Development of Markowitz Portfolio Optimization Model Considering Time Factor and Skewness and kurtosis of Returns

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ABSTRACT

The investment portfolio optimization problem is a topic that has always been of interest to financial researchers and capital market activists. The aim of this research is to develop the two-dimensional Markowitz portfolio optimization model into a five-dimensional model considering the mean, variance, skewness, kurtosis, and time factor, and then identify the optimal portfolio for investment. In this regard, the return of each share was identified using daily stock price information. Then, the average return, return variance, skewness, and kurtosis of the stock returns of the companies under study were identified in the short-term, medium-term, and long-term time periods, and then the optimal portfolio was identified based on the calculated values and the efficient utility function. To test the model, five main industries of the Tehran Stock Exchange were used, including chemical industries, petroleum products and coke, basic metals, cement, lime and gypsum, and pharmaceuticals. The most profitable company from each industry was selected, including Persian Gulf Petrochemical Industries with the trading symbol Fars, Isfahan Oil Refining with the trading symbol Shapna, Isfahan Mobarakeh Steel Company with the trading symbol Foolad, Tehran Cement Company with the trading symbol Setran, and Pars Daru Company with the trading symbol Depars. The results of the study indicate that the proposed model is able to identify optimal portfolios in different time periods and investors choose a portfolio consisting of Foolad and Setran stocks to obtain maximum utility in the short term. In the medium term, they choose Foolad and Setran stocks and invest in Fars and Foolad stocks in the long term.

Keywords: Mean, Variance, Skewness, Kurtosis, Time.

1. Introduction

Selecting an optimal set of assets has always been a topic of interest for researchers in finance and practitioners in the capital market. In the field of microeconomics, since an investor defers today's returns in the hope of achieving higher returns in the future, investment decisions are highly important. The decision for better investment depends on the utility of each individual, which is determined by their beliefs, assumptions, mental models, and personal preferences, and this utility is not necessarily identical across individuals; however, the utility of each person in relation to selecting an optimal investment portfolio can be quantified using specific criteria.

In reality, considering that different asset classes—including securities, cash, land, gold, etc.—have varying risks and returns, and that individuals' risk tolerance differs, asset returns are unpredictable. Due to this uncertainty in the market, asset diversification becomes particularly important. On the other hand, the capital market in our country lacks sufficient efficiency, and to achieve reasonable returns, one cannot rely solely on existing studies. Therefore, the risk of entering and investing in this market is high.One of the ways to manage and reduce investment risk is forecasting price trends. However, some researchers believe that the future cannot simply be predicted, but must be created. Therefore, success in forecasting requires intervening in the formation of realities in an optimal way (Pierdzioch et al., 2012). The primary goal of stock price forecasting is to assist investors in selecting an optimal portfolio, as an optimal portfolio can significantly reduce risk and maximize individual returns. The purpose of portfolio formation is to distribute investment risk across several stocks, so that the profit from one stock can offset losses from others (Khudameradi et al., 2013).

Portfolio formation is influenced by various factors, and several models have been proposed for the problem of portfolio selection and optimization. Each of these models considers a subset of factors affecting portfolio selection, but none are fully comprehensive. From a theoretical standpoint, the most important factors in portfolio selection are the risk and return of the constituent assets and ultimately the risk and return of the overall portfolio. However, in reality, other factors beyond risk and return—both quantitative and qualitative—may affect portfolio selection.In this study, we have attempted to incorporate additional factors such as return skewness, return kurtosis, and time, alongside the two main factors of mean return and standard deviation, in the Markowitz portfolio optimization problem. Therefore, the aim of this research is to develop the Markowitz portfolio optimization model considering time, skewness, and kurtosis of returns.

In the following sections, we present the theoretical foundations and literature review, then describe the research methodology, followed by data analysis and results. Finally, we provide a summary of the discussion and offer necessary recommendations.

2- Theoretical Foundations and Literature Review

Investment theories have made significant progress over the past decades and have developed numerous practical formulas throughout their historical evolution. With the introduction of the Markowitz model in 1952, this model was widely used as a useful tool for portfolio optimization. This model, considering the risk and return criteria of the selected set, was introduced as the modern portfolio theory (MPT). Prior to this theory, despite being familiar with the concepts of risk and return, investors were unable to quantitatively measure them. In fact, Markowitz was the first to expand and formalize the concept of diversification in investment portfolios.

The modern portfolio theory provides a holistic view of the stock market. Unlike technical and fundamental methods, this theory focuses on the set of stocks in the portfolio. In formulating his mean-variance model, Markowitz placed particular emphasis on the investment objective. According to him, a rational investor seeks investments that offer higher returns and lower risk. He did not consider the risk of an investment solely in terms of its standard deviation but also examined the relationship between different assets in the portfolio and its impact on the total portfolio risk. Another concept introduced by Markowitz was the efficient portfolio, which refers to the optimal combination of securities such that the portfolio risk is minimized for a given return, or the portfolio return is maximized for a given level of risk. The set of such portfolios is known as the efficient frontier and can be identified through solving a nonlinear parametric problem.

