Examining the Role of Markov Switching in Predicting Volatility in the Tehran Stock Exchange Based on Regional Market Fluctuations Using Heterogeneous Autoregressive (HAR) Models

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ABSTRACT

This study investigates the role of Markov switching in predicting volatility in the Tehran Stock Exchange (TSE), incorporating the influence of international market fluctuations (with a focus on regional markets) using Heterogeneous Autoregressive (HAR) models. The research employs 30-minute interval data from the TEPIX index and international indices (including Saudi Arabia, Russia, Dubai, and Turkey) over the period 2022–2024 to model volatility through a Markov regime-switching approach .Key findings reveal that volatility regime shifts in the TSE follow a persistent pattern: a low-volatility regime exhibits a 92% probability of persistence, whereas a high-volatility regime shows only a 25–35% chance of stability. Regional stock market fluctuations play a moderating role during high-volatility regimes, mitigating the amplification of domestic volatility. Negatively correlated markets (e.g., Saudi Arabia) highlight geopolitical influence. The results indicate that Markov regime-switching models, combined with international variables, serve as an effective tool for forecasting TSE volatility. These insights can assist policymakers in risk management and help investors optimize trading strategies.

Keywords: Markov Switching | Volatility Forecasting | HAR Model | International Volatility Spillovers

1. Introduction

Investors in financial markets are willing to accept a certain level of risk, making it essential to measure and predict this risk accurately (Liu & Morley, 2009). Additionally, selecting appropriate securities requires reliable forecasting tools. Such a system must provide precise information about the stock market's behavior for the following day. Consequently, numerous studies have focused on predicting stock market volatility, reflecting the ongoing search for models capable of forecasting future volatility trends (Moradi et al., 2015).

Leong et al. (2000) note that the variety of tools for predicting financial indicators alongside economic growth has expanded in recent years, creating broader global investment opportunities for both individual and institutional investors. Lo & MacKinlay (1988), using data from stable markets like the U.S., Western Europe, and Japan, argue that significant evidence suggests stock returns are somewhat predictable. Thus, forecasting stock index volatility is crucial for developing effective trading strategies (Moradi et al., 2015).

To guide investors and optimize risk management, various linear and nonlinear algorithms can be applied to predict volatility in stock market time series at different intervals. However, most researchers agree that financial and economic issues often involve nonlinear relationships, particularly in stock markets. As a result, conventional linear models yield inadequate predictions for future market conditions (Yu et al., 2009).

In response, efforts have been made to develop financial engineering models using emerging computational intelligence techniques. These engineering methods provide researchers with robust tools for forecasting and deriving precise solutions in complex, noisy data environments.

Forecasting volatility and uncertainties in investment and risk management is a crucial part of the investment process. Volatility forecasting is performed using mathematical modeling. Recent advancements in financial econometrics have led to the development of models that can describe investors' strategies regarding risk, expected returns, and market volatility. Today, financial analysts use econometric time-series models to analyze and explain the behavior of return volatility in stock markets (Falahpour et al., 2015).

Much financial research in recent years has focused on improving traditional linear and nonlinear models to achieve more accurate estimations and forecasts. In this regard, various neural networks and hybrid models have been proposed. For modeling return volatility, autoregressive conditional heteroskedasticity (ARCH) models are commonly used, as they are grounded in financial and economic theory. However, the application of these models, particularly in studies conducted in Iran, has not always been fully effective (Arab Mazar Yazdi et al., 2010). Therefore, integrating these theoretical foundations with heterogeneous autoregressive (HAR) models can enhance the accuracy of forecasting systems. This study aims to develop more suitable models for return volatility modeling by combining HAR models with Markov regime-switching.

Financial markets, as part of the production factor markets, facilitate investment financing and channel idle savings toward optimal economic pathways. Investor behavior in these markets often changes abruptly. While some of these changes may be temporary, asset price fluctuations frequently persist for extended periods. For example, the mean, volatility, and correlation patterns in stock markets shift significantly during such changes, often leading to financial crises. Market regime shifts—some short-term and others persistent—are common in financial markets such as bonds, stocks, exchange rates, and certain macroeconomic variables (Abtahee et al., 2012).

Regime-switching models can identify these sudden changes in investor behavior and the dynamic price movements following periods of transition. In other words, these models can analyze financial markets' tendency for abrupt shifts due to changes in investor behavior and financial variables under new conditions. The identified regimes, detected through econometric methods, may directly correlate with legal, political, and regulatory changes.

In equilibrium models, regimes influence the dynamic characteristics of financial asset prices, leading to nonlinear risk-return trade-offs. Additionally, regime transitions can have broad implications for optimal portfolio selection. On the other hand, global stock markets have historically experienced upward and downward trends, giving rise to common terminology describing different market regimes. In these markets, investors are categorized into bull regimes (optimistic investors expecting price increases) and bear regimes (pessimistic investors anticipating declines). However, an optimist may not remain so indefinitely—political and economic risks can shift their outlook. This classification stems from systematic and unsystematic risks in the market. Moreover, regime shifts in stock markets can be triggered by shocks in other financial markets (e.g., gold or forex), which may spill over into equities.

As noted, stock market volatility plays a fundamental role in asset pricing, allocation, and risk management. Consequently, a growing body of research focuses on forecasting stock market volatility. With increasing financial globalization, policymakers and investors also monitor international equity markets. Following Peng et al. (2018), Lei et al. (2018), and Liu et al. (2019), this study incorporates international stock market volatility to forecast Tehran Stock Exchange (TSE) volatility. The research builds on the Heterogeneous Autoregressive Realized Volatility (HAR-RV) model, pioneered by Corsi (2009), by integrating realized volatility (RV) from N international stock markets, thus constructing N extended models enriched with global volatility data.

The core objective of this study is to examine the role of Markov regime-switching in forecasting TSE volatility. The rationale for using Markov switching lies in the limitations of the standard HAR-RV model: while it captures key volatility features (e.g., long memory, multi-scaling behavior, and leverage effects), it assumes a single regime over extended periods—an often unrealistic premise (Duan et al., 2018). Hamilton (1989) argues that financial time-series behavior varies significantly across business cycle regimes.

Thus, the central research question is:

Does the Markov-switching-augmented HAR-RV model outperform its non-regime-switching counterpart in forecasting Tehran Stock Exchange volatility under international market conditions?

