithm. *Specialty Journal of Accounting and Economics*, 5(2), 1-13.
- Khajezadeh, Samiran, Shahvardiani, Shadi, Daneshvar, Amir, Madanchi Zaj, Mehdi. (2020). Prediction of optimal stock portfolio, meta-heuristic algorithm approach and Markov decision process. *Decision making and operations research*, 5(4), 426-445.
- Ramoz, Najmeh; Zahra Akbari Aqhamshhodi and Ali Reza Atefat Dost, (2020), choosing the optimal portfolio using consensus planning model in Tehran Stock Exchange, *Financial Management Strategy* 8 (1),
- Rostgar, Mohammad Ali, Okinejad, Mohammad. (2017). The effect of investment horizon in optimizing the investment portfolio using wavelets and Garch-Copola. *Risk modeling and financial engineering*, 2(3), 340-361.

- Rezaei Mohammad Hossein, Qahtarani Alireza, Najafi Amir Abbas. Application of robust optimization in the problem of choosing an investment portfolio with capital loss subject to conditional risk. *Operations research in its applications*. 2017; 14 (2): 81-93
- Asgari Mohammad Ali (2016) "Investment portfolio modeling for the optimal selection of the investment portfolio of companies listed on the Tehran Stock Exchange (the meta-evolutionary approach)" doctoral thesis of Urmia University
- Farid, Dariush; Abolfazl Dehghani Firouzabadi and Hamidreza Mirzaei, 2019, application of fuzzy logarithmic preference planning approach in choosing investment portfolio, 4th International Industrial Management Conference, Yazd, Yazd University - Iranian Industrial Management Scientific Association.
- Fallah Shams, Mirfaiz, Abdullahi, Ahmed, Moghads, Motaheera. (2013). Investigating the performance of different risk criteria in selecting and optimizing the investment portfolio using the Ant algorithm in companies listed on the Tehran Stock Exchange. *Financial Management Strategy*, 1(2), 15-32.
- Sina, Afsanah, Fallah Shams, Mirfiz. (2019). Optimizing the investment portfolio with Frein's value theory approach in Tehran Stock Exchange. *Financial Engineering and Securities Management*, 10(40), 184-200.
- Safari Saeed, Mohammad Javad Sheikh, Youssef Mushtaghi (2016) "Choosing the optimal investment portfolio using ideal modeling and the hierarchical analysis process of the economic value added approach" *Perspective of Financial Management*, No. 13 Spring 2015, pp. 37-52
- Mohammadi, Parisa, (2019), multi-criteria model for the optimal selection of Mitani investment portfolio based on Gordon's approach, the first international conference on knowledge management, blockchain and economics, Tehran, Iran Knowledge Management Association,
- Faezy, F. & Shahabi, V. (2020). Forming the stock optimized portfolio using model Grey based on C5 and the Shuffled frog leap algorithm. *Journal of Statistics and Management Systems*; 19: 397-421.
- Fernandez A., Gomez S, (2019), Portfolio selection using neural networks; *Computer& Operations Research*.s, 34.
- Gulpinar, N. Rustem, B, 2019. Worst-case robust decisions for multi-period mean–variance portfolio optimization. *European Journal of Operational*, Volume 183, pp. 981-1000.
- Gupta, P. Mehlawat, M.K, Yadav, S. Kumar, A, 2020. Intuitionistic fuzzy optimistic and pessimistic multi-period portfolio Intuitionistic fuzzy optimistic and pessimistic multi-period portfolio. *Soft Computing*.
- Hassanlou, K., 2017. A multi period portfolio selection using chance constrained programming. *Decision Science Letters*, pp. 221-232
- Liagkouras, K. Metaxiotis, K, 2018. Multi-period mean–variance fuzzy portfolio optimization model with transaction costs. *Engineering Applications of Artificial Intelligence*, Volume 67, pp. 260-269.
- Liu, Y. Zhang, W. Gupta, P, 2019. Multi-period Portfolio Performance Evaluation Model Based on Possibility Theory. *TRANSACTIONS ON FUZZY SYSTEMS*.
- Najafi, A.A. Mushakhian, S, 2015. Multi-stage stochastic mean–semivariance– CVaR portfolio Multi-stage stochastic mean–semivariance–CVaR portfolio. *Applied Mathematics and Computation*, Volume 256, pp. 445-458.
- Sun, Yufei. AW, G. Teo, k. Zhu. Y. Wang, X, 2016. Multi-period Portfolio Optimization Under Probabilistic Risk Measure. *Finance Research Letters*, Volume 18, pp. 60-66.
- Wei Zhou, Zeshui Xu 2018 " Portfolio selection and risk investment under the hesitant fuzzy environment" *Knowledge-Based Systems* 144 (2018) 21–31
- Zhang, J. Li, Q, 2019. Credibilistic Mean-Semi-Entropy Model for Multi-Period Portfolio Selection with Background Risk. *Entropy*, Volume