Although the modern portfolio theory is widely applied in the financial industry, its fundamental assumptions have been challenged (Xidonas et al., 2012). Due to the limitations of the Markowitz model, Treenor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966) independently developed the Capital Asset Pricing Model (CAPM), which provides a set of predictions about the expected equilibrium returns of risky assets. However, CAPM also faced criticisms regarding its assumptions. To address the limitations of both models, Ross (1970) introduced the Arbitrage Pricing Theory (APT). The core concept of APT is the logic of a "single price", meaning that it is impossible to sell two assets with the same risk and return at different prices (Raei & Pouyanfar, 2010).Many st udies on portfolio selection theory have been conducted based on the first two moments of return distributions, i.e., mean and variance, showing how investor thinking shifted from intuitive and judgment-based decision-making to scientific and mathematical methods. The Markowitz mean-variance model assumes that returns are normally distributed (Naqvi et al., 2017). However, subsequent researchers, including Konno & Suzuki (1995), argued that return distributions are not always normal and may follow non-normal distributions, exhibiting skewness and kurtosis. Considering the third (skewness) and fourth (kurtosis) moments can lead to more precise decisions in optimal portfolio selection.Contrary to Markowitz's assumption, other researchers such as Lai (2006), Gotoh et al. (2018), Metaxiotis (2019), and Lu et al. (2019) have stated that stock returns are not normally distributed and can exhibit positive or negative skewness and excessive kurtosis. According to Khan et al. (2020), stock returns with negative skewness indicate a higher probability of negative returns than positive returns. Many researchers, including Chen et al. (2020), Marques & Benasciutti (2020), and Khan et al. (2020), have considered third and fourth moments—skewness and kurtosis—when selecting the best portfolio. Naqvi et al. (2017) and Diaz et al. (2022) also highlighted the importance of skewness and kurtosis in portfolio formation.

The model that integrates skewness and kurtosis with the basic Markowitz model is called the mean-variance-skewness-kurtosis model, whose main objective is to minimize risk and excess kurtosis while maximizing return and skewness. By adding the dimensions of skewness and kurtosis to the basic Markowitz model, this model aims to identify the optimal portfolio based on all four dimensions: mean return, return variance, skewness, and kurtosis.On the other hand, the time horizon is of great importance to investors. As the investment period increases, the investment risk also rises, and accordingly, the expected return should increase. Fahmi (2019) incorporated the time factor into the basic Markowitz mean-variance model and found that as the investment horizon increases, the investor's return increases proportionally with their risk. He extended the two-dimensional Markowitz model into a three-dimensional model consisting of mean return, return variance, and time. Based on the theoretical foundations outlined, numerous empirical studies have been conducted on portfolio optimization considering various factors. In the following, a brief review of some of these studies is presented.

Gubu and Hilmi (2024), in a study titled Beyond the Markowitz Mean-Variance Model: Comparing the Mean-Variance-Skewness-Kurtosis Model with the Fixed Mean-Variance Model, concluded that stock return distributions in capital markets are not normally distributed and exhibit skewness and kurtosis. Furthermore, the returns of portfolios identified using the fixed mean-variance model were higher compared to the classical Markowitz model and the mean-variance-skewness-kurtosis model. In addition, the mean-variance-skewness-kurtosis model was found to be more efficient than the traditional mean-variance model.

In another study, Goncalves et al. (2022) examined portfolio optimization with higher-order moments considering the role of information entropy. They analyzed the mean-variance-kurtosis-skewness-entropy model and compared it with the classical Markowitz mean-variance model. The results showed that this model serves as a robust financial management tool for decision-making and is a powerful instrument for quantitative analysis, asset selection, and allocation. Compared to the classical Markowitz model, it provides better performance in asset analysis and allocation and allows investors to make decisions with greater confidence.

Nasrollahi et al. (2025), in a study titled Comparing the Efficiency of Option Pricing Models under Jump, Skewness, and Non-Normal Kurtosis, found that when data distributions are non-normal, it is necessary to consider skewness and kurtosis in addition to mean and variance for more efficient decision-making. They also showed that different models exhibit varying levels of efficiency.

Rezaei et al. (2024), in a study titled Comparing the Performance of Optimal Portfolios Based on Skewed Normal and Skewed Laplace-Normal Distributions with a Mean-Absolute Deviation-Entropy Approach, concluded that returns of some stocks in the Iranian capital market do not follow a normal distribution and exhibit skewness and kurtosis, and thus the normal distribution is not suitable for modeling stock returns. They also found that the skewed Laplace-normal distribution, which considers both skewness and kurtosis, is more efficient than the skewed normal distribution.