Literature Review and Background

The term "volatility" in financial discussions refers to price changes over a specific time period. By this definition, standard deviation is typically considered as a measure of volatility, although in reality, volatility and standard deviation are not exactly the same concepts. In financial discussions, return volatility is of great importance. This volatility is used in asset pricing, portfolio-related decisions, and risk management through value-at-risk calculations. There are two main types of volatility: implied volatility and realized volatility (historical volatility). Implied volatility is often used in options pricing discussions. Implied volatility is derived from the market price of options based on option pricing models and is considered as the market's view of the future volatility of the asset. Implied volatility is usually represented by the symbol σ in most sources. On the other hand, realized volatility measures what has actually happened in the past. This type of volatility is calculated from the sum of squared logarithmic returns at a specific frequency. According to this definition, using higher frequencies can yield more information. Increasing the sampling frequency means using intraday data and entering the realm of high-frequency data (Falahpour et al., 2017). Below are some of the studies conducted in this field:

Swan (2020), in his study titled "Measuring Adverse Risk in Regime-Switching Stochastic Volatility," claimed that while regime-switching models have been used to measure adverse risk, these models have been limited in their ability to capture the dynamics of stochastic volatility and thus have constrained risk measurement. In his study, he proposed a regime-switching model that can measure risk more realistically. He presented adverse risk measurement along with the regime-switching model for measuring risk under conditions of stock price jump risk and proved that his proposed regime-switching model converges to a continuous-time asset pricing model, making his risk measurement method robust.

Chen et al. (2020), in their study titled "Forecasting Oil Price Volatility Using High-Frequency Data: New Evidence," considered conditional volatility for modeling and forecasting future oil price volatility based on the HAR-RV model and its various extensions. Their empirical results show several noteworthy observations. First, the in-sample results indicate that the residuals of HAR-RV models of the ARCH type show significant effects. Second, the out-of-sample results show that compared to linear HAR-RV models, HAR-RV-type models, including FIGARCH-structured models, can generally produce higher forecasting accuracy when predicting short-term volatility. Third, when forecasting mediumand long-term volatility, the proposed model, HAR-S-RV-J-FIGARCH, can demonstrate higher predictive ability.

Duan et al. (2018), in their study titled "Leverage Effect, Economic Policy Uncertainty, and Regime-Changing Realized Volatility," first examined the effects of leverage and economic policy uncertainty on future volatility within a regime-switching framework. The out-of-sample results show that HAR-RV incorporating leverage effects and economic policy uncertainty with regimes can achieve higher forecasting accuracy compared to classical RV and GARCH models. Further results show that these factors within a regime-switching framework can significantly improve the forecasting performance of HAR-RV.

Ansari Nasab et al. (2019), in their study titled "Examining the Nonlinear Behavior of Exchange Rates in Iran: Evidence from Markov Switching Models," showed that exchange rates in Iran exhibit nonlinear and asymmetric behavior and that exchange rates behave differently in three different regimes. The behavior of exchange rates in these three regimes depends on the period in which they are situated, which can be important for economic policymaking in the exchange rate domain.

Mirzaei Ghazani (2018), in his study titled "Analysis of Realized Volatility Behavior in Tehran Stock Exchange Based on Heterogeneous Autoregressive Models," examined the behavior of realized volatility in relation to high-frequency data of the Tehran Stock Exchange index from from April28th, 2012 to August 8th, 2018. To achieve the research objective and analyze the behavior of realized volatility, three different types of heterogeneous autoregressive models were used: HAR-RVJ, HAR-RV, and HAR-RV-CJ. The results obtained from the three different models show that the estimated realized volatility in the market is well explained by traders who operate daily within the HAR-RVJ model framework. Additionally, based on the well-known heterogeneous market hypothesis, it can be concluded that across all studied time horizons, the values related to the four evaluation metrics (including RMSE, MAE, etc.) in the aforementioned model are lower than those in the HAR-RV-CJ and HAR-RV models.

Research Methodology

Given the unique geopolitical factors affecting the Tehran Stock Exchange (TSE), analyzing the impact of global market fluctuations on this market requires considering markets of countries that have significant economic, trade, political, or financial ties with Iran. Since the TSE is heavily influenced by commodity markets, four key commodity markets have been selected: the Saudi Stock Exchange (Tadawul), the Moscow Exchange (MOEX), the Dubai Financial Market (DFM), and the Turkish Stock Exchange (BIST).

To examine the proposed models and test the research hypotheses, the TEPIX index—representing the average market movements—has been judiciously selected from the calculated indices of the TSE as the sample. The performance of the proposed models in forecasting the realized volatility of the TSE's overall index will be evaluated. After aligning the days when all international stock markets in the sample had active trading, observations for each market were obtained. The data used spans two years (from 03/26/2022 to 08/21/2024) in 30-minute intervals during trading hours (9:00 AM to 12:30 PM). Additionally, to determine errors and assess model efficiency, data from 08/24/2024 to 03/18/2025 (a six-month period) in 15-minute intervals during the same trading hours were used.

For evaluating international stock indices of developed countries, data from Yahoo Finance was used, with daily values of selected countries' indices over the past two years assessed in alignment with Iranian trading times.

Following Tsay (2005), using lagged data is a standard method for non-synchronous markets (e.g., Iran and global markets). Similarly, Bekaert et al. (2000) employed time lags to study the influence of developed markets on emerging markets due to differing trading hours and days. Diebold et al. (2009) also examined cross-market spillover measurement methods, emphasizing the need for appropriate lags in markets with different schedules. Since the TSE operates on Saturdays and Sundays while global markets (typically) close on these days, global market fluctuations on Fridays are attributed to Iran's Saturday and Sunday trading sessions.

The Refinitiv Eikon software was used to automatically align international data with time lags, synchronizing it with Iranian data.

Research Steps for Data Alignment and Lag Usage

1. Standardizing Time Periods and Defining Common Intervals

First, the common time frame across all data was identified. To precisely align Iranian trading days with global markets (Turkey, Dubai, Russia, Saudi Arabia), tables were designed accounting for time lags (Lag=1,...,5, except in special cases). These tables outline working days for each market and their lagged impact (1–5 days) on the TSE.

For aligning foreign market timelines with Iran's, weighting logic was based on "time intervals" and "critical market points." Consistent with Andersen et al. (1998), market spillover effects from the latest trades decay exponentially, justifying higher weights for more recent days. Engle et al. (2002) noted that in markets with longer closures (e.g., Fridays and Saturdays), the last trading day (Thursday) most strongly impacts the first post-holiday trading day (Sunday). Kaminsky et al. (2003) similarly argued against equal weighting, as the last trading day disproportionately influences post-holiday sessions. Additionally, major economic news often breaks on weekends, prompting investors to adjust positions pre-holiday (French et al., 1980).

