Designing an optimal supply chain model for price determination in the steel industry based on market structure with a Neural Network approach and game theory

Mohamad Ali Afshar kazemi
Professor, Faculty of Management, Islamic Azad University Central Tehran Branch, Tehran, Iran.
M_afsharkazemi@iauec.ac.ir

Mina Kazemian
PhD in Industrial Management, Graduated from Islamic Azad University, Science and Research Branch, Tehran, Iran.
(Corresponding Author)
mina.kazemian@srbiau.ac.ir

Submit: 13/10/2024 Accept: 09/04/2025

ABSTRACT

The main issue in the steel industry and supply chain management is to identify and model fluctuations in this market. Considering the vertical chain in this industry and the interaction between players, game theory is used to model the optimal price. On the other hand, players need to interact with and repeat the game to reach a balance, for which neural network models were employed. In the following, according to the specific conditions of the country that is facing severe sanctions in the metal industry, the sanctions variable is considered as an adjustment factor in the price modeling of this industry.

The research method is practical in terms of purpose. The research period of seasonal data is from 2011 to 2020, and MATLAB software is used.

Based on the explanations, a hybrid model based on neural networks and game theory was presented. To predict steel prices, three Bayesian neural networks, support vectors, and cross diffusion were used .The results indicate that the cross-emission model of Grossberg is more accurate in predicting steel prices Then the predicted price was entered into the game theory process and the Nash equilibrium point of the model was determined. The results indicate that the presence of sanctions in the model has increased the price and decreased production in the steel industry.

Keywords:

Optimal Price, Neural Network, Game Theory, Steel Industry.

1. Introduction

The supply chain is a complex system composed of multiple enterprises that realizes organizational alliance of all links between enterprises based on information flow [26] .It governs the circulation of products from manufacturers to customers [14]. The supply chain is not only a multi-organizational network for improving circulation efficiency, but also a business network with value-added at its core, and information flow, logistics flow and capital flow under its framework. With continuous development of the supply chain, a series of problems have occurred during numerous transactions on the part of participants, ranging from high trust costs, tricky transaction disputes, to especially information asymmetry, which has heightened complexity and fragmentation and made demand forecasting increasingly difficult [34,9]. The supply chain is also faced with the risk of overcapacity due to low demand realization rate, and excess inventory holdings have generated additional costs, which in turn have seriously damaged the interests of the nodes in the supply chain and restricted its overall development [23].

Information sharing refers to data exchange and delivery between different organizations in the transaction or cooperation process [13,23]. High-quality information sharing is considered the basis for stable and efficient operation of the supply chain [36,23]. The results of qualitative research show that information sharing can reduce the risks caused by information asymmetry, such as alleviating the bullwhip effect, shortening the order lead time, reducing costs, and improving operating efficiency [11,7]. At the same time, some studies have proved through quantitative analysis that selecting information sharing strategies among members is beneficial to saving costs and increasing overall profits [35]. By comparing the changes in expected value of revenues before and after sharing demand and cost information, it is concluded that the total revenues of the supply chain has increased after information is shared [31].

In the steel industry, the supply chain, apart from actual production, is an extremely complex task, requiring the consideration of numerous factors and objectives. Sharply fluctuating demand, raw materials supply and uncertain prices produce a negative impact on steel production. At the same time, the supply chain of the steel industry has to consider multiple objectives and multiple stages of steel production and supply chain simultaneously in a global market. It requires an optimized supply chain alternative by extending visibility of demand based on economy and market, raw material supply based on transportation, and suppliers and their price [29]. The steel industry is the core of industrial growth, and it has an indispensable role in the development of countries. Steel is a highly recyclable product, meaning that it can be reused infinitely, increasing the significance of its reverse logistics [22].

Lack of supply chain management of the steel industry has faced serious problems. Problems such as fluctuations and sharp price differences in different areas are in this category. Accordingly, the problem of the present study is to determine the optimal price with an intelligent decision-making system with a game theory approach (steel industry) in the presence and absence of sanctions. After the introduction, in the second part, the theoretical foundations and research background will be examined. In the third part of the research method; In the fourth section, model estimation, and finally in the fifth section, we will summarize and present policy proposals.

2-Theoretical Foundations and Research Background

In the history of the supply chain the process, firms use supply chains in different ways at the different time (Figure 1). While the frame of the supply chain revolution has traced and passed the following phases, the revolution of logistical military operational solutions, transportation management systems, the logistics, and physical distribution, fragmental logistics process from 1940 s-1960 s were worked and used. In this stage better focus was also placed on warehousing and materials handling process by the organizations. While in the early1980 s (1970 s-1980) ineffective supply chain decisions were taken from principally functional perspective. Since this period was a transformation process and planning and operation process done in an isolated manner. This is the tradition systems of supply chain and logistics. While the transformation and supply chain incremental phase (1980 s-1990 s) were the benefits of aligning organizations, along with the associated business objectives [8]. This stage is the transition period for contemporary supply chain system and the negative effects of individual business underlined by corporate leaders. In addition, for executives and underlying business processes performance incentives, cost reduction through the technological application and the importance of IT base planning were developed [12]. This era is also, the era of specialization, companies improve their overall competence in the same way that outsourcing manufacturing and distribution has done and focused on their core competencies and assemble networks on best class domains specific partners to contribute the overall value chain and improve the overall performances of the systems. Following in 2000 s own wards the era of globalization and specializations, the business process is done using digitalizing and internet-based business process to facilitate ordering, production, logistics, planning, inspection and warehousing systems.

Meanwhile, individual business are participating and connecting their business process in to the global economy directly, using digital platforms to participate, to learn how doing business, then doing business, develop competitive systems, formulate cooperation with firms and to attain profit economic benefits [4,10,26]. As well in this era business organizations implement ERP systems, cloud computing, and inert of thing in their supply chain systems.

To sum up, the progress of supply chain management and supply chain integration changes in four major stage focus, single, arrow concepts traditional activities, multicultural traditional management systems, transformation mutual and computer based systems to multi-dimensions, more improve the drivers and enabling strategies targets. Though supply chain advancements improve throughout the evolution's in a year from simple, traditional, blocked and individual systems to make multidimensional web-based controlling and management systems to make things within unlimited bounder's both globalized and specialization supply chain integration systems. Since the evolution and revolution of supply chain occurs due to various drivers of supply chains.

Evolutionary game theory--a mathematical method used to study and predict the evolution of social interactions--considers individuals to be rational and then analyzes individual policy choices and game equilibria [21,6]. In the evolutionary game, it is important to determine that the concept behind the game equilibrium is the evolutionarily stable strategy (ESS), which is equivalent to the Nash equilibrium but can also be applied to the evolution of individual policies. When a state can be maintained under slight disturbances caused by the dynamic system, it is called a steady state. In addition to the concept of an evolutionarily stable strategy, evolutionary game theory also considers replicator dynamics (RD). According to conclusions derived from the replication dynamic model, the trend of individual strategy selection in the population can be better predicted. The mathematical formula for competitive growth dynamics in RD is a differential equation that simulates the individual participating in the game, so it can better describe the effective rational trend of an individual's behavior in the population. Some scholars have used the idea of games to conduct research on data sharing.

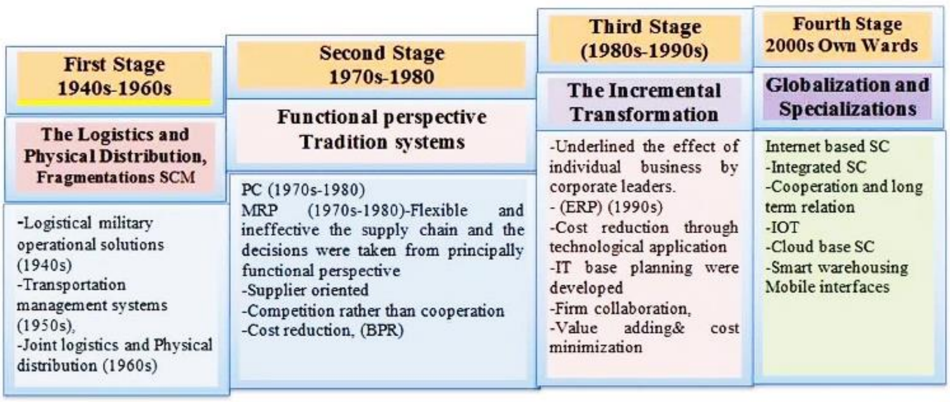


Figure 1. The evolution and Revolution of Supply chain, the author adopted from[3,8]

A three-echelon supply chain consisting of multiple suppliers, a single manufacturer, and multiple retailers was presented, where suppliers sell components to the manufacturer. The manufacturer produces products and wholesales them to retailers and, finally, sell the products to end customers. Fig. 2 shows the relations among the supply chain members.