In another study, Mohammadi et al. (2020) optimized a three-stock portfolio considering higher-order moments, semi-moments, and entropy, comparing the mean-semi-variance-skewness-semi-kurtosis-entropy method with the mean-variance-skewness-kurtosis-entropy method. They concluded that using higher-order moments, semi-moments, and entropy improves portfolio efficiency.

Finally, Aghamohammadi et al. (2017), in a study titled Skewness-Mean-Variance Testing in Optimal Portfolio Selection and Skewed Normal Distribution, concluded that the core assumption of the Markowitz model is the normal distribution of returns, and if the return distribution is not normal, the model loses efficiency. They showed that the skewness-mean-variance model is more efficient than the classical mean-variance model.

As evident from the literature, numerous studies have focused on portfolio optimization based on the Markowitz mean-variance model and on adding factors such as skewness and kurtosis. Some studies have also incorporated the time factor alongside mean and variance. However, among the reviewed studies, no model was found that simultaneously considers the five factors of mean return, return variance, skewness, kurtosis, and time. Therefore, in this study, we aim to develop a Markowitz portfolio optimization model that incorporates these five factors simultaneously.

3- Research Methodology

This research is from an applied perspective in terms of objective and descriptive-survey in terms of method. The statistical population of this study consists of all companies listed on the Tehran Stock Exchange. To select the sample, in the first step, among all active industries in the Tehran Stock Exchange, five major industries that constitute the bulk of market transactions were selected (Al Ali et al., 2022). These five industries are: chemical products, petroleum products and coke, basic metals, cement, lime and plaster, and pharmaceutical industries.

In the second step, after determining the target industries, the most profitable company in each industry was selected according to the following criteria:

The company should not be an investment company.

Since the company's listing on the stock exchange or over-the-counter market, its shares must have been traded on at least 60% of trading days, or its symbol should have remained active (Bajlan et al., 2018).

The company's fiscal year ends on March 20, 2024 (Esfand 2023).

After applying the above restrictions, the companies under study are as follows:

Persian Gulf Petrochemical Company (FARS) in the chemical products industry

Isfahan Oil Refining Company (SHAPNA) in the petroleum products and coke industry

Mobarakeh Foolad Company (FOOLAD) in the basic metals industry

Tehran Cement Company (SETRAN) in the cement, lime, and plaster industry

Pars Darou Company (DPARS) in the pharmaceutical industry

After collecting data on the closing prices of the selected companies, the normality or non-normality of the sample was first examined using the quartile test. Then, the mean, variance, skewness, and kurtosis of the returns of these stocks were calculated for short-term (1-year), medium-term (5-year), and long-term (10-year) periods.

Subsequently, various two-stock portfolios were formed from the selected companies with equal weights, and the mean, variance, skewness, and kurtosis of these two-stock portfolios were calculated for the 1-year, 5-year, and 10-year periods. Finally, using the CARA utility function, the optimal portfolio for the investor was identified over different time periods, allowing investors to select the best portfolio according to their risk aversion, risk tolerance, and investment horizon.

The research variables include stock returns, variance of returns, kurtosis, skewness, and time. The calculation method for these variables is explained below:

Stock Return: Stock return represents the investor's gain or loss from holding the stock over a given period. The return of each stock consists of two components: dividends received and capital gain or loss from changes in the stock price. It can be calculated as follows:

$$R_{i,t} = \frac{(1+\alpha+\beta)P\ i,t - (Pi,t-1 + c\alpha) + DPS\ i,t}{Pi,t-1 + c\alpha}$$

Where:

$P_{i,t}$: The price of stock i at the end of year t

Pi,t-1:: The price of stock i at the end of year t−1

Di,t:: The cash dividend paid

α: The percentage of capital increase from receivables and cash contributions

β: The percentage of capital increase from retained earnings

C: The nominal amount paid by the investor for the capital increase from cash contributions (Shokrkhah et al., 2017).

In this study, to calculate stock returns, cash dividends, percentage of capital increase from claims and cash contributions, and percentage of capital increase from reserves, as well as the nominal amount paid by the investor for the capital increase from cash contributions are ignored. The stock return is calculated based on the price change of the stock on day t relative to its price on the previous day. In other words, the return for each stock is calculated using the following formula:

$$R_i = \frac{P\ i,t\ - Pi,t-1}{Pi,t-1}$$

To calculate the portfolio return, after identifying the return of each stock and its weight in the portfolio, the following formula is used (Goncalves et al., 2022):

$$E(Rp) = \sum_{i=1}^{n} \mathrm{Ri} * \mathrm{Wi}$$

Variance of Stock Returns: The variance of stock returns is a factor for identifying the risk of each stock (Gobo et al., 2024). To calculate the variance of each stock's return, the following formula is used:

$$\sigma\ 2 = \frac{1}{n-1} \sum_{i=0}^{n} (Pi\ -\ n)$$

Also, to calculate the variance of a two-stock portfolio return, the following formula is used (Goncalves et al., 2022):

Variance of Stock Returns $= \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{Wi\ Wj\ Cov\ i, j}$

Where and represent the weights of stocks wi and wi, respectively, and denotes the covariance between the two stocks.