2. Aligning the TSE with the Turkish Stock Exchange (XU100.IS)

Given the TSE's closure on Thursdays and Fridays and BIST's inactivity on Saturdays and Sundays, the effects of Wednesday, Thursday, and Friday trading sessions in Turkey manifest on Iran's Saturday, Sunday, and Monday sessions. Weighted or cumulative averages of these three days' BIST data were used to measure their impact on Iran's trading days. The table below combines global data from these three days with weighted coefficients.

Table 1: Weighted average calculation of trading days in Turkey's market

lag	Description	Calsulation Function	Trading day in Iran
Friday (lag=1), Thursday (lag=2), Wednesday (lag=3)	The highest weight is given to Friday as it is the trading day closest to Saturday in Iran	Weighted Average = (0.15 × Wed return) + (0.35 × Thu return) + (0.5 × Fri return)	Saturday
Friday lag=2 ،Thursday lag=3	Focus on Thursday	Weighted Average = (0.3 × Thu return) + (0.7 × Fri return)	Sunday
Lag=3	Focus on Friday	Friday's return	Monday
Lag=1	-	Monday's return	Tuesday
Lag=1	-	Tuesday's return	Wednesday

In the Turkish stock market, no trading occurs on Saturdays and Sundays.
Source: Empirical study results

2. Comparison of the Iranian Stock Exchange with the Dubai Financial Market (DFM)

In the Dubai Financial Market, no trading takes place on Fridays, Saturdays, and Sundays. Based on the logic of exponential decay of influence, the table for calculating the weighted average of DFM trading days and the considered delays is as follows:

The weight assigned to Wednesday was decreased (0.2) because of its greater temporal distance [from Iran's active trading days]. The Dubai Financial Market (DFM) remains closed on weekends (Friday-Sunday) in accordance with regional conventions. Research shows that the last trading day (Thursday) has an impact even with long intervals and extended holidays (French & Roll, 1986).

Table 2: Calculation Table for Weighted Average of Trading Days in DFM (Dubai)

lag	Description	Calsulation Function	Trading day in Iran
Thursday lag=2 ، Wednesay lag=3	The highest weight is assigned to Thursday as the trading day closest to Iran's Saturday.	(0.65 × Thursday's return) + (0.35 × Wednesday's return)	Saturday
Thursday lag=3	Focus on Thursday	(1.0 × Thursday's return)	Sunday
Thursday lag=4 ، Wednesday lag=5	Focus on Thursday	(0.8 × Thursday's return) + (0.2 × Wednesday's return)	Monday
Lag=1	-	Monday's return	Tuesday
Lag=1	-	Tuesday's return	Wednesday

Source: Empirical study results

3. Alignment of the Iranian Stock Market with the Russian Stock Market (MOEX)

Based on the logic of exponential reduction of influence, the calculation table for the weighted average of trading days in the Russian stock market is similar to that of Turkey, and the considered delays are as follows:

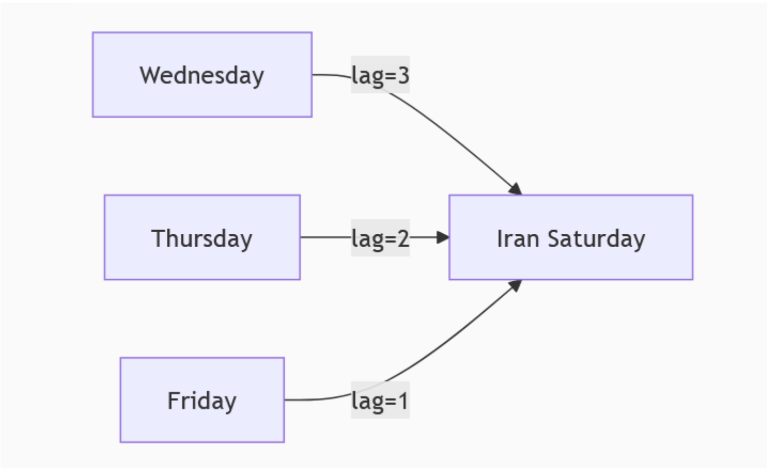
"Moscow Exchange (MOEX) operates Monday-Friday, remaining closed on weekends (Saturday-Sunday) in accordance with Russian financial regulations.

$$\text{Adjusted Return} = \sum_{i=\text{Wed}}^{\text{Fri}} w_i \times R_{t-\text{lag}i}$$

Table (3): Calculation of Weighted Average for Trading Days in the Russian Market

lag	Description	Calsulation Function	Trading day in Iran
Friday (lag=1), Thursday (lag=2), Wednesday (lag=3)	The highest weight is assigned to Friday as it is the trading day closest to Iran's Saturday.	(0.15 × Wednesday's return) + (0.35 × Thursday's return) + (0.5 × Friday's return)	Saturday
Friday lag=2 ،Thursday lag=3	Focus on Thursday	(0.3 × Thursday's return) + (0.7 × Friday's return)	Sunday
Lag=3	Focus on Friday	Friday's Return	Monday
Lag=1	-	Monday's Return	Tuesday
Lag=1	-	Tuesday's Return	Wednesday

Source: Empirical study data"



The lag days in the Russian market context

4. Alignment of the Iranian Stock Market with the Saudi Stock Market (Tadawul)

Since the Saudi stock market is closed on Fridays and Saturdays, the influence of trading days on Wednesday and Thursday will be reflected on Iran's trading days of Saturday and Monday. For this purpose, the weighted or cumulative average of the two mentioned trading days from the Russian stock market can be used to impact Iran's trading days of Saturday and Monday. The following table shows the weighting coefficients used for the trading days of the Saudi stock market:

Cross-Market Return=$w_{wed}R_{t-3} + w_{Tue}R_{t-2} + \varepsilon$

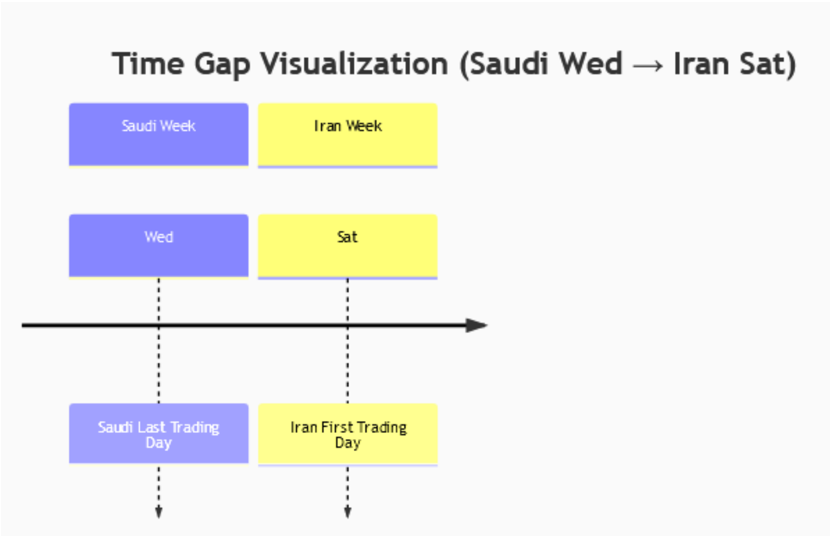
Where R_{t-n}= returns with n-day lag to Iran.