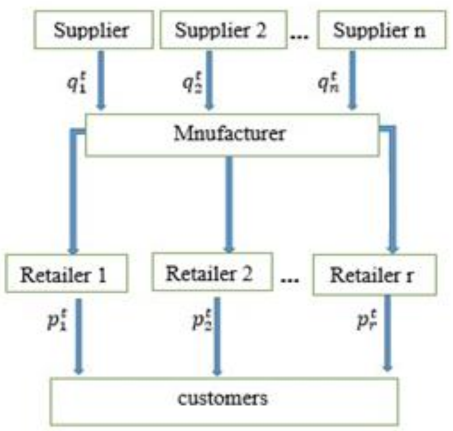


Fig. 2. supply chain model

We consider demand Q_i that is similar to the demand function used in the literature [27, 24, 29]. The following notations in Table 1 are used. We define the general demand function for product i that captures product and service competition as follows:

Table 1: Notations of parameters and variables.

Symbol	Description
D_i	Market demand of product i i϶{1,2}
Q_i	Quantity of products ordered by retailer from manufacturer i
a_i	The market base of product i
b_p	Price elasticity on market demand
b_s	Service elasticity on market demand
θ_p	Intensity of price competition
θ_s	Intensity of service competition
η_1	Service cost coefficient of retailer
η_2	Service cost coefficient of manufacturer 2
C_i	Manufacturer i's product cost
W_i	Wholesale price of product i
P_i	Retailer price of product i
S_1	Service level provided by retailer
S_2	Service level provided by manufacturer 2
πM_i	Profit function of manufacturer i
πR	Profit function of retailer

$Q_i \ (P_1, P_2, S_1, S_2) = a_i - b_{pi} + \theta_p(p_j - p_i) + b_{si} - \theta_s(s_j - s_i)$

where $a_i > 0$, $b_p > 0$, $b_s > 0$, $\theta_p > 0$, $\theta_s > 0$ and i , j ∈ {1.2}, i ≠j (1)

The first manufacturer carries the production cost and the second manufacturer carries the production cost and service cost. The cost of providing S_2 units of service by the second manufacturer is $\eta_2 S_2^2/2$, where η_2 is the ultimate cost of service and the cost of providing S_1 units of service by the retailer is $\eta_1 S_1^2/2$, where η_1 is the service cost coefficient of the retailer as is given in the literature [17, 16, 5, 23, 15, 29, 21, 25, 24, 28]. Therefore, the profit functions for two manufacturers are:

$$\pi M_1 = (W_1 - C_1)\, Q_1 \quad (2)$$

$$\pi M_2 = (W_2 - C_2)\, Q_2 - \frac{\eta_2 S_2^2}{2} \quad (3)$$

The costs to the retailer include wholesale prices and retail services. Therefore, the retailer's profit function is given as follows, Retailer Stackelberg

$$\pi R = (P_1 - W_1)\, Q_1 - \frac{\eta_1 S_1^2}{2} + (P_2 - W_2)\, Q_2 \quad (4)$$

The Retailer Stackelberg game occurs in markets where the size of the retailers is large compared with their suppliers or manufacturers. For example, the size of the retailers such as Walmart is large compared with their suppliers and they can influence the sales of each product by lowering price and they are leader in the market. In a Stackelberg game, according to the follower's response function, the leader makes a decision to maximize his own profit. First, the optimal reaction functions for the two manufacturers are obtained, given that the manufacturers have observed the decisions made by the retailer. Then, the retailer's equilibrium solutions are obtained when he knows how the manufacturers would react to his decisions.

Manufacturers Reaction Functions: The first manufacturer must choose wholesale price w_1^* and the second manufacture must choose wholesale price w_2^* and service level s_2^* to maximize their equilibrium profit. Then, the reaction functions of the manufactures are:

$$W_i^* \in argmaxw_i \pi M_i\left(w_i . w_j^* . s_2^* | p_1 . p_2 . s_1\right)$$
(5)

$$S_2^* \in argmaxs_2 \pi M_2(w_1^* . w_2^* . s_2 | p_1 . p_2 . s_1)$$
(6)

where πM_i $(w_1, w_2, s_2 | p_1 . p_2 . s_1)$ given by E_q (2) and E_q (3) denote the profit to the manufacturers at this stage when they set the wholesale prices w_1, w_2 and the second manufacturer sets service level s_2, given earlier decisions on retail price and service level p_1, p_2 and s_1 by the retailer. The Proposition 1 gives the results.

Proposition 1. In the Retailer Stackelberg game, for a given retail prices p_1, p_2 and s_1, the manufacturers reaction functions are obtained as:

$$W_1^* = I_2 - G_2 P_1 - H_2 P_2 - K_2 S_1$$
(7)

$$W_2^* = J_2 - M_2 P_1 - L_2 P_2 - N_2 S_1$$
(8)

$$S_2^* = O_2 - V_2 P_1 - U_2 P_2 - Y_2 S_1$$
(9)

where I_2, G_2, H_2, K_2, J_2, M_2, L_2, N_2, O_2, V_2, U_2 and Y_2 are defined in s_1 Appendix. The proof of Proposition 1 is given in s_1 Appendix.

Retailer Decision: By using the reaction functions of manufacturers, we can obtain the retailer's optimal retail prices and services. The retailer's best response functions for retail prices p_1, p_2 and service level s_1 are obtained by maximizing retailer's profit, given w_i^* and s_2^* in E_q (7), E_q (8) and E_q (9), respectively. This leads to

$$P_i^* \in argmaxp_i \pi R\left(p_i . p_j^* . s_1^*\right)$$
(10)

$$S_1^* \in argmaxs_1 \pi R\left(p_i . p_j^* . s_1^*\right)$$
(11)

The Proposition 2 gives the results.

Proposition 2. In the Retailer Stackelberg game, the retailer's optimal retail prices and the optimal retail service level, denoted as p_1^*, p_2^* and s_1^* are given as follows:

$$P_1^* = \frac{\sigma_2 \tau_2 - \varepsilon_2 \epsilon_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2}$$
(12)

$$P_2^* = \frac{\delta_2 \varepsilon_2 - \vartheta_2 \tau_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2}$$
(13)

$$S_1^* = \frac{\gamma_2 \sigma_2 \tau_2 - \varepsilon_2 \epsilon_2 \gamma_2 + \emptyset_2 \delta_2 \epsilon_2 - \emptyset_2 \gamma_2 \tau_2 + \delta_2 \sigma_2 \beta_2 - \varepsilon_2 \vartheta_2 \beta_2}{-2\lambda_2(\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2)}$$
(14)

where σ_2, τ_2, ε_2, ϵ_2, δ_2 ϑ_2, γ_2, $\emptyset_2$, β_2 and λ_2 are defined in s_1 Appendix. The proof of Proposition 2 is given in s_1 Appendix.

By substituting E_q (12), E_q (13) and E_q (14) into E_q (7), E_q (8) and E_q (9), the manufactures optimal wholesale prices and the second manufacturer optimal service level are obtained as follows:

$$W_1^* = I_2 - G_2 \frac{\sigma_2 \tau_2 - \varepsilon_2 \epsilon_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2} - H_2 \frac{\delta_2 \varepsilon_2 - \vartheta_2 \tau_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2} - K_2 \frac{\gamma_2 \sigma_2 \tau_2 - \varepsilon_2 \epsilon_2 \gamma_2 + \emptyset_2 \delta_2 \epsilon_2 - \emptyset_2 \gamma_2 \tau_2 + \delta_2 \sigma_2 \beta_2 - \varepsilon_2 \vartheta_2 \beta_2}{-2\lambda_2(\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2)}$$
(15)

$$W_2^* = J_2 - M_2 \frac{\sigma_2 \tau_2 - \varepsilon_2 \epsilon_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2} - L_2 \frac{\delta_2 \varepsilon_2 - \vartheta_2 \tau_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2} - N_2 \frac{\gamma_2 \sigma_2 \tau_2 - \varepsilon_2 \epsilon_2 \gamma_2 + \emptyset_2 \delta_2 \epsilon_2 - \emptyset_2 \gamma_2 \tau_2 + \delta_2 \sigma_2 \beta_2 - \varepsilon_2 \vartheta_2 \beta_2}{-2\lambda_2(\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2)}$$
(16)

$$S_2^* = O_2 - V_2 \frac{\sigma_2 \tau_2 - \varepsilon_2 \epsilon_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2} - U_2 \frac{\delta_2 \varepsilon_2 - \vartheta_2 \tau_2}{\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2} - Y_2 \frac{\gamma_2 \sigma_2 \tau_2 - \varepsilon_2 \epsilon_2 \gamma_2 + \emptyset_2 \delta_2 \epsilon_2 - \emptyset_2 \gamma_2 \tau_2 + \delta_2 \sigma_2 \beta_2 - \varepsilon_2 \vartheta_2 \beta_2}{-2\lambda_2(\delta_2 \sigma_2 - \varepsilon_2 \vartheta_2)}$$
(17)