Skewness of returns: The degree of asymmetry in a distribution is called skewness, which indicates the extent to which a distribution deviates from normal and symmetrical form. Skewness is calculated using the following formula (Shokrkhah et al., 2017):

$$Si = \frac{\frac{1}{N}\sum_{i=1}^{N}(ri - \bar{r}i)3}{\frac{1}{N}\sum_{i=1}^{N}(ri - \bar{r}i)3/2}$$

Where S_i represents the skewness of the daily stock returns, r_i is the daily return of the stock, and $\bar{r}$ denotes the mean daily return of the stock, which is obtained by dividing the total sum of returns over the period by the number of trading days considered.

The skewness of the stock portfolio returns is calculated using the following formula:

$$Sp = \frac{(n1*AS1*\sigma1\text{^}3) + (n2*AS2*\sigma2\text{^}3)+(n1*(M1-M)\text{^}3) + n2*(M2-M)\text{^}3}{((n1+n2)*\sigma\text{^}3}$$

Kurtosis of returns: Kurtosis indicates the height or peakedness of a distribution compared to the normal distribution. It is calculated using the following formula (Shokrkhah et al., 2017):

$$Ki = \frac{\frac{1}{N}\sum_{i=1}^{N}(ri - \bar{r}i)4}{[\frac{1}{N}\sum_{i=1}^{N}(ri - \bar{r}i)2]2}$$

To calculate the kurtosis of the stock portfolio, the following formula is used:

$$Kp = \frac{(n1-1) * k1+(n2 - 1) * k2}{(n1 + n2 + 2)}$$

Time: The time variable in this research is divided into three periods: short-term or one-year, medium-term or five-year, and long-term or ten-year. The idea of using time in any kind of activity, including production activities and others, goes back to the studies of

Gossen (1854) and Becker (1965). The application of the time factor in the development of the mean-variance model of Markowitz was raised in the research of Fahmi (2019). In this model, a portfolio consisting of n risky assets, each with an expected rate of return x, is formed in a time period T which is the optimal time for the investment trading strategy.

At the beginning of the period, that is t = 0, when the investor identifies the optimal portfolio, according to the weight of each stock, an amount is invested in that stock. The end of the investment period, that is t = n, is the time when the investor, after the time period n, has gained a positive or negative return from the investment. In the time interval between t = 0 and t = n, that is the medium-term or the optimal and expected time, the behavior of the investor and their trading strategy can be inferred.

For example, an investor who has formed a portfolio of two stocks A and B for a one-year period, after a severe financial shock in the fourth month, reacts while irrational investors overreact to new information and in the same month adjust their portfolio of stocks. In other words, the investment time period of such investors is considered short-term. On the other hand, there are rational investors who may adjust their portfolio of stocks after the market returns to a stable condition, meaning that their trading strategy is more long-term.

In this case, investors must consider two factors:

The first factor is a weight vector representing the weight of each stock in the portfolio (asset allocation), and the second factor, according to the realized rate of return, is to identify the optimal trading time (time allocation).

At time t = 0, only the weight factor of each stock is considered in such a way that the individual's return is maximized and the investment risk is minimized. Then, the individual enters a specific and optimal time period for investment which can range from one day to n years. Suppose that the individual's utility in the lowest investment state is equal to m and in the highest state the individual's utility is equal to M. Also, the individual's utility can be between the two limits m and M, that is, it can be considered as a, b, and c.$Um, Ua, Ub, Uc, \ldots\ldots, UM$

For a rational investor who prefers long-term strategies, if Ub is greater than Ua, then tb is also greater than ta. In this regard, the following principles are considered:

First principle:Regarding the time periods a and b, it can be said that either time b is greater than a, or they are equal, or time a is less than b, and it is shown as follows:

$$ta < tb \ , \ ta > tb \ , ta = tb$$

Second principle:If the time period c is greater than b, and the time period b is also greater than a, then we have:

$$tc > tb \, , tb > ta \ \rightarrow \ tc > ta$$

Third principle:The time period on the vector T is continuous. For every time point t within the total investment period T, the investor prefers the time period t + 1 over the time period t.

Fourth principle:The time utility system is uniform. This means that the utility of one unit of time is the same, regardless of when it occurs.

Neumann and Morgenstern (1947) showed that if principles one, two, three, and four are considered, a continuous utility curve is obtained that represents the preference order with respect to time. These principles guarantee the existence of a continuous utility function U(t). However, since time is not always stable, if a utility function exists, it must satisfy the law of diminishing marginal utility of time duration.