Saudi Arabia's Tadawul exchange operates Sunday-Thursday, remaining closed on weekends (Friday-Saturday) in alignment with Islamic financial conventions.

Table (4): Weighted Average Calculation for Trading Days in Saudi Arabia's Market

lag	Description	Calsulation Function	Trading day in Iran
Wednesday (lag=3), Thursday (lag=2)	The highest weight has been assigned to Thursday as the trading day.	× ۰,۳)Wednesday's return) + (0.7 × Thursday's return(	Saturday
Lag=2	Focus on Thursday	* ۰,۲)Wednesday's return) + (0.8 * Thursday's return(	Sunday
Lag=1	-	Sunday's return	Monday
Lag=1	-	Monday's return	Tuesday
Lag=1	-	Tuesday's return	Wednesday

Source: Empirical study data



Saudi-Iran Calendar Alignment

Alignment of Data Time Frames with Tehran

To precisely match the trading hours of the Tehran Stock Exchange (9:00 AM to 12:30 PM Iran time) with other exchanges in a 30-minute timeframe, the following table presents the time overlap in minutes:

It should be noted that the stock exchanges operating simultaneously with Iran (9:00 AM - 12:30 PM Tehran time) include Turkey, Dubai, Russia, and Saudi Arabia, which have complete overlap in trading hours.

The best time overlap is from 11:00 AM to 12:30 PM Tehran time, which is the peak period of simultaneous activity for regional stock exchanges.

Based on the tables aligning the time frames and trading days of the data, the initial number of observations is shown in the following table:

Table (5):Time Zone Alignment Table (Relative to Tehran Time)

Tehran Time (UTC+3:30)	Turkey Exchange (UTC+3)	Dubai Exchange (UTC+4)	Russia Exchange (UTC+3)	Saudi Exchange (UTC+3)
09:00 – 09:30	–	–	–	–
09:30 – 10:00	–	–	–	–
10:00 – 10:30	10:00 – 10:30 (Active)	10:00 – 10:30 (Active)	10:00 – 10:30 (Active)	10:00 – 10:30 (Active)
10:30 – 11:00	10:30 – 11:00 (Active)	10:30 – 11:00 (Active)	10:30 – 11:00 (Active)	10:30 – 11:00 (Active)
11:00 – 11:30	11:00 – 11:30 (Active)	11:00 – 11:30 (Active)	11:00 – 11:30 (Active)	11:00 – 11:30 (Active)
11:30 – 12:00	11:30 – 12:00 (Active)	11:30 – 12:00 (Active)	11:30 – 12:00 (Active)	11:30 – 12:00 (Active)
12:00 – 12:30	12:00 – 12:30 (Active)	12:00 – 12:30 (Active)	12:00 – 12:30 (Active)	12:00 – 12:30 (Active)

Source: Empirical study data

Table (6): of the best trading intervals based on volatility

Tehran Time Interval	Active Markets	Key Considerations
10:30 – 11:30	Simultaneous Start: Turkey / Dubai / Russia / Saudi Arabia	Peak volatility: Short-term trades (scalping) in communication-related symbols (e.g., refining group stocks)
11:30 – 12:30	All markets + Regional closing auction	Positioning for tomorrow: Order placement based on capital outflows from Dubai/Saudi exchanges

Country Data overview Table (7):

Country	Raw Data	Minute-Level Data	Test Data
Iran	573	4,011	600
Turkey	600	3,000	600
Dubai	482	2,410	600
Russia	612	3,060	600
Saudi Arabia	586	2,930	600

Due to the Dubai Financial Market's (DFM) shorter trading week, linear interpolation was employed to estimate values for non-trading days, ensuring compatibility with markets operating on different schedules.
Methodology: Applied to closing prices with time-weighted adjustments.
To examine the impact of fluctuations in international markets on the fluctuations of the Tehran Stock Exchange, the Heterogeneous Autoregressive Realized Volatility (HAR-RV) model has been used. Due to differences in the number of observations and trading days between the Tehran Stock Exchange and the studied international markets, data alignment was performed as follows:

1. Temporal Alignment of Data

First, common trading days between the Tehran Stock Exchange and each of the international markets were identified. For this purpose, the Temporal Intersection method was used, where only days with trading activity in all studied markets were retained, and the rest were removed. This process was implemented using Inner Join operations in the Python programming environment.

2. Preliminary Statistical Tests

After data alignment, the following tests were conducted to ensure the quality of the matched data:

a) Augmented Dickey-Fuller (ADF) Test
- To check the stationarity of the realized volatility time series.
- Where non-stationarity was detected, appropriate differencing was applied.

b) Correlation Test:
- Calculation of Pearson correlation coefficients between the volatility of the Tehran Stock Exchange and each of the international markets.
- Examination of the statistical significance of observed correlations.

c) Granger Causality Test:
- To investigate causal relationships between international market volatilities and the Tehran Stock Exchange.
- Determination of the direction of influence and optimal lag lengths.

3. Handling Missing Data
In limited cases where data for certain markets were missing on common trading days:
- Linear interpolation using weighted averages of preceding and succeeding days was applied.
- This method was only used for short gaps (maximum of 2 consecutive days).