3-Research method

Our previous work in various areas (e.g., artificial intelligence, soft computing, reasoning under uncertainty, and neuroscience) identified that many cooperations between two agents (artificial or natural) can be interpreted or bear some of the characteristic features of a game. For example, the main concepts in a game are the players in a game, a set of rules by which the game is played, and an outcome in the form of a reward or a punishment (more generally referred to as a payoff) for the players in the game. In addition, a so-called payoff matrix is a common scheme to represent the dynamic behavior of a game.

Figure 1 applies these key concepts to a coupled neuron system where the neurons are modeled to calculate their strategies according to their individual payoff matrix. (The scopes for game theory and neural networks are extremely wide. The paper therefore uses several abstractions and simplifications (e.g., the neuronal circuit models presented in this text are relatively basic, and in terms of game theory this paper concentrates on static games and dynamic games of complete/perfect information). At large, the paper does not suffer from this reductionism as the findings mentioned in the paper are relevant in a wider sense [17], for instance, emphasize that the contribution of single neurons to computation in the brain has long been underestimated and that there is a need to investigate novel mechanisms that allow individual neurons to implement elementary computations.) Imagine that the two neurons in Figure 3(a) shall generate the following global behavior: if Neuron-1 fires, then Neuron-2 shall fire, and if Neuron-1 is at rest (not firing), then Neuron-2 shall be at rest (it is possible to assume an information exchange, unidirectional or bidirectional, via biochemical substances or electrical signals between Neuron-1 and Neuron-2). Figure 3(b) presents this behavior in a payoff matrix. The payoff matrix assigns a payoff (illustrated as a reward R or a punishment P) to each neuron for each combination of strategies (Fire, Rest). For instance, if Neuron-1 fires and Neuron-2 also fires, then each neuron obtains a rewarding payoff. (Traditionally, the payoff for Neuron-1 would be the left value in a matrix cell, and the payoff for Neuron-2 would be the right value in a cell. Note also that the payoffs in a cell need not be identical.) If the two neurons correspond with different strategies (e.g., Neuron-1 fires and Neuron-2 remains at rest or vice versa), then each neuron receives a punishment payoff P. Thus, if the goal for the two neurons in Figure 3(a) is to eventually demonstrate the global behavior Fire/Fire, Rest/Rest, then it is possible to assume the following: (i) if the two neurons demonstrate the desired behavior (Fire/Fire, Rest/Rest), then no action is required, and (ii) in case the two neurons do not demonstrate this desired way of interaction, then some corrective action has to be taken to achieve the desired global behavior. Again, this paper is not interested in the exact description of the biochemical processes (which are not known in their entirety anyway) that may achieve this mode of operation in biological neurons—the motivation here is to describe this global interaction via game theoretic concepts, perhaps involving additional models and abstractions for the two neurons in Figure 3(a). On the other hand, it is crucial to understand that the payoff matrix in Figure 1 is a crude generalization. In reality, it is very difficult to find and specify exactly a payoff function for a game, which is a critical task in game theory (i.e., approximations are the norm rather than the exception).

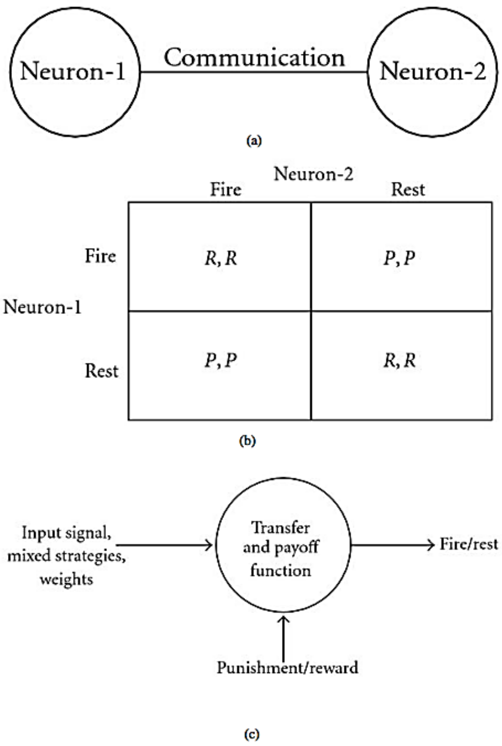


Figure 3 :Relationships between (a) biological neurons, (b) game theory, and (c) artificial neurons.

Laying this issue aside, it is possible to provide a rather straightforward mathematical description for the modeling of the global behavior desired for the two neurons in Figure 3(a). To begin with Figure 3(a), it is necessary to understand that the communication between the two neurons in Figure 3(a) is a relatively simple, one-dimensional, linearly separable, and supervised learning classification task. Neuron-1 can either fire or be at rest, and Neuron-2 has to respond accordingly. It is possible to imagine a function f(x) where a value $x \in R$ above a certain threshold value $t \in R$ represents the firing state for Neuron-1 and, a value $x \leq t$ represents the resting state for this neuron (1) as:

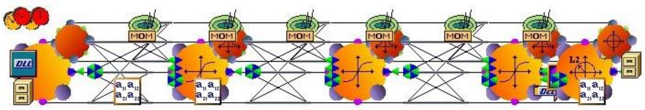
$$f_x = \begin{cases} fire & if \quad x > t. \\ Rest & otherwise \end{cases}$$

Collectively, it is possible to think of Neuron-1 and Neuron-2 as a simple input-output unit that behaves similar to a switch. In terms of its global behavior, a perceptron can be interpreted exactly in the same way. (It is not necessary to elaborate on the perceptron learning algorithm in great detail as this information is widely available in the neural network literature (e.g., in [4, pages 43–54]).) This does not mean, however, that the payoff matrix in Figure 3(b) can be implemented by a traditional perception. Figure 3(c) illustrates a model that is similar to a perceptron but incorporates elements from game theory that may allow this model to demonstrate the behavior illustrated by the payoff matrix in Figure 3(b). It is clear from Figure 3(c) that the decision-making process for this model involves some form of an input, an output, a transfer function, and a reward/punishment mechanism, all based on concepts from game theory. The current focus is to describe the intuitive relationship between game theory, biological neurons, and artificial neural networks just mentioned in more detail and to elaborate on the various (fundamental) challenges involved in this relationship [1]. In the present study, models Grossberg , Supporting vectors and Bayesian have been used.

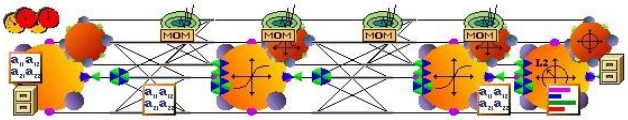
4-Model estimation

In this model, to predict the price of steel, three approaches of Bayesian, Grassberg and support vectors will be used to predict the price of steel. The neural network structure of the above models is presented in Figure (6) (a: SVM network structure, b: Bayesian network structure and c: Grossberg anti-emission network structure).

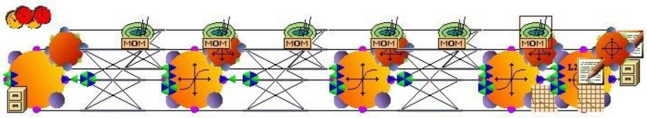
Based on the results of Table (3); Is observed; The Grossberg method has a higher accuracy in predicting the price of steel, so the price forecast is based on the Grossberg method. In Figure (5); Steel price forecasting is done using Grossberg method.