Fifth principle:The marginal utility of the investment time period has a negative relationship with the difference between the total utility value at time t and the estimated maximum satisfaction level at time t, which is shown as follows:

$$\frac{d}{dt}U(t) \ = \ -c[U(t) \ - \ U(\bar{t})] \, , c > 0$$

where c represents the proportionality parameter.

Sixth principle:If principles one to five hold, then the utility curve represents the utilities that preserve the time preference order of an investment portfolio over the time period T, and finally, the expected utility can be calculated as follows:

$$U(t) \ = \ M + [m - M] \, e^{-ct} \, , c > 0$$

where m represents the utility at day zero, and M also indicates the utility at a specific day of the investment. Finally, if all the aforementioned principles hold, the following figure can be drawn:

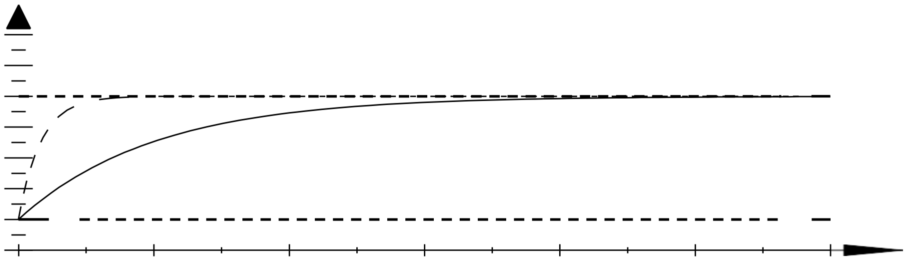

Figure 1. Time Factor

As can be inferred from Figure 1, if it is assumed that the optimal investment period is 12 months, the return in each month compared to the previous month increases (the period t + 1 provides higher utility than period t) and over time, this increase follows a decreasing rate. When the individual reaches the maximum utility (month 12), the decreasing rate causes a reduction in the portfolio return, and at this point, the investor needs to adjust their portfolio (Fahmi, 2019).

Finally, to identify the optimal portfolio, after calculating the mean, variance, skewness, and kurtosis of returns and considering the time factor, the efficient utility function can be used to determine the optimal portfolio. This function is defined as follows:

$$\mathrm{CARA} = \mu_k - \frac{m\,\sigma_k^2}{2} + \frac{m^2\,\sigma_k^3 (S_k)}{6} - \frac{m^3\,\sigma_k^4\,(K_k - 3)}{720}$$

If the skewness and kurtosis values for the portfolios under consideration are zero, or in other words, if the distribution is normal, the results of this formula are identical to those obtained using the Markowitz model. After calculating the efficiency coefficient, the portfolio with the highest coefficient is considered the optimal portfolio. In this formula, μ_k represents the mean return of portfolio K, m represents the time period, and σ_k is the standard deviation of portfolio K. S_k represents the skewness of portfolio K, and K_k denotes the kurtosis of portfolio K (Di Pierro & Mosevic, 2008).

4- Research Findings

Considering that in this study the assumption of data normality is omitted (by removing this assumption, the calculation of skewness and kurtosis becomes meaningful), it is first necessary to examine whether the data are normal or non-normal. To this end, the Jarque-Bera test is employed. The results of this test for the research data are presented in Table 1.

Table 1. Results of the Jarque–Bera Test

period		Jarque–Bera Test	p-value
One-year	Fars	۵۴۰۹۲۰	۰/۰۰۰
	Shapan	۳۳۱۰۰۶	۰/۰۰۰
	Foolad	۴۶۹۰۷	۰/۰۰۰
	Setran	۸۰۲۰	۰/۰۰۰
	Depars	۸۰۸۹۳	۰/۰۰۰
Five-year	Fars	۶۴۶۲۴۳۳	۰/۰۰۰
	Shapna	۱۲۸۴۵۶۶۴	۰/۰۰۰
	Foolad	۳۴۲۹۴۰۵	۰/۰۰۰
	Setran	۵۴۱	۰/۰۰۰
	Depars	۱۸۰۸۶۹۰	۰/۰۰۰
Ten-year	Fars	۱۰۴۱۵۲۶۰	۰/۰۰۰
	Shapna	۱۸۲۵۳۹۷۳	۰/۰۰۰
	Foolad	۳۴۵۷۳۴۳	۰/۰۰۰
	Setran	۵,۳۳	۰/۰۰۰
	depars	۱۰۹۸۴۹۰۲	۰/۰۰۰

Source: Researcher's findings

As observed, the calculated P-values for all companies in the sample are less than 0.05; therefore, it can be stated that the distribution of the data significantly differs from a normal distribution and is not normal. Consequently, the calculation of skewness and kurtosis is meaningful.