4. Implementation of the HAR-RV Model
After ensuring data quality, the HAR-RV model was estimated separately for each international market. In this modeling:

Dependent variable: Realized volatility of the Tehran Stock Exchange on day t (RV_t)

Independent variables:
- Previous day's volatility of the Tehran Stock Exchange (RV_{t-1})
- Weekly volatility of the Tehran Stock Exchange ($RV_{t-5:t-1}$)
- Monthly volatility of the Tehran Stock Exchange ($RV_{t-22:t-1}$)
- Previous day's volatility of the relevant international market ($RV_{i,t-1}$)

HAR-RV Model
With the availability of high-frequency data, daily stock returns can be used to measure the daily realized variance proposed by Andersen and Bollerslev (1998). Statistically, realized volatility is obtained by aggregating the squared returns from high-frequency data observed at very short intervals during market trading hours, defined as follows:

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \quad (1)$$

in the above relation:

RV_t :Realized volatility on trading day t

$r_{t,j}$:Represents the j-th intraday [1]return on day t

$M = 1/\Delta$: Equals the daily frequency and sampling frequency

To predict the volatility of the Tehran Stock Exchange, we begin with the benchmark HAR-RV model originally proposed by Corsi (2009). This benchmark model has several desirable features. First, it appropriately describes some stylized facts in asset return volatility, such as long memory and multi-scale behavior. Additionally, the HAR-RV model is easy to implement as it only includes three predictors: daily, weekly, and monthly realized volatilities. The specification of the HAR-RV model can be written as follows:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + \varepsilon_t \quad (2)$$

$$RV_{t-h:t-1} = (1/h)(RV_{t-h} + \cdots + RV_{t-1}) \quad (3)$$

In the above relation:

RV_t: The realized volatility of Tehran Stock Exchange on trading day t

$RV_{t-5:t-1}$and $RV_{t-22:t-1}$ Represent the weekly and monthly realized volatilities, respectively.

Since an increasing number of studies (Lei et al., 2018; Peng et al., 2018; Liu et al., 2019) rely on global stock market volatilities to predict domestic stock market volatility, we also follow them to predict Tehran Stock Exchange volatility by separately adding (N) international market volatilities to the benchmark model. Consequently, we will have N extended models as follows:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + \beta_i RV_{i,t-1} + \varepsilon_t \quad (4)$$

In the above relation:

i represents global stock markets, and $RV_{i,t-1}$ is the realized volatility of the i-th international stock market on trading day t-1.

Markov Regime-Switching Model

To examine the role of regime switching in predicting Tehran Stock Exchange volatility in international markets, we use the Markov regime-switching model

[1] Intraday return is one of the two key components of a stock's daily return. It measures the return generated by a stock during regular trading hours based on its price change from the start to the end of a trading day.

proposed by Hamilton (1989) and develop other related models with regime changes. The extended benchmark HAR-RV model with regime switching can be expressed as follows:

$$RV_t = \beta_0 + \beta_{d,s_t} RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + \omega_t \qquad (5)$$

In the above relations:
s_t represents the regime at time t,
β_{d,s_t} is the regression slope of daily realized volatility in regime s_t with normal error $N(0,\sigma_{s_t}^2)$ and $\sigma_{s_t}^2$ is the evolved variance from regime s_t .
Following the studies of Raggi et al. (2012), Shi et al. (2015), Wang et al. (2017), and Duan et al. (2018), we consider two regimes. Specifically:
- Regime St = 0 represents the low-volatility regime (stable period)
- Regime St = 1 represents the high-volatility regime (unstable/turbulent period)
Thus, St follows a two-state Markov process with the following transition probability matrix:

$$p = \begin{pmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{pmatrix}$$

In the above relation $p_{kl} = p(S_{t+1} = l| S_t = k)$ is the transition probability from state k to state l, where $(k, l \in \{0,1\})$ and $p_{10} + p_{11} = 1$ and $p_{00} + p_{01} = 1$
To predict Iran's stock market volatility using global market volatilities, the regime-switching models are expressed as follows:

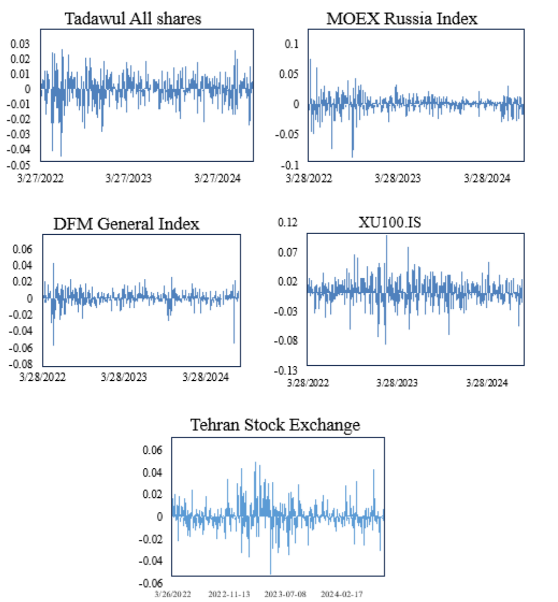
$$RV_t = \beta_0 + \beta_{d,s_t} RV_{t-1} + \beta_w RV_{t-5:t-1} + \beta_m RV_{t-22:t-1} + \beta_{i,s_t} RV_{i,t-1} + \omega_t \qquad (6)$$

In the above relation:
β_{i,s_t}: Regression slope of daily realized volatility from the i-th international stock market in state s_t
p_{00}: Probability of remaining in the low-volatility regime
p_{01}: Probability of transitioning from high-volatility to low-volatility regime
St = 0: Low-volatility regime state

Research Findings

In-Sample Estimation Results

Figure 1 displays the stock market returns, and Figure 2 illustrates the average realized volatility of the examined international markets and the Iranian stock market during the period from 2022-03-26 to 2024-08-21.



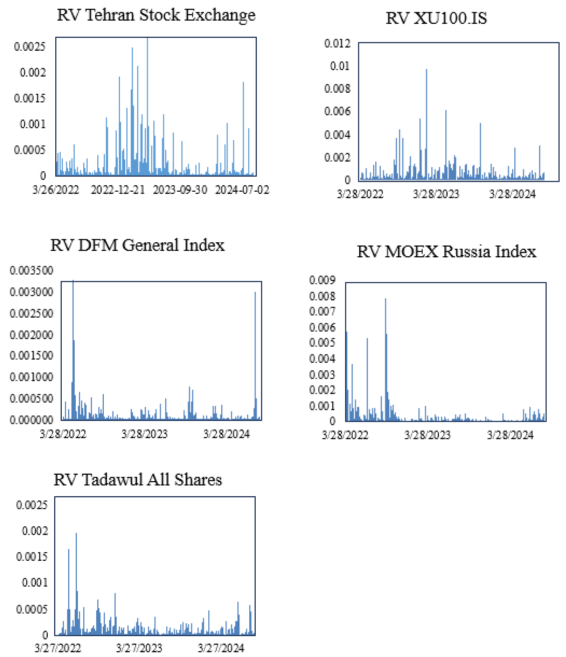


Table 8 presents the descriptive statistics of the return variable, while Table 9 shows the descriptive statistics of the realized volatility variable.