A: The structure of the SVM Network



B: The structure of the Bayesian Network



C: The structure of Grossberg Counter-Propagation Network

Figure 4: The structure of the Neural Network Used in the Research

Table 2: Error rates in different neural network models

	Model	MAFE	MSFE
No sanctions	Grossberg	0/076	0/009
	Supporting vectors	0/083	0/010
	Bayesian	0/088	0/011
sanctions	Grossberg	0/077	0/010
	Supporting vectors	0/092	0/012
	Bayesian	0/081	0/010

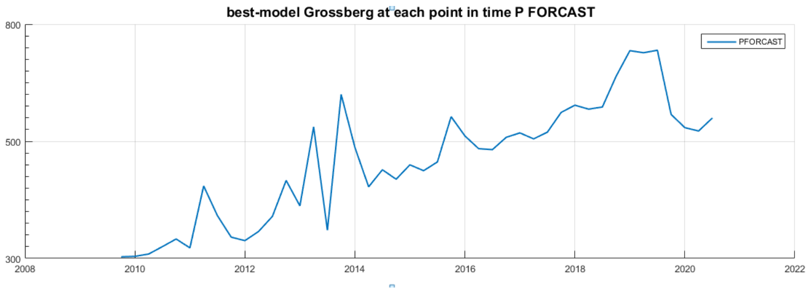


Figure 5: Steel Price Prediction with Grossberg Anti-Emission Neural Network Model

The proposed game theory model of the present study includes two categories of retailers and manufacturers. The proposed game theory model of the present study includes two categories of retailers and manufacturers. The set of decisions Δα is the rate of change in steel production for producers and Δβ is the rate of change in the purchase of steel by retailers [19]. If α_0 is the current production of steel and β_0 is the current purchase amount. We have the following relationships:

$$\Delta\alpha = \alpha - \alpha_0$$
$$\Delta\beta = \beta - \beta_0$$

So α is the amount of production and β is the amount of purchasing decisions made by the actors. For up manufacturers (Δα, Δβ) and for us buyers (Δα, Δβ) is the consequence function, which is introduced as follows.

$$up(\Delta\alpha.\Delta\beta) = f_{net}(\alpha.\beta) - f_0$$
$$us(\Delta\alpha.\Delta\beta) = -up(\Delta\alpha.\Delta\beta)$$

So that f_0 current currents are steel and f_{net} (α, β) is the predictive function of steel in the previous section, which was calculated by the anti-emission neural network of Grasberg [18]. If producers increase their steel production to α and buyers increase their purchase to β; The price of steel is then determined by this function and the result for the players from the previous relationship. In the diagram above us (Δα, Δβ), the output function of the purchase shows. Since the following relation is established in the presented game model; So the game model is zero.

$$us(\Delta\alpha.\Delta\beta) + up(\Delta\alpha.\Delta\beta) = 0$$

In a game the sum is zero; The balance of the game shows the optimal decisions that the parties do not want to deviate from. In the game we suggest $(\Delta\alpha^*.\Delta\beta^*)$is a Nash equilibrium; If and only if the following relationships are established.

$$us(\Delta\alpha^*.\Delta\beta^*) \geq us(\Delta\alpha \ \ .\Delta\beta^*) \qquad \forall\Delta\alpha \in [-A.A]$$
$$up(\Delta\alpha^*.\Delta\beta^*) \geq up(\Delta\alpha^*.\Delta\beta \ \ ) \qquad \forall\Delta\alpha \in [-B.B]$$

Where in them; The ceiling is possible in increasing production and the ceiling is possible for buyers to buy. To obtain the Nash equilibrium, the Minimax and Maxmin algorithms can be used according to the following equations.

$$(\Delta\alpha^*.\Delta\beta^*) = argmax_{\Delta\alpha}\,(rgmax_{\Delta B}(us(\Delta\alpha \ \ .\Delta\beta \ \ )))$$
$$(\Delta\alpha^*.\Delta\beta^*) = argmax_{\Delta\beta}\,(rgmax_{\Delta\alpha}(us(\Delta\alpha \ \ .\Delta\beta \ \ )))$$

If the output value of the two relations is equal; Playing with values $(\Delta\alpha^*, \Delta\beta^*)$has a pure Nash equilibrium. The points that balance Nash are the optimal decisions that neither party is willing to deviate from, and deviating from them will be to the detriment of each. Nash equilibrium resulting from the above relations using numerical methods that build Nash equilibrium are optimal decisions, decisions that neither party is willing to deviate from it and deviation from it causes damage to each.

In the diagram above, changes in sales of 2,000 tons per production increase the price of steel by $ 55 per ton.

Given that our country has always been subject to sanctions and these sanctions affect the optimal behavior of players; The effect of sanctions on the optimal balance is then evaluated. In order to estimate the sanctions index and the macro-model, a total of about 10 variables and resources used in data collection are presented.

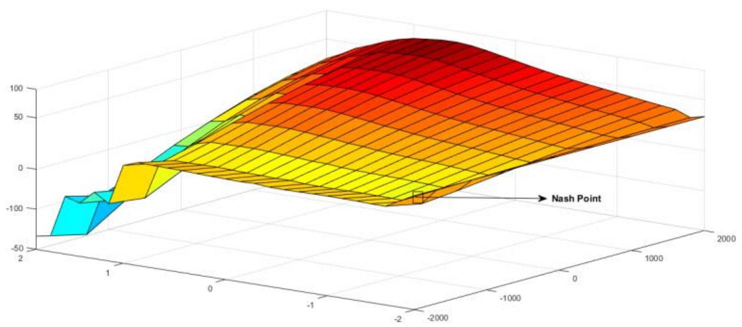


Chart 6: Nash balance in the absence of sanctions

Table 3: Introduction of variables used in heuristic factor analysis

Rows	Variable affected by sanctions	Notes	Source
1	Price Index of Imported Goods (PM)	This is the base year of 2011. For the data of the last two months, the implicit index of national accounts imports was used.	Central bank time series information and central bank national accounts
2	Export price index (PX)	Basic year 1390	Central Bank Time Series Information and Central Bank Indicators
3	Exchange relationship (PX PM)	It is obtained from the ratio of the price index of exported goods to imported goods.	———
4	Country's share of world crude oil production (OILPS)	The ratio of crude oil production in Iran to world production	Global Energy Statistics (BP Company)
5	Country's share of foreign direct investment (FDI)	The share of foreign direct investment in Iran in the world each year	UNCTAD time series database
6	US Share of Iran Foreign Trade (USIRITR)	The ratio of Iran's foreign trade with the United States to the total volume of Iran's trade	Statistics Center of America
7	Exchange Rate Premium (PEREX)	The ratio of the official exchange rate difference from the informal exchange rate to the official exchange rate	Central Bank Time Series Information and Central Bank Indicators
8	Exchange rate variance (VAREX)	The variance of the formal and informal exchange rate differences based on quarterly exchange rate information	Central Bank Time Series Information and Central Bank Indicators
9	Ratio of non-oil trade balance to GDP (TDNOIL)	Calculated from the division of the real non-oil trade balance into GDP	National Central Bank Accounts

Source: [25, 2]

The method in the software has been used to index the sanctions from the above variables.

The number of components extracted in each model is equal to the number of variables that are examined; But a certain number of these components can be selected. Usually the first two or three components take into account a significant amount of data scatter; Therefore, selecting the first two or three components is sufficient to continue the work; But in some cases it is necessary to consider other criteria to find the required number of components. These criteria are:

Scree Screen Test:

Draws eigenvalues against related basic components. In this diagram, the change in the importance of eigenvalues for each basic component is specified. The fracture point indicates the maximum number of key components to be considered. A less than a number indicating a fracture may also be appropriate. Based on this, in Figure (7), the first component or the first two components can be selected.

Exclusive value:

Considers components whose eigenvalues are greater than one and ignores the other components.

Variance:

Components that explain a higher percentage of dispersion are sufficient to continue the work, usually the first component taking the most variance.

According to the results, a principal vector can be identified based on which we will extract the sanctions index. In this case, the sanctions index creates the total weight of each variable multiplied by the said variable and the amount of the mentioned index for each period.

Comparing the results of Figure (8) with Figure (6), which shows the Nash equilibrium, it can be seen that the presence of sanctions in the model has increased prices and reduced production in the steel industry. In the chart above, the sanctions increase the price of steel by $ 48 per ton. This reduces the competitiveness of production at the international level.