After identifying the type of data distribution in the study, and considering that the research data are asymmetric, the coefficients of mean, variance, skewness, and kurtosis of returns were calculated and are presented in Table 2.

After identifying the mean, variance, skewness, and kurtosis coefficients of returns for all the companies under study, it is possible to calculate each of these coefficients for different portfolios. To calculate the portfolio

variance, which represents the risk of each portfolio, in addition to the aforementioned coefficients, the correlation coefficients between each pair of stocks are also required. Table

3 shows the correlation coefficients among different stocks across various time periods.

After calculating the correlation coefficients, mean, variance, skewness, and kurtosis of each stock's returns, the mean return, return variance, return skewness, and return kurtosis coefficients for each portfolio can be calculated. The calculated values for each portfolio are presented in Table 4.

After calculating the mean return, return variance, skewness, and kurtosis, the optimal portfolio for short-term, medium-term, and long-term periods was identified using the efficient utility function. The computed coefficients are presented in Table 5:

The calculated values for the efficient utility function are presented in Table 5. The higher this coefficient, the higher the portfolio's rank. The final ranking of portfolios across different time periods is presented in Table 6.

Table 2. Results of the Jarque–Bera Test

period	index	depars	setran	foolad	shapna	fars
One-year	Mean return	۰/۰۰۵۱	۰/۰۰۰۶	۰/۰۰۱۴	۰/۰۰۱۵	۰/۰۰۳۸
	Return variance	۰/۰۰۷۸	۰/۰۰۰۵۲	۰/۰۰۰۸۳	۰/۰۰۴۷	۰/۰۰۳۹
	Return skewness	۷/۷۰	۲/۸۶	۶/۶	۱۲/۴۲	۱۴/۹۸
	Return kurtosis	۹۰/۶۷	۳۰/۴۳	۶۹/۵۰	۱۸۱/۳۴	۲۳۱/۲۴
Five-year	Mean return	۰/۰۰۱۲	-۰/۰۰۱۸	-۰/۰۰۰۲۱	۰/۰۰۰۶۳	۰/۰۰۰۸۵
	Return variance	۰/۰۰۴۴	۰/۰۰۰۷	۰/۰۰۱۲	۰/۰۰۳۳	۰/۰۰۱۷
	Return skewness	۱۰/۶۶	۰/۲۱	۱۳/۴۱	۲۰/۲۹	۱۷/۳۷
	Return kurtosis	۱۹۰/۹	۶/۲۵	۲۶۲	۵۰۵/۳۰	۳۵۸/۷۴
Ten-year	Mean return	۰/۰۰۱۲	۰/۰۰۲۶	۰/۰۰۰۲۶	/۰۰۰۸	۰/۰۰۰۷۱
	Return variance	۰/۰۰۴	۰/۰۳۲	۰/۰۰۱۴	۰/۰۰۲۷	۰/۰۰۱۹
	Return skewness	۱۴/۳	۴۷/۵	۱۰/۶۹	۱۷	۱۵/۹
	Return kurtosis	۳۳۲/۴	۲۳۰۴/۹	۱۸۷/۳۲	۴۲۷/۹	۳۲۳/۵

Source: Researcher's findings

Table 3 – Correlation Coefficient Matrix of the Companies under Study

depars	setrab	foolad	shapna	fars		period
۰/۰۰۵۴	-۰/۰۰۱	-۰/۰۰۳۸	۰/۰۳	***	Fars	One-year
۰/۰۴۱	۰/۰۲۲	۰/۰۳۰	***	۰/۰۳	Shapna	
۰/۰۲۶	۰/۲۱	***	۰/۰۳۰	-۰/۰۰۳۸	Foolda	
۰/۰۴۲	***	۰/۲۱	۰/۰۲۲	-۰/۰۰۱	Setran	
***	۰/۰۴۲	۰/۰۲۶	۰/۰۴۱	۰/۰۰۵۴	Depars	
۰/۰۱۵	۰/۰۵۱	۰/۰۴۵	۰/۰۴۴	***	Fars	Five-year
۰/۰۱۵	۰/۰۴۴	۰/۰۵۷	***	۰/۰۴۴	Shapna	
۰/۰۱۰	۰/۰۹۴	***	۰/۰۵۷	۰/۰۴۵	Foolad	
۰/۰۵۶	***	۰/۰۹۴	۰/۰۴۴	۰/۰۵۱	Setrab	
***	۰/۰۵۶	۰/۰۱۰	۰/۱۵	۰/۰۱۵	Depars	
۰/۰۰۲	۰/۰۰۷	۰/۱۱	۰/۰۲۵	***	Fars	Ten-year
۰/۰۹	۰/۰۰۸	۰/۰۱۵	***	۰/۰۲۵	Shapna	
۰/۰۳۷	۰/۰۱۷	***	۰/۰۱۵	۰/۱۱	Foolda	
۰/۰۱	***	۰/۰۱۷	۰/۰۰۸	۰/۰۰۷	Setran	
***	۰/۰۱	۰/۰۳۷	۰/۰۹	۰/۰۰۲	Depars	