Table 8: Descriptive statistics of Return

Variable	Mean	Median	Std. Dev.	Skewness	Kurtosis	ADF
Tehran Stock Exchange	0.000715	-0.00010	0.01139	0.384128	6.45957	-10.40488***
XU100.IS	0.002735	0.002298	0.019615	-0.116684	5.599092	-23.78915***
DFM General Index	0.0004822	0.000770	0.008981	-1.003727	10.35144	-21.13059***
MOEX Russia Index	0.0002420	0.000895	0.013593	-0.719284	11.19391	-20.57278***
Tadawul All Shares	-6.93E-05	0.000406	0.008939	-0.535435	4.924627	-20.49018***

*MacKinnon (1996) one-sided p-values.

Table 9: Descriptive statistics of International Realized Variances

Variable	Mean	Median	Std. Dev.	Skewness	Kurtosis	Q(10)	ADF
Tehran Stock Exchange	0.00013	2.57E-05	0.000306	4.421341	26.78511	1987.420***	-18.24105394***
XU100.IS	0.00039	0.000127	0.000825	5.719476	48.34858	1175.893***	-12.55408695***
DFM General Index	8.05E-05	1.80E-05	0.000243	9.388008	110.9234	802.661***	-16.0355187***
MOEX Russia Index	0.000184	2.96E-05	0.000587	8.159707	84.68831	1722.308***	-9.34290329***
Tadawul All Shares	7.97E-05	2.76E-05	0.000158	5.892653	56.12090	1089.774***	-9.64705801***

According to Tables 8 and 9, the descriptive statistics of two key variables including daily returns and realized variance of Tehran Stock Exchange and other countries have been examined. The descriptive statistics results for Tehran Stock Exchange indicate that:

- The average daily return is close to zero with a relatively high standard deviation. This indicates significant fluctuations in market returns without a clear long-term trend. Such characteristics may suggest unpredictable market behavior and high investment risk.

- The average realized volatility is at a high level with a considerable standard deviation. This shows that Tehran Stock Exchange has faced structural volatility during the study period, with a high probability of existence of both high and low volatility regimes.
- The skewness and kurtosis of both variables deviate from normal distribution. Positive skewness in realized volatility indicates that most data are concentrated in lower volatility ranges, while high kurtosis suggests heavy tails and outliers in the distribution. These characteristics limit the use of normal-based models and emphasize the need for nonlinear and non-normal models such as Markov switching models.
- The ADF test shows that both realized volatility and return variables are stationary and suitable for time series modeling, and can therefore be used directly without further transformations.
- The Q(10) test also confirms the existence of autocorrelation in the data, indicating time dependency in market volatility behavior. This characteristic justifies the use of heterogeneous autoregressive (HAR) models.

The in-sample estimation results for all regime-switching models are reported in Table 10. This table presents the in-sample estimation results of regime-switching models. In this study, the basic HAR-RV model has been used as the main framework for predicting realized volatility (RV) of Iran's stock market. To develop this model, several international indices have been considered, each forming a separate extended model by adding the daily volatility of the relevant stock market to the basic model, where the name of each international stock market index represents the extended model.

In the current analysis, regression coefficients have been calculated separately for two volatility regimes:
The coefficients $\beta_{d,0}$ and $\beta_{d,1}$ represent the impact of daily volatility of Iran's stock market in low-volatility (St=0) and high-volatility (St=1) regimes, respectively.
The coefficients $\beta_{i,0}$ and $\beta_{i,1}$ show the effect of daily volatility from each international stock market in these two regimes.
t-statistics are presented in parentheses next to each coefficient. Transition probabilities are also calculated:
- p00: probability of remaining in low-volatility regime (transition from state 0 to state 0)
- p10: probability of switching from high-volatility to low-volatility regime (transition from state 1 to state 0)

The symbols ***, **, and * indicate significance at 99%, 95%, and 90% confidence levels (two-tailed), respectively.
For model quality assessment, the Akaike Information Criterion (AIC) has been calculated and reported.

The key finding is that both transition probabilities p00 and p10 are statistically significant at the 5% level. This evidence indicates that regime switching is essential for the in-sample predictability of realized volatility (RV) in Iran's stock market. Furthermore, the low-volatility regime (St=0) is more likely to transition from the low-volatility state to another state compared to the high-volatility regime. This conclusion is based on comparing the values of p00 (probability of remaining in the low-volatility regime) and p10 (probability of transitioning from high-volatility to low-volatility regime) in the results table below.

Table 10: In-sample estimation results for regime switching models

AIC	p10	p00	$\beta_{i,1}$	$\beta_{i,0}$	$\beta_{d,1}$	$\beta_{d,0}$	Benchmark switching models
2472.8	0.236*** (6.932)	0.921*** (8.112)	−0.934*** (4.215)	0.598*** (6.874)	0.981*** (9.102)	0.512*** (7.321)	Main model
2468.3	0.269*** (6.801)	0.908*** (7.998)	−0.918*** (4.087)	0.582*** (6.701)	0.964 *** (8.934)	0.498*** (7.045)	XU100 index (Istanbul)
2470.1	0.312*** (6.874)	0.915*** (8.045)	−0.926*** (4.176)	0.590*** (6.790)	0.997 *** (9.301)	0.505*** (7.210)	DFM General Index (Dubai)
2471.6	0.267*** (6.850)	0.913*** (8.021)	−0.920*** (4.132)	0.595*** (6.832)	0.972 *** (9.045)	0.509*** (7.278)	MOEX Russia Index
2473.4	0.340*** (6.890)	0.917*** (8.067)	−0.928*** (4.198)	-0.588*** (6.765)	0.988 *** (9.187)	0.503*** (7.189)	Tadawul All share Index (Saudi)

High p00 values across all models:
The p00 values in all rows of the table are close to 1 and statistically significant (0.921 for the baseline model, 0.908 for Istanbul stock exchange, and averaging above 0.92). This indicates that if the market is in a low-volatility regime, there is a very high probability it will remain in this regime. In other words, the probability of exiting the low-volatility regime is very low (approximately 3-8%).
Lower p10 values compared to p00:

In contrast, p10 (probability of transitioning from high-volatility to low-volatility regime) averages around 0.25-0.35 (e.g., 0.236 for the baseline model, 0.269 for Istanbul stock exchange). This means the market in a high-volatility regime has only a 25-35% probability of returning to a low-volatility regime. Therefore, the probability of persisting in the high-volatility regime (1-p10) is much higher (65-75%).