In the following, we will examine the effect of each of the factors affecting the change in steel demand and its effect on the Nash equilibrium in the presence or absence of sanctions. In this section, the results of violating the condition us (Δα, Δβ) + up (Δα, Δβ) = 0 and increasing the bargaining power of the parties on the balance of the game will be examined. Applying both conditions to the steel market means monopolizing trading markets and distancing oneself from competitive markets.

According to the results, it can be seen that the violation of us (Δα, Δβ) + up (Δα, Δβ) = 0 and increasing the bargaining power of the parties shifts the balance of the game and the balance of the game in favor of the parties. Has changed from a market with higher bargaining power (in this game manufacturers), In other words, applying the non-zero-sum bet condition has been to the detriment of the buyers of this market. The results also indicate the fact that the presence of sanctions by applying the above conditions has degraded the situation of producers and buyers at the same time.

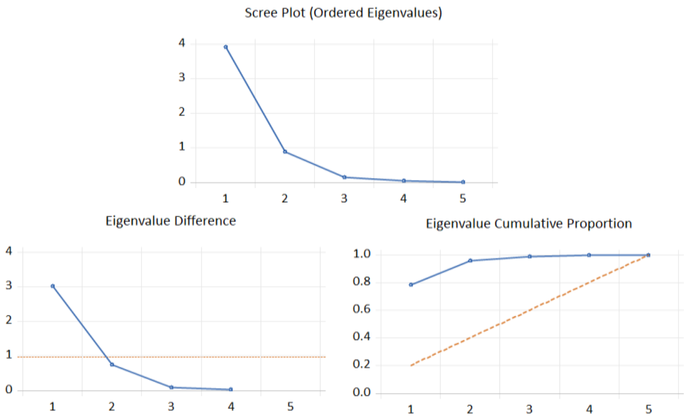


Figure 7: PCA model results between sanctions model variables

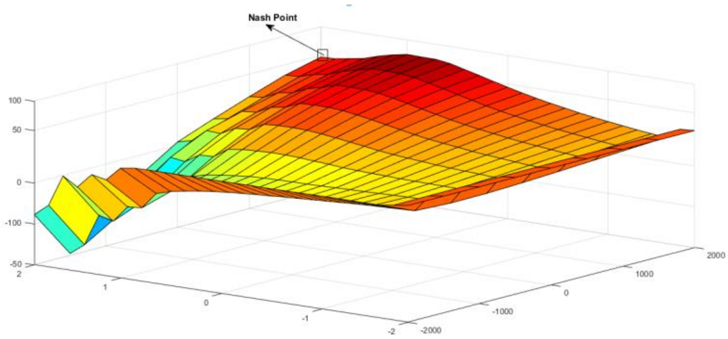
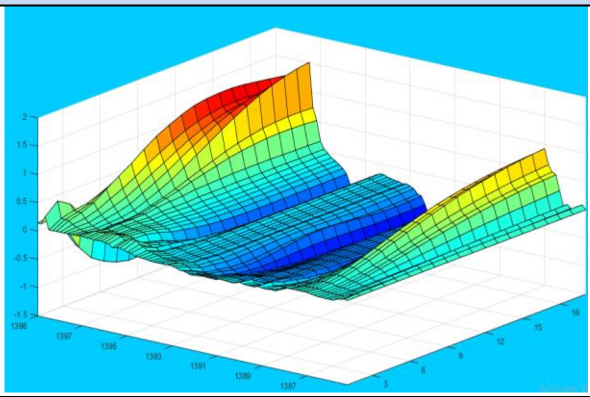
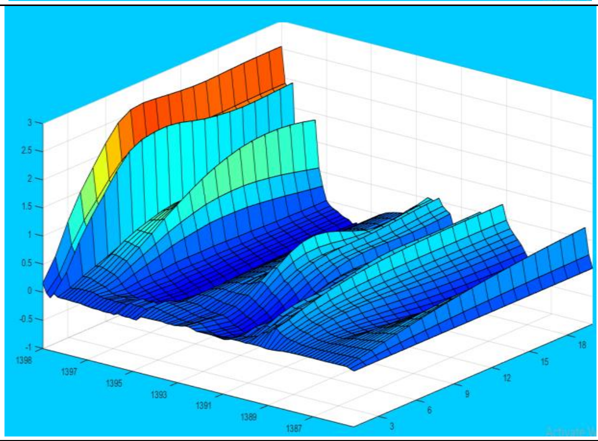


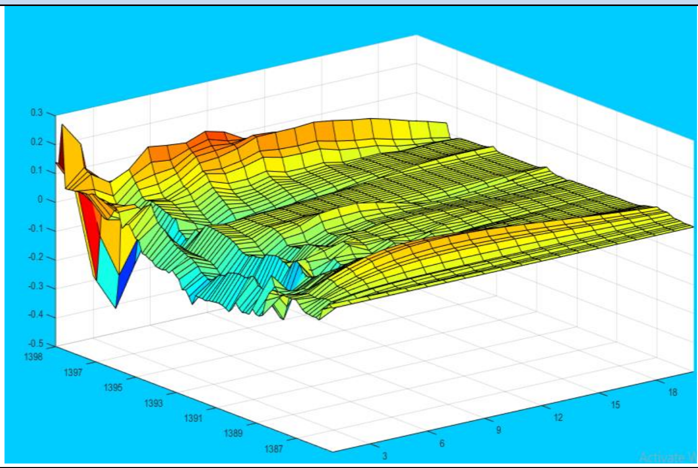
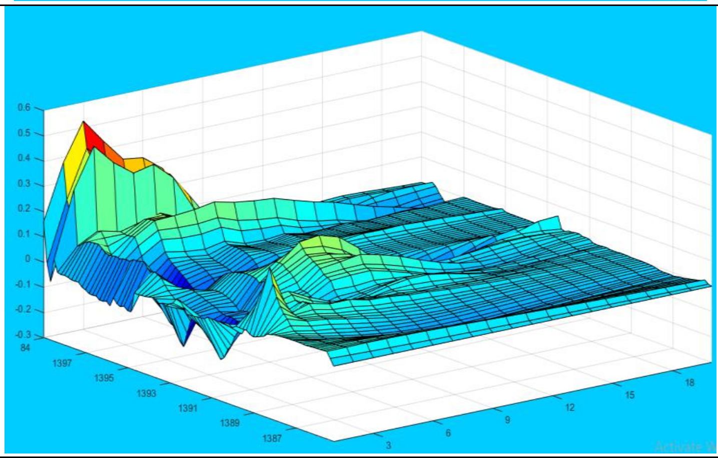
Figure 8: Nash equilibrium in the presence of sanctions

Table 4: The effect of changing parameters affecting Nash equilibrium on changes in demand

Chart	variable name
	Absence of zero-sum game on Nash balance
	Increase bargaining power

Source: Research calculations

Table 5: The effect of changing the parameters affecting the Nash equilibrium in the presence of sanctions on changes in demand

Chart	variable name
	Absence of zero-sum game on Nash balance
	Increase bargaining power

Source: Research calculations

5- Research summary

Considering the importance of price and time decisions in a two-channel supply chain, in this study, using two-stage optimization technique and Stackelberg game, optimal decisions for price and production in a decentralized supply chain were investigated .This research addressed a key issue. It was a matter of examining how the interaction between producers and retailers in the steel industry affects price changes and optimal quantity. To answer this question, in this study, a hybrid model based on artificial neural networks and game theory is presented to help steel industry actors in determining the price level and optimal production. In this model, a neural network is used to learn the effect of steel manufacturers' decisions in determining the level of steel supply on its price.

The trained neural network is then used to create a consequence function for a game model between manufacturers and retailers.

The proposed model is used to determine the best decision for the amount of steel production to determine the optimal price. To predict steel prices, three Bayesian neural networks, support vectors and cross-emission antipressor were used. The results indicate the fact that the cross-emission model of Grossberg is more accurate in forecasting stock prices. Then the predicted price entered the game theory process and the equilibrium point of the model was determined. Then, according to the specific circumstances of the country, the variable of sanctions was entered in the game theory model. The results indicate the fact that the presence of sanctions in the model has led to higher prices and reduced production in the steel industry.

Resources

Alfons Schuster, Yoko Yamaguchi, "Application of Game Theory to Neuronal Networks", Advances in Artificial Intelligence, vol. 2010, Article ID 521606, 12 pages, 2010. https://doi.org/10.1155/2010/521606

Caruso, P, (2003). "The impact of International Economic Sanctions on Trade. An Empirical Analysis". Peace Economics, Peace Science and Public Policy, vol. 9, no.2.