Source: Researcher's findings

Table 4– Coefficients for Each Portfolio

period	Portfolio	Return kurtosis	Return skewness	Return variance	Mean-return
One-year	Fars/shapna	206.2896	-0.991633341	0.002231773	0.00271
	Fars/foolad	150.363	-0.993954739	0.001181178	0.002643
	Fars/setran	130.8364	-0.992773458	0.001105241	0.002232
	Fars/depars	160.954	-0.994701374	0.002909985	0.004468
	Shapna/foolad	125.4194	-0.984333034	0.001396078	0.001519
	Shapna/setran	105.8928	-0.970997034	0.001318138	0.001108
	Shapna/depars	136.0104	-0.99241124	0.003122883	0.003344
	Foolad/setran	49.96611	-0.99231572	0.000338666	0.001041
	Foolad/depars	80.08373	-0.993804947	0.002143411	0.003277
	Setran/depars	60.55712	-0.992862579	0.002065471	0.002866
Five year	Fars/shapna	432.0189	-0.201007519	0.001265517	0.000741
	Fars/foolad	310.3672	-0.198947306	0.000728352	0.000316
	Fars/setran	182.4923	0.201795723	0.000608365	-0.0005
	Fars/depars	274.8148	-0.201390743	0.001523719	0.001015
	Shapna/foolad	383.6472	-0.181389205	0.001123394	0.000206
	Shapna/setran	255.7723	0.202014495	0.001003408	-0.00061
	Shapna/depars	348.0948	-0.201118135	0.001918761	0.000904
	Foolad/setran	134.1206	0.201659787	0.000466243	-0.00103
	Foolad/depars	226.443	-0.199933021	0.001381596	0.00048
	Setran/depars	98.56818	0.203401787	0.00126161	-0.00033
Ten-year	Fars/shapna	375.6774	-0.101560839	0.001133451	0.000735
	Fars/foolad	255.4009	-0.101511674	0.000795297	0.000482
	Fars/setran	1314.598	-0.101304978	0.008523529	0.001631
	Fars/depars	327.9306	-0.101582671	0.001448911	0.000948
	Shapna/foolad	307.6228	-0.101486502	0.001008516	0.000515
	Shapna/setran	1366.35	-0.101323503	0.008736749	0.001664
	Shapna/depars	380.1009	-0.101579016	0.00166213	0.000981
	Foolad/depars	1246.543	-0.101140076	0.008398594	0.001411
	Foolad/depars	259.876	-0.101550123	0.001323976	0.000728
	Setran/depars	1318.632	-0.1014108	0.009052209	0.001877

Source: Researcher's findings

Table 5 – Coefficients for Each Portfolio

portfolio	One-year ranking coefficients	five-year ranking coefficients	ten-year ranking coefficients
Fars/shapna	-21.99807157	-1710.392896	-9313.134082
Fars/foolad	-4.74509323	-406.5970447	-3106.779448
Fars/setran	-3.687681503	-164.7072777	-1852587.637
Fars/depars	-29.23393178	-1572.084138	-13268.66871
Shapna/foolad	-5.556611614	-1196.101128	-6027.829362
Shapna/setran	-4.276753207	-631.4562226	-2023242.931
Shapna/depars	-28.60677091	-3162.527888	-20262.44923
Foolad/setran	-0.212342284	-70.61566695	-1705353.36
Foolad/depars	-8.479237384	-1063.229772	-8760.246587
Setran/depars	-6.197917001	-376.5231897	-2095955.142

Source: Researcher's findings

Table 6. Final Ranking of Portfolios

rank	One-year portfolio	five-year portfolio	ten-year portfolio
۱	Foolad/setran	Foolad/setran	Fars/foolad
۲	Fars/setran	Fars/setran	Shapna/foolad
۳	Shapna/setran	Setran/depars	Foolad/depras
۴	Fars/foolad	Fars/foolad	Fars/shapna
۵	Shapna/foolad	Shapna/setran	Fars/depars
۶	Setran/depars	Foolad/depars	Shapna/depars
۷	Foolad/depars	Shapna/foolad	Foolad/setran
۸	Fars/shapna	Fars/depars	Fars/setran
۹	Shapna/depars	Fars/shapna	Shapna/setran
۱۰	Fars/depars	Shapna/depras	Setran/depars