The significant difference between high p00 and low p10 demonstrates that the stability of the low-volatility regime is much greater than that of the high-volatility regime. This pattern is repeated across all extended models, even after adding international variables, indicating a structural characteristic of Iran's stock market. The findings suggest that low-volatility periods in Iran's market are longer and more stable, while high-volatility periods are typically shorter with a higher probability of exit. This may be due to regulatory policies of Iranian authorities or investor behavior in this market that tends to return to stability more quickly.

On the other hand, the regression coefficients of Iran's daily realized volatility (RV) are positive and statistically significant at the 1% level in both low- and high-volatility regimes. As expected, the regression coefficients of Iran's daily RV in the high-volatility regime are significantly larger than those in the low-volatility regime across all regime-switching models. This result is entirely logical because the expected realized volatility during high-volatility periods should be greater than during low-volatility periods. The high-volatility regime is typically associated with market shocks, macroeconomic uncertainties, or sharp price changes. Under such conditions, Iran's RV response to explanatory variables (daily RV) naturally intensifies. In contrast, in the low-volatility regime, the market is more stable, and thus smaller, more moderate coefficients are observed.

Additionally, the regression coefficients for daily realized volatility of most international stock markets are negative in the high-volatility regime, and many are statistically significant. This suggests that daily volatility in international markets likely moderates overestimations of Iran's realized volatility during high-volatility periods. As evident, the Saudi market shows a negative relationship, which may stem from geopolitical rivalries.

In the Markov switching model, the coefficients for daily realized volatility of international markets (βi_0) in Iran's high-volatility regime are predominantly negative and statistically significant. This finding indicates that global market volatilities play a moderating role in estimating Iran's market volatility. In other words, during periods when Iran's market experiences excitement or severe fluctuations, the model prevents exaggeration of domestic volatility by accounting for international market behavior.

In these circumstances, global market volatilities act as a "realistic signal" that helps the model determine whether Iran's volatility stems from fundamental global factors or merely domestic speculation. Therefore, the negative βi_0 coefficients reflect the fact that external volatilities can reduce the intensity of estimated volatilities in Iran's market during high-volatility regimes and help moderate them.

Out-of-Sample Estimation Results
Out-of-sample forecasting results are more important than in-sample estimation results. Therefore, we have generated out-of-sample forecasts for Iran's market volatility based on a rolling estimation window. Specifically, the entire research sample is divided into two parts:
- In-sample section: Consisting of the first 600 observations
- Out-of-sample section: Consisting of the remaining 200 observations

Each time an out-of-sample forecast was made, the estimation window moved forward—adding one new observation and removing the first observation from the previous window.
The main objective of this research** is to examine the role of Markov switching in predicting Tehran Stock Exchange volatility using international market volatilities based on heterogeneous autoregressive (HAR) models. For this purpose, the forecasting performance of non-regime-switching models must be

directly compared with regime-switching models. Therefore, the one-to-one comparison method is highly suitable for this study. Accordingly, we have used the DM test (Diebold & Mariano, 1995), a widely used pairwise comparison method employed in many similar studies (e.g., Patton & Sheppard, 2015; Buncic & Gisler, 2017; Gong & Lin, 2018; Zhang, Ma, & Wei, 2019). Before conducting the DM test, two loss functions are introduced:

1. Heteroskedasticity-adjusted Mean Squared Error (HMSE)
2. Heteroskedasticity-adjusted Mean Absolute Error (HMAE)

Statistically, these loss functions are defined as follows:

- HMSE is a measure that evaluates the difference between the market's actual variance (RV□) and the model's forecast (RV̂□), accounting for heteroskedasticity.
- HMAE is a similar measure but based on absolute error.

$$HMSE: \left(1 - \frac{\widehat{RV_t}}{RV_t}\right)^2 \qquad (7)$$

$$HMAE: \left|1 - \frac{\widehat{RV_t}}{RV_t}\right| \qquad (8)$$

$$DM\ statistic = \frac{\bar{d}}{\sqrt{var(d)}}, \qquad (9)$$

$$\bar{d} = \frac{1}{q}\sum_{t=m+1}^{m+q} d_t, d_t = L(\widehat{RV_{nrs}}, RV_t) - L(\widehat{RV_{rs,t}}, RV_t)$$
(10)

$Var(d)$ is theVariance of $\begin{matrix} m+q \\ t=m+1 \end{matrix}\{d_t\}$ and $L(\widehat{RV_{nrs}}, RV_t)$ and $L(\widehat{RV_{rs,t}}, RV_t)$ are the loss functions of the non-regime-switching and regime-switching forecasting models, respectively, and m and q are the lengths of the in-sample and out-of-sample estimation periods, respectively.

According to the definition of the DM statistic in equation (10), it can be inferred that:

- A positive DM statistic value indicates better performance of the regime-switching model compared to the baseline non-regime-switching model, since the forecasting error of the regime-switching model under a given loss function is smaller.
- Conversely, a negative DM statistic value indicates the opposite result.

Furthermore, since the distribution of the DM statistic is asymptotically standard normal, the significance level of the DM test can be easily calculated.

Table 11: Out-of-sample forecasting performance based on the Diebold-Mariano test

Forecasting models	HMSE		HMAE	
	DM statistic	*p*-value	DM statistic	*p*-value
Main model	8.502***	0.000	13.508***	0.000
XU100 index (Istanbul)	8.721***	0.000	13.606***	0.000
DFM General Index (Dubai)	4.031***	0.000	9.145***	0.000
MOEX Russia Index	7.326***	0.000	11.208***	0.000
Tadawul All share Index (Saudi)	2.625***	0.000	12.067***	0.000

Table 11 presents the DM test results comparing regime-switching models with conventional (non-regime-switching) models in the out-of-sample period. A notable finding in this table is that all DM statistic values are significantly positive. This result indicates that regime-switching models demonstrate substantially better performance in forecasting Iran's stock market volatility compared to traditional models without this feature. Indeed, the Markov regime-switching approach proves to be a highly effective solution for enhancing the prediction accuracy of Iran's stock market volatility within the framework of international conditions.

Conclusion

This study set out to investigate the predictive power of Markov regime-switching models, integrated with international volatility spillovers within a Heterogeneous Autoregressive (HAR) framework, for forecasting volatility in the Tehran Stock Exchange (TSE). The empirical findings, derived from a meticulous data alignment process accounting for the TSE's unique trading calendar and geopolitical context, offer compelling evidence for the superiority of this modeling approach.

The core conclusion is that volatility dynamics in the TSE are fundamentally characterized by distinct and persistent regimes. The analysis reveals a stark asymmetry in regime persistence: the low-volatility regime exhibits remarkable stability, with a high probability (approximately 92%) of persisting. In contrast, the high-volatility regime is significantly less stable, with a much lower probability (25-35%) of remaining and a high likelihood of transitioning back to calm. This pattern suggests that while periods of stability in the TSE can be prolonged, turbulent phases are inherently transient, potentially due to swift regulatory interventions or the conservative, risk-averse behavior of domestic investors.