Cheng, H. C., Chen, M. C., & Mao, C. K. (2010). The evolutionary process and collaboration in supply chains. Industrial Management and Data Systems, 110(3), 453–474. https://doi.org/10.1108/02635571011030079.

Dametew, A. W., & Ebinger, F. (2017). Technological innovations as a potential vehicle for supply chain integration on basic metal industries. International Journal of Swarm Intelligence and Evolutionary Computation, 06, 02. https://doi.org/10.4172/2090-4908.1000159.

Dan B, Xu G, Liu C. Pricing policies in a dual-channel supply chain with retail services. *International Journal of Production Economics*. 2012. September 30; 139(1):312–20.

De, S., Nau, D. S., and Gelfand, M. (2016). *Using Game Theory to Study the Evolution of Cultural Norms*. CoRR. Available at: https://arxiv.org/pdf/1606.02570.pdf.

Devika, K., Jafarian, A., Hassanzadeh, A., and Khodaverdi, R. (2016). Optimizing of Bullwhip Effect and Net Stock Amplification in Three-Echelon Supply Chains Using Evolutionary Multi-Objective Metaheuristics. *Ann. Oper. Res.* 242 (2), 457–487. doi:10.1007/s10479-013-1517-y

Esposito, E., & Passaro, R. (2009). Evolution of the supply chain in the Italian railway industry. Supply Chain Management, 14(4), 303–313. https://doi.org/10.1108/13598540910970135

Fernández, C. P., Trucco, P., and Huaccho, H. L. (2019). Managing Structural and Dynamic Complexity in Supply Chains: Insights from Four Case Studies [J]. *Prod. Plann. Control.* 30 (8), 611–623. doi:10.1080/09537287.2018.1545952

James, M., Richard, D., & Jonathan, W. (2016). Digital globalization: the new era of global flows. McKinsey Global Institute (MGI).

Jeong, K., and Hong, J.-D. (2019). The Impact of Information Sharing on Bullwhip Effect Reduction in a Supply Chain. *J. Intell. Manuf* 30 (4), 1739–1751. doi:10.1007/s10845-017-1354-y

Kitaw, D., & Goshu, Y. Y. (2017). Performance measurement and its recent challenge: A literature review. International Journal of Business Performance Management, 18(4), 381. https://doi.org/10.1504/IJBPM.2017.10007477

Lee, H. L., Padmanabhan, V., and Whang, S. (1997). Information Distortion in a Supply Chain: The Bullwhip Effect. *Management Sci.* 43 (4), 546–558. doi:10.1287/mnsc.43.4.546

Leng, K., Bi, Y., Jing, L., Fu, H.-C., and Van Nieuwenhuyse, I. (2018). Research on Agricultural Supply Chain System with Double Chain Architecture Based on Blockchain Technology. *Future Generation Computer Syst.* 86, 641–649. doi: 10.1016/j.future.2018.04.061

Lu JC, Tsao YC, Charoensiriwath C. Competition under manufacturer service and retail price. *Economic Modelling*. 2011. May 31; 28(3):1256–64.

Lu Q, Liu N. Pricing games of mixed conventional and e-commerce distribution channels. *Computers & Industrial Engineering*. 2013. January 31; 64(1):122–32.

London and Häusser, “Dendritic computation,” Annual Review of Neuroscience, vol. 28, pp. 503–532, 2005.

Lotfi, E., Navidi, H. (2012). “A decision support system for OPEC oil production level based on game theory and ANN”. Advances in Computational Mathematics and its Applications, Vol. 2, No. 1, pp. 253-258

Navidi. N, Rahimi. R. (2011). Intermediate performance impacts of advanced manufacturing technology systems: An empirical investigation, Decision Sciences, 30 (4),993-1020.

Pei Z, Yan R. Do channel members value supportive retail services? Why? *Journal of Business Research*. 2015. June 30; 68(6):1350–8.

Phelps, S., and Wooldridge, M. (2013). Game Theory and Evolution. *IEEE Intell. Syst.* 28 (4), 76–81. doi:10.1109/mis.2013.110

Pourmehdi M, Paydar M, Ghadimi P, Azadnia A.(2022). Analysis and evaluation of challenges in the integration of Industry 4.0 and sustainable steel reverse logistics network. Computer and Industrial Engineering.vol.163

Qian T, Zhang Z, Yuan Z and Li Z (2022) The Game Analysis of Information Sharing for Supply Chain Enterprises in the Blockchain.Front. Manuf. Technol 2:88033210.doi:.3389/fmtec.2022.880332

Tsay AA, Agrawal N. Channel dynamics under price and service competition. *Manufacturing & Service Operations Management*. 2000. October; 2(4):372–91.

Torbat, A. (2005). "Impacts of the US Trade and Financial Sanctions on Iran" The World Economy, Vol. 28, No. 3, pp. 407-434.

Wellman, K. M. (2016). Computer grading of introductory organic chemistry laboratory results. Journal of Chemical Education, 47(2), 142. https://doi.org/10.1021/ed047p142.

Wu CH. Price and service competition between new and remanufactured products in a two-echelon supply chain. *International Journal of Production Economics*. 2012. November 30; 140(1):496–507.

Xiao T, Xu T. Coordinating price and service level decisions for a supply chain with deteriorating item under vendor managed inventory. *International Journal of Production Economics*. 2013. October 31; 145(2):743–52.

Xiao T, Yang D. Price and service competition of supply chains with risk-averse retailers under demand uncertainty. *International Journal of Production Economics*. 2008. July 31; 114(1):187–200.

Xue, X., Dou, J., and Shang, Y. (2021). Blockchain-driven Supply Chain Decentralized Operations - Information Sharing Perspective. *Bpmj* 27 (1), 184–203. doi:10.1108/bpmj-12-2019-0518

Yang, Z., Aydın, G., Babich, V., and Beil, D. R. (2009). Supply Disruptions, Asymmetric Information, and a Backup Production Option. *Management Sci.* 55 (2), 192–209. doi:10.1287/mnsc.1080.0943

Yao DQ, Liu JJ. Competitive pricing of mixed retail and e-tail distribution channels. *Omega*. 2005. June 30; 33(3):235–47.

Yao DQ, Yue X, Liu J. Vertical cost information sharing in a supply chain with value-adding retailers. *Omega*. 2008. October 31; 36(5):838–51.

Yu, K., Cadeaux, J., Luo, N., Qian, C., and Chen, Z. (2018). The Role of the Consistency between Objective and Perceived Environmental Uncertainty in Supply Chain Risk Management. *Imds* 118 (7), 1365–1387. doi:10.1108/imds-09-2017-0410

Yu, Z., Yan, H., and Edwin Cheng, T. C. (2001). Benefits of Information Sharing with Supply Chain Partnerships. *Ind. Manag. Data Syst.* 101 (3), 114–121. doi:10.1108/02635570110386625

Zelbst, P. J., Green, K. W., Sower, V. E., and Baker, G. (2010). RFID Utilization and Information Sharing: The Impact on Supply Chain Performance. *J. Business Ind. Marketing* 25 (8), 582–589. doi:10.1108/08858621011088310.

S1 Appendix

Proof of Proposition 1. By solving the first conditions $\frac{\partial \pi_{Mi}}{\partial w_i} = 0$ and $\frac{\partial \pi_{M2}}{\partial s_2} = 0$, the manufacturers reaction functions are obtained as:

$$(b_p + \theta_p)w_1 + \theta_s s_2 = a_1 - (b_p + \theta_p)p_1 + \theta_p p_2 + (b_s + \theta_s)s_1 + (b_p + \theta_p)c_1 \quad (1)$$

$$(b_p + \theta_p)w_2 - (b_s + \theta_s)s_2 = a_2 - (b_p + \theta_p)p_2 + \theta_p p_1 - \theta_s s_1 + (b_p + \theta_p)c_2 \quad (2)$$

$$(b_s + \theta_s)w_2 - \eta_2 s_2 = (b_s + \theta_s)c_2 \quad (3)$$

By assuming $p_1 = w_1 + m_1$, $p_2 = w_2 + m_2$, $A = -2(b_p + \theta_p)$ $B = (b_s + \theta_s)$, $D_2 = a_1 + 0.5Ap_1 + \theta_p p_2 + Bs_1 - 0.5Ac_1$, $E_2 = a_2 + 0.5Ap_2 + \theta_p p_1 - \theta_s s_1 - 0.5Ac_2$,equations (1), (2), and (3) reduces to equations (4), (5), and (6) respectively:

$$-0.5Aw_1 + \theta_s s_2 = D_2 \quad (4)$$

$$-0.5Aw_2 - Bs_2 = E_2 \quad (5)$$

$$Bw_2 - \eta_2 s_2 = Bc_2 \quad (6)$$

By solving (4), (5) and (6), simultaneously, we get:

$$w_1 = \frac{-0.5A\eta_2 D_2 - D_2 B^2 - B\theta_s E_2 - 0.5AB\theta_s c_2}{(0.5A)(0.5A\eta_2 + B^2)} \quad (7)$$

$$w_2 = \frac{B^2 c_2 - \eta_2 E_2}{0.5A\eta_2 + B^2} \quad (8)$$

$$s_2 = \frac{-BE_2 - 0.5ABc_2}{0.5A\eta_2 + B^2} \quad (9)$$

By assuming $F_2 = (0.5A\eta_2 + B^2)$, $I_2 = \left(1/_{0.5AF_2}\right)(-a_1F_2 + 0.5AF_2c_1 - a_2B\theta_s)$, $G_2 = \left(1/_{0.5AF_2}\right)(0.5AF_2 + B\theta_s\theta_p)$, $H_2 = \left(1/_{0.5AF_2}\right)(F_2\theta_p + 0.5AB\theta_s)$, $K_2 = \left(1/_{0.5AF_2}\right)(F_2B - B\theta_s^2)$,
$J_2 = \frac{B^2c_2 - a_2\eta_2 + 0.5Ac_2\eta_2}{F_2}$, $L_2 = \frac{0.5A\eta_2}{F_2}$, $M_2 = \frac{\eta_2\theta_p}{F_2}$, $N_2 = -\frac{\eta_2\theta_s}{F_2}$, $O_2 = -\frac{a_2B}{F_2}$, $U_2 = \frac{0.5AB}{F_2}$, $V_2 = \frac{B\theta_p}{F_2}$, $Y_2 = -\frac{B\theta_s}{F_2}$, equations (7), (8), and (9) reduces to equations (9), (10), and (11) respectively:

$$w_1^* = I_2 - G_2p_1 - H_2p_2 - K_2s_1 \quad (10)$$

$$w_2^* = J_2 - M_2p_1 - L_2p_2 - N_2s_1 \quad (11)$$

$$s_2^* = O_2 - V_2p_1 - U_2p_2 - Y_2s_1 \quad (12)$$

Taking the second-order partial derivatives of π_{M2} with respect to w_2 and s_2, we have optimal second condition and the Hessian Matrix, respectively,

$$\frac{\partial^2\pi_{M2}}{\partial w_2{}^2} = -(b_p + \theta_p) < 0 \ , \quad H = \begin{bmatrix} \frac{\partial^2\pi_{M2}}{\partial w_2{}^2} & \frac{\partial^2\pi_{M2}}{\partial w_2\partial s_2} \\ \frac{\partial^2\pi_{M2}}{\partial s_2\partial w_2} & \frac{\partial\pi_{M2}}{\partial s_2} \end{bmatrix} = \begin{bmatrix} -(b_p + \theta_p) & (b_s + \theta_s) \\ (b_s + \theta_s) & -\eta_2 \end{bmatrix}$$

$(H_{11} = -(b_p + \theta_p) < 0)$, $Det(H) = \left((b_p + \theta_p)\eta_2 - (b_s + \theta_s)^2\right) > 0$

Proof of Proposition 2. In this stage, by substituting Eqs. (10), (11), (12) in π_R and solving the first conditions $\partial\pi_R/_{\partial p_i} = 0$ and $\partial\pi_R/_{\partial s_1} = 0$, the retailer's reaction functions are obtained as:

$$\frac{\partial\pi_R}{\partial p_1} = R_2 + 2T_2p_1 + X_2p_2 + \gamma_2 s_1 = 0 \quad (13)$$

$$\frac{\partial\pi_R}{\partial p_2} = Z_2 + X_2p_1 + 2\alpha_2p_2 + \phi_2 s_1 = 0 \quad (14)$$

$$\frac{\partial\pi_R}{\partial s_1} = \beta_2 + \gamma_2p_1 + 2\lambda_2s_1 + \phi_2p_2 = 0 \quad (15)$$

By assuming $R_2 = [a_1(1 + G_2) + I_2(b_p + \theta_p) - (1 + G_2)\theta_s O_2 - I_2\theta_s V_2 + a_2M_2 - J_2\theta_p + M_2(b_s + \theta_s)O_2 + J_2(b_s + \theta_s)V_2]$, $T_2 = [-(1 + G_2)(b_p + \theta_p) + (1 + G_2)\theta_s V_2 + M_2\theta_p - M_2(b_s + \theta_s)V_2]$, $X_2 = [-H_2(b_p + \theta_p) + (1 + G_2)\theta_p + (1 + G_2)\theta_s U_2 + H_2\theta_s V_2 - M_2(b_p + \theta_p) + (1 + L_2)\theta_p - M_2(b_s + \theta_s)U_2 - (1 + L_2)(b_s + \theta_s)V_2]$, $\gamma_2 = [-K_2(b_p + \theta_p) + K_2\theta_s V_2 + (1 + G_2)\theta_s Y_2 + N_2\theta_p - N_2(b_s + \theta_s)V_2 - M_2(b_s + \theta_s)Y_2]$, $Z_2 = [a_1H_2 - I_2\theta_p - H_2\theta_s O_2 - I_2\theta_s U_2 + a_2(1 + L_2) + J_2(b_p + \theta_p) + (1 + L_2)(b_s + \theta_s)O_2 + J_2(b_s + \theta_s)U_2]$, $\alpha_2 = [H_2\theta_p - H_2\theta_s U_2 - (1 + L_2)(b_p + \theta_p) - (1 + L_2)(b_s + \theta_s)U_2]$, $\phi_2 = [K_2\theta_p + H_2(b_s + \theta_s) + K_2\theta_s U_2 + H_2\theta_s Y_2 - N_2(b_p + \theta_p) - N_2(b_s + \theta_s)U_2 - (1 + L_2)(b_s + \theta_s)Y_2]$, $\beta_2 = [a_1K_2 - I_2(b_s + \theta_s) - K_2\theta_s O_2 - I_2\theta_s Y_2 + a_2N_2 + N_2(b_s + \theta_s)O_2 + J_2(b_s + \theta_s)Y_2]$,
$\lambda_2 = \left[K_2(b_s + \theta_s) + K_2\theta_s Y_2 - \frac{\eta_1}{2} - N_2(b_s + \theta_s)Y_2\right]$

By solving (13), (14) and (15) simultaneously, and assuming $\delta_2 = X_2\gamma_2 - 2T_2\phi_2$, $\xi_2 = 2\alpha_2\gamma_2 - \phi_2 X_2$, $\tau_2 = R_2\phi_2 - Z_2\gamma_2$, $\vartheta_2 = \phi_2\gamma_2 - 2\lambda_2 X_2$, $\sigma_2 = \phi_2^2 - 4\alpha_2\lambda_2$, $\varepsilon_2 = 2\lambda_2 Z_2 - \phi_2\beta_2$, $\delta_2 p_1 + \xi_2 p_2 = \tau_2$, $\vartheta_2 p_1 + \sigma_2 p_2 = \varepsilon_2$, we get:

$p_1 = \frac{\sigma_2\tau_2 - \xi_2\varepsilon_2}{\delta_2\sigma_2 - \xi_2\vartheta_2}$, $p_2 = \frac{\delta_2\varepsilon_2 - \vartheta_2\tau_2}{\delta_2\sigma_2 - \xi_2\vartheta_2}$

$$s_1 = \frac{\gamma_2\sigma_2\tau_2 - \xi_2\varepsilon_2\gamma_2 + \phi_2\delta_2\varepsilon_2 - \phi_2\vartheta_2\tau_2 + \delta_2\sigma_2\beta_2 - \xi_2\vartheta_2\beta_2}{-2\lambda_2(\delta_2\sigma_2 - \xi_2\vartheta_2)}$$