Source: Researcher's findings

As observed, the Foolad/Setran portfolio in the short-term period is the best portfolio for investment. This portfolio in the medium-term period is also the optimal portfolio for investment, however, in the long-term period, this portfolio ranks seventh. Regarding this portfolio, it can be concluded that the investor's utility until the end of the medium-term period from this portfolio is increasing, and after that, the investor's utility in the long-term period will be decreasing. Therefore, medium-term investment in this portfolio is recommended.Regarding the Fars/Setran portfolio, it can be stated that this portfolio is considered optimal in the short-term period. This portfolio in the medium-term period is also suitable for investment, but in the long-term period, this portfolio ranks eighth. Regarding this portfolio, it can be concluded that the investor's utility until the end of the medium-term period from this portfolio is increasing, and after that, the investor's utility in the long-term period is decreasing. Therefore, medium-term investment in this portfolio is recommended.Regarding the Shapna/Setran portfolio, it should be noted that this portfolio ranks third in the short-term period. In the medium-term period, this portfolio ranks fifth, and in the long-term period, it ranks ninth. Regarding this portfolio, it can be said that this portfolio is a short-term portfolio, and the investor's utility from the short-term to the long-term period is decreasing, an after the medium-term period and invest in other portfolios.Regarding the Foolad/Depars portfolio, this portfolio ranks seventh in the short-term period, sixth in the medium-term perio d the investor needs to adjust this portfolio after the short-term period. Regarding the Fars/Foolad portfolio, it can be said that this portfolio ranks fourth in the short-term period, fourth in the medium-term period, and first in the long-term period. Therefore, long-term investment in this portfolio is recommended. Regarding the Shapna/Foolad portfolio, it can be stated that this portfolio ranks fifth in the short-term period, seventh in the medium-term period, and second in the long-term period. The medium-term utility of this portfolio is decreasing, and the investor can, by maintaining and adjusting the portfolio and passing through the medium-term period, hold the portfolio long-term and benefit from increasing utility.Regarding the Setran/Depars portfolio, it can be stated that this portfolio ranks sixth in the short-term period, third in the medium-term period, and tenth in the long-term period. After investing in this portfolio and adjusting it in the short-term period and passing the short-term period, the investor can enter the medium-term period and achieve increasing utility, and considering that in the long-term this portfolio is loss-making, it is better to sell this portfolio d, and third in the long-term period. The investor can, by investing in this portfolio and maintaining it until the end of the long-term period, achieve increasing utility. Regarding the Fars/Shapna portfolio, this portfolio ranks eighth in the short-term period, ninth in the medium-term period, and fourth in the long-term period. It can be stated that the investor, by investing in this portfolio and holding it long-term, can achieve higher utility. Regarding the Shapna/Depars portfolio, this portfolio ranks ninth in the short-term period, tenth in the medium-term period, and sixth in the long-term period, and the investor can achieve increasing utility by long-term investment.Regarding the Fars/Depars portfolio, it can be said that this portfolio ranks tenth in the short-term period, eighth in the medium-term period, and fifth in the long-term period. Therefore, it is a long-term portfolio, and by long-term investment in it, higher utility can be achieved. Another scenario for how to invest in these portfolios can be considered, which is that the investor invests in the Foolad/Setran portfolio in the short-term and one-year period, then continues this investment until the medium-term, and finally, for long-term investment, invests in the Fars/Foolad portfolio to achieve the highest utility.

5- Conclusion and Recommendations

The aim of this research was to develop the Markowitz optimal portfolio selection model. To this end, by rejecting one of the most important assumptions of this model, namely the normality of data, the third and fourth moments, i.e., skewness and kurtosis of returns, were added to the model. After examining whether the data were normal or non-normal, it was found that the data for the short-term, medium-term, and long-term periods were non-normal at a significance level of 0.000. The results of this part of the study are consistent with the findings of Gobo & Hilmi (2024), Goncalves et al. (2022), Nasrollahi et al. (2025), Rezaei et al. (2024), Mohammadi et al. (2020), and Aghamohammadi et al. (1396).

Given the non-normal distribution of the data, the mean-variance-skewness-kurtosis model is more efficient compared to the basic Markowitz optimal portfolio selection model. The results of this part of the study are also aligned with those of Gobo & Hilmi (2024), Goncalves et al. (2022), Mohammadi et al. (2020), and Aghamohammadi et al. (2017).

Then, by calculating the efficient utility coefficient, the optimal portfolios were identified. It was found that in the short-term, the most efficient portfolio is Foolad/Setran, in the medium-term it is also Foolad/Setran, and in the long-term, it is Fars/Foolad. The change in the optimal portfolio across different periods reflects changes in the investor's final utility over different time horizons, which aligns with the findings of Fahmi (2019).

Ultimately, investors can increase their utility from short-term to long-term periods by adjusting and modifying their portfolios. For future research, it is recommended that researchers also examine portfolios with three or more stocks and consider very short-term investment horizons (daily or even hourly). Moreover, in future studies, the short-term, medium-term, and long-term periods should be examined in greater detail so that optimal portfolios for investment can be identified on a daily basis.

6- Conflict of Interest

"No conflict of interest was reported by the authors."

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