A pivotal finding of this research is the nuanced role of international market fluctuations. Contrary to simply amplifying domestic volatility, volatilities from key regional markets (Saudi Arabia, Russia, Dubai, and Turkey) act as a moderating force, particularly during high-volatility regimes. The significantly negative coefficients for international variables in these turbulent periods indicate that global volatility signals help correct for potential overestimations of domestic risk, providing a more realistic assessment. The negative correlation observed with markets like Saudi Arabia further underscores the profound influence of geopolitical factors, where regional rivalries may lead to divergent market behaviors.

The out-of-sample forecasting results robustly confirm the empirical value of incorporating regime-switching behavior. The Diebold-Mariano test statistics, based on heteroskedasticity-adjusted loss functions, were unanimously positive and significant. This demonstrates that Markov-switching HAR models consistently and significantly outperform their single-regime counterparts in predicting TSE volatility. This establishes the Markov-switching HAR framework as a powerful and effective tool for volatility forecasting in complex, emerging markets like Iran's.

Practical Implications:

- **For Policymakers and Regulators:** The findings provide a quantifiable framework for risk management. Recognizing the high persistence of low-volatility regimes, authorities can focus on policies that reinforce macroeconomic stability—such as controlling inflation and enhancing policy transparency—to extend these calm periods. During high-volatility episodes, understanding the moderating effect of international spillovers allows for more calibrated interventions, preventing knee-jerk reactions and avoiding the amplification of panic.

For Investors and Portfolio Managers: The model offers a superior tool for optimizing trading and hedging strategies. Investors can tailor their strategies based on the identified regime: adopting more aggressive positions during stable, low-volatility periods and shifting to defensive assets or using international diversification (especially with negatively correlated markets) as a hedge during turbulent times. The time-alignment tables can directly inform intraday trading decisions, identifying windows of peak simultaneous activity and volatility.

References

Abtahi, S. Y., & Nikfetrat, H. (2012). Identifying regime switching of stock market returns in Iran. Economic Modeling, 20, 41–56.

Andersen & Bollerslev (1998). "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts". International Economic Review.

ansarinasab, M. and m, Z. (2019). Investigation of Nonlinear Exchange Rate Behavior in Iran: Evidence from Markov Switching Model. Journal of Iranian Economic Issues, 6(1), 21-40. doi: 10.30465/ce.2019.4916

Arab Mazar, M. and Ghasemi, M. (2010). Pricing Initial Public Offerings: combining Artificial Neural Networks and Genetic Algorithm. Accounting and Auditing Review, 16(4)

Bekaert, G., & Harvey, C. R. (2000). "Foreign Speculators and Emerging Equity Markets". Journal of Finance

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics, 7, 174–196.

Diebold, F. X., & Yilmaz, K. (2009). "Measuring Financial Asset Return and Volatility Spillovers". *Journal of Econometrics

Duan, Y., Chen, W., Zeng, Q., & Liu, Z. (2018). Leverage effect, economic policy uncertainty and realized volatility with regime switching. Physica A: Statistical Mechanics and its Applications, 493, 148–154.

Duan., Yinying, Chen., Wang, Zeng., Qing, Liu., Zhicao (2018), Leverage effect, economic policy uncertainty and realized volatility with regime switching, Physica A: Statistical

Mechanics and its Applications, volume 493, page 148-154.

Engle, R. F. (2002). "Dynamic Conditional Correlation". Journal of Business & Economic Statistics.

Fallahpour, S., & Motaharinia, V. (2016). Including jump components in modeling and forecasting realized volatility: Evidence from Tehran Stock Exchange. Financial Engineering and Portfolio Management, 7(2), 171–190.

French (1980). "Stock Returns and the Weekend Effect". Journal of Financial Economics.

Gibbons, M. R., & Hess, P. (1981). "Day of the Week Effects and Asset Returns". Journal of Business.

Hansen, P. R., & Lunde, A. (2005). "A Forecast Comparison of Volatility Models". Journal of Applied Econometrics.

Kaminsky, G., & Schmukler, S. (2003). "Short-Run Pain, Long-Run Gain: The Effects of Financial Liberalization". NBER Working Paper.

Leung, Mark T., Daouk, H., & Chen, An-Sing (2000). Forecasting stock indices: A comparison of classification and level estimation models. International Journal of Forecasting, 16, 173-190.

Liu, J., Ma, F., & Zhang, Y. (2019). Forecasting the Chinese stock volatility across global stock markets. Physica A: Statistical Mechanics and its Applications, 525,466–477.

Liu, Jia, Chen, Zhiping (2018), Time consistent multi-period robust risk measures and portfolio selection models with regime-switching, European Journal of Operational Research, Volume 268, Issue 1, Pages 373-385.

Lo, A. W., & MacKinlay, A. C. (1988). Stock market prices do not follow random walks: Evidence from a simple specification test, Review of Financial Studies, 1, 41-66.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. Journal of Applied Econometrics, 11(6), 601-618.

Mirzaee, M. (2018). Analysis of Realized Volatility in Tehran Stock Exchange using Heterogeneous Autoregressive Models Approach. Financial Research Journal, 20(3), 365-388. doi: 10.22059/frj.2018.255568.1006657

Moradi, M., Sedoughi Yazdi, H., & Abdollahian, J. (2015). A New Engineering Approach to Forecast Tehran Securities Exchange Indices Volatility. Journal of Accounting Advances, Shiraz University, 7(2).

Raggi, D., & Bordignon, S. (2012). Long memory and nonlinearities in realized volatility: A Markov switching approach. Computational Statistics & Data Analysis, 56,3730–3742.

Shi, Y., & Ho, K.-Y. (2015). Long memory and regime switching: A simulation study on the Markov regime-switching ARFIMA model. Journal of Banking & Finance, 61, S189–S204.

Swan. Mitra (2020), Downside risk measurement in regime switching stochastic volatility, Journal of Computational and Applied Mathematics (2020), doi: https://doi.org/10.1016/j.cam.2020.112845

Tsay, R. S. (2005). *Analysis of Financial Time Series*. Wiley.

Yu, Lean, Wang, Shouyang, & Lai, K. K. (2009). Intelligent computational methods for financial engineering. Journal of Applied Mathematics and Decision Sciences, Article ID 394731, doi:10.1155/2009/394731, 2 pages