Taking the second-order partial derivatives of π_R with respect to p_1, p_2 and s_1, respectively, we have the Hessian Matrix:

$$H = \begin{bmatrix} \frac{\partial^2\pi_R}{\partial {p_1}^2} & \frac{\partial^2\pi_R}{\partial p_1\partial p_2} & \frac{\partial^2\pi_R}{\partial p_1\partial s_1} \\ \frac{\partial^2\pi_R}{\partial p_2\partial p_1} & \frac{\partial^2\pi_R}{\partial {p_2}^2} & \frac{\partial^2\pi_R}{\partial p_2\partial s_1} \\ \frac{\partial^2\pi_R}{\partial s_1\partial p_1} & \frac{\partial^2\pi_R}{\partial s_1\partial p_2} & \frac{\partial^2\pi_R}{\partial {s_1}^2} \end{bmatrix} = \begin{bmatrix} 2T_2 & X_2 & \gamma_2 \\ X_2 & 2\alpha_2 & \phi_2 \\ \gamma_2 & \phi_2 & 2\lambda_2 \end{bmatrix}$$

Since, the second order optimization conditions, can be derived:

if $\left(H_{11} = \frac{\partial^2\pi_R}{\partial {p_1}^2} = 2T_2 < 0\right)$, $\det\left(\begin{bmatrix} 2T_2 & X_2 \\ X_2 & 2\alpha_2 \end{bmatrix}\right) = 4\alpha_2 T_2 - X_2^2 > 0$ and $\det(H) = [2T_2(4\alpha_2\lambda_2 - \phi_s^2) - X_2(2X_2\lambda_2 - \gamma_2\phi_2) + \gamma_2(X_2\phi_2 - 2\alpha_2\gamma_2)] < 0$, π_R is strictly concave in p_1, p_2 and s_1.

Proof of Proposition 3. Similar to Proof of Proposition 1, By solving the first conditions $\frac{\partial\pi_{M2}}{\partial w_2} = 0$ and $\frac{\partial\pi_{M2}}{\partial s_2} = 0$, the manufacturers reaction functions are obtained as:

$$w_1 = \rho p_1 \quad (16)$$

$$w_2 = \frac{B^2 c_2 - \eta_2 E_2}{0.5A\eta_2 + B^2} \quad (17)$$

$$s_2 = \frac{-BE_2 - 0.5ABc_2}{0.5A\eta_2 + B^2} \quad (18)$$

Equations (16), (17), and (18) reduces to equations (19), (20), and (21) respectively:

$$w_1^* = \rho p_1 \quad (19)$$

$$w_2^* = J_2 - M_2 p_1 - L_2 p_2 - N_2 s_1 \quad (20)$$

$$s_2^* = O_2 - V_2 p_1 - U_2 p_2 - Y_2 s_1 \quad (21)$$

The second-order partial derivatives of π_{M2} with respect to w_2 and s_2 are Similar to Proof of Proposition1.

Proof of Proposition 4. In this stage, by substituting Eqs. (19), (20), (21) in π_R and solving the first conditions ${\partial\pi_R}/{\partial p_i} = 0$ and ${\partial\pi_R}/{\partial s_1} = 0$, the retailer's reaction functions are obtained as:

$$\frac{\partial\pi_R}{\partial p_1} = R_3 + 2T_3 p_1 + X_3 p_2 + \gamma_3 s_1 = 0 \quad (22)$$

$$\frac{\partial\pi_R}{\partial p_2} = Z_3 + X_3 p_1 + 2\alpha_3 p_2 + \phi_3 s_1 = 0 \quad (23)$$

$$\frac{\partial\pi_R}{\partial s_1} = \beta_3 + \gamma_3 p_1 + 2\lambda_3 s_1 + \phi_3 p_2 = 0 \quad (24)$$

By assuming $R_3 = [a_1(1-\rho) - (1-\rho)\theta_s O_2 + a_2 M_2 - J_2\theta_p + M_2(b_s+\theta_s)O_2 + J_2(b_s+\theta_s)V_2]$, $T_3 = [-(1-\rho)(b_p+\theta_p) + (1-\rho)\theta_s V_2 + M_2\theta_p - M_2(b_s+\theta_s)V_2]$, $X_3 = [(1-\rho)\theta_p + (1-\rho)\theta_s U_2 - M_2(b_p+\theta_p) + (1+L_2)\theta_p - M_2(b_s+\theta_s)U_2 - (1+L_2)(b_s+\theta_s)V_2]$, $\gamma_3 = [(1-\rho)(b_s+\theta_s) + (1-\rho)\theta_s Y_2 + N_2\theta_p - N_2(b_s+\theta_s)V_2 - M_2(b_s+\theta_s)Y_2 - M_2\theta_s]$, $Z_3 = [a_2(1+L_2) + J_2(b_p+\theta_p) + (1+L_2)(b_s+\theta_s)O_2 + J_2(b_s+\theta_s)U_2]$, $\alpha_2 = [-(1+L_2)(b_p+\theta_p) - (1+L_2)(b_s+\theta_s)U_2]$, $\phi_2 = [-N_2(b_p+\theta_p) - N_2(b_s+\theta_s)U_2 - (1+L_2)(b_s+\theta_s)Y_2 - (1+L_2)\theta_s]$, $\beta_2 = [a_2 N_2 + N_2(b_s+\theta_s)O_2 + J_2(b_s+\theta_s)Y_2 + J_2\theta_s]$, $\lambda_2 = \left[-\frac{\eta_1}{2} - N_2(b_s+\theta_s)Y_2 - N_2\theta_s\right]$

By solving (22), (23) and (24) simultaneously, and assuming $\delta_3 = X_3\gamma_3 - 2T_3\phi_3$, $\xi_3 = 2\alpha_3\gamma_3 - \phi_3 X_3$, $\tau_3 = R_3\phi_3 - Z_3\gamma_3$, $\vartheta_3 = \phi_3\gamma_3 - 2\lambda_3 X_3$, $\sigma_3 = \phi_3^2 - 4\alpha_3\lambda_3$, $\varepsilon_3 = 2\lambda_3 Z_3 - \phi_3\beta_3$, $\delta_3 p_1 + \xi_3 p_2 = \tau_3$, $\vartheta_3 p_1 + \sigma_3 p_2 = \varepsilon_3$, we get:

$p_1^* = \frac{\sigma_3\tau_3 - \xi_3\varepsilon_3}{\delta_3\sigma_3 - \xi_3\vartheta_3}$

(25)

$p_2^* = \frac{\delta_3\varepsilon_3 - \vartheta_3\tau_3}{\delta_3\sigma_3 - \xi_3\vartheta_3}$

(26)

$s_1^* = \frac{\gamma_3\sigma_3\tau_3 - \xi_3\varepsilon_3\gamma_3 + \phi_3\delta_3\varepsilon_3 - \phi_3\vartheta_3\tau_3 + \delta_3\sigma_3\beta_3 - \xi_3\vartheta_3\beta_3}{-2\lambda_3(\delta_3\sigma_3 - \xi_3\vartheta_3)}$

(27)

Taking the second-order partial derivatives of π_R with respect to p_1, p_2 and s_1, respectively, we have the Hessian Matrix:

$$H = \begin{bmatrix} \frac{\partial^2\pi_R}{\partial {p_1}^2} & \frac{\partial^2\pi_R}{\partial p_1 \partial p_2} & \frac{\partial^2\pi_R}{\partial p_1 \partial s_1} \\ \frac{\partial^2\pi_R}{\partial p_2 \partial p_1} & \frac{\partial^2\pi_R}{\partial {p_2}^2} & \frac{\partial^2\pi_R}{\partial p_2 \partial s_1} \\ \frac{\partial^2\pi_R}{\partial s_1 \partial p_1} & \frac{\partial^2\pi_R}{\partial s_1 \partial p_2} & \frac{\partial^2\pi_R}{\partial {s_1}^2} \end{bmatrix} = \begin{bmatrix} 2T_3 & X_3 & \gamma_3 \\ X_3 & 2\alpha_3 & \phi_3 \\ \gamma_3 & \phi_3 & 2\lambda_3 \end{bmatrix}$$

Since, the second order optimization conditions, can be derived:

if $(H_{11} = \frac{\partial^2\pi_R}{\partial {p_1}^2} = 2T_3 < 0), \det\left(\begin{bmatrix} 2T_3 & X_3 \\ X_3 & 2\alpha_3 \end{bmatrix}\right) = 4\alpha_3 T_3 \quad - X_3^2 > 0$ and $\det(H) = [2T_3(4\alpha_3\lambda_3 - \phi_3^2) - X_3(2X_3\lambda_3 - \gamma_3\phi_3) + \gamma_3(X_3\phi_3 - 2\alpha_3\gamma_3)] < 0$, π_R is strictly concave in p_1, p_2 and s_1.