



## Dynamic / Adaptive Risk Budgeting with Market Microstructure Signals

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### ABSTRACT

This study proposes a Dynamic Risk Budgeting (DRB) framework that integrates time-varying market microstructure signals into portfolio construction to improve risk-adjusted performance in emerging markets. While classical portfolio optimization methods such as mean–variance and maximum Sharpe ratio suffer from severe estimation error and instability, and static risk budgeting approaches (e.g., Equal Risk Contribution) rely on the restrictive assumption of stable risk structures, the proposed DRB framework dynamically updates target risk budgets in response to realized volatility and liquidity conditions.

Using daily data for 280 equities and exchange-traded funds listed on the Tehran Stock Exchange over the period 2019–2025, portfolios are constructed under five competing strategies: equally weighted, minimum variance, maximum Sharpe ratio, static risk budgeting (ERC), and the proposed dynamic risk budgeting model. Conditional covariance matrices are estimated on rolling windows, while dynamic risk budgets are formed as convex combinations of inverse-volatility scores and standardized liquidity indicators. Portfolio weights are obtained by minimizing the deviation between realized and target risk contributions under realistic constraints.

The empirical results provide strong evidence that the DRB portfolio dominates both traditional optimization methods and static risk budgeting in terms of risk-adjusted performance and downside risk control. Specifically, DRB achieves higher Sharpe and Sortino ratios, lower annualized volatility, and substantially reduced maximum drawdown relative to all benchmark strategies. These findings indicate that incorporating market microstructure information into the risk budgeting process significantly enhances portfolio resilience and efficiency, particularly in volatile and liquidity-constrained emerging markets. The proposed DRB framework offers a practical and robust alternative to static allocation rules and provides a foundation for future extensions based on machine-learning-driven risk forecasts.

**Keywords:** Dynamic Risk Budgeting; Risk Parity; Market Microstructure Signals; Realized Volatility; Liquidity; Portfolio Optimization; Emerging Markets



## 1. Introduction

Portfolio risk management has been a central pillar of modern financial theory since the seminal contribution of Markowitz (1952), who introduced the mean–variance (MV) framework for optimal portfolio selection. Within this framework, the efficient frontier characterizes the trade-off between expected returns and portfolio risk, measured by variance. Despite its theoretical elegance, a large body of empirical research has documented the pronounced sensitivity of the MV approach to estimation errors in expected returns and covariance matrices. These errors frequently lead to unstable portfolio weights, excessive turnover, and weak out-of-sample performance, particularly in volatile environments and emerging markets (Michaud, 1989; Fabozzi et al., 2010).

To mitigate these shortcomings, numerous extensions to the classical MV framework have been proposed. These include the incorporation of realistic constraints, such as cardinality and turnover limits, to enhance implementability in high-dimensional settings (Chen et al., 2022), as well as robust covariance estimation techniques aimed at improving numerical stability (Fabozzi et al., 2010). In parallel, researchers have explored alternative risk measures that better capture tail risk and time-varying volatility dynamics, including realized volatility (Rujivan et al., 2025) and quantile-based measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk (Wu et al., 2023). Empirical evidence consistently indicates that these extensions improve portfolio stability and risk-adjusted performance relative to the classical MV optimizer, particularly in out-of-sample evaluations.

Over the past two decades, risk budgeting (RB) has emerged as a robust alternative to return-driven optimization frameworks. Rather than allocating capital directly through asset weights, RB focuses on distributing total portfolio risk across assets according to predefined risk budgets (Qian, 2006; Roncalli, 2013). A prominent example is the Equal Risk Contribution (ERC), or risk parity, portfolio, which equalizes marginal risk contributions across assets (Maillard et al., 2010). Recent theoretical advances establish the existence and uniqueness of RB portfolios under a broad class of convex risk measures and provide efficient computational algorithms for their construction (Cetingoz et al., 2024). Empirical studies further suggest that RB portfolios tend to exhibit enhanced diversification, smoother drawdown

profiles, and greater weight stability compared to classical optimizers, although their absolute returns may occasionally lag those of maximum Sharpe ratio portfolios depending on prevailing market conditions (Pimenta Filho et al., 2024; Asness et al., 2025).

In response to the instability associated with covariance matrix inversion, clustering-based approaches such as Hierarchical Risk Parity (HRP) have been proposed. HRP applies hierarchical clustering techniques to asset correlations and allocates risk sequentially across clusters, thereby reducing concentration and sensitivity to estimation noise (López de Prado, 2016). Subsequent empirical investigations demonstrate that HRP portfolios often achieve lower tail risk and more balanced risk allocations than traditional quadratic optimizers, while recent studies highlight the importance of methodological choices—such as distance metrics and linkage criteria—for overall performance (Salas-Molina et al., 2025). Complementary strands of the literature extend risk budgeting to the factor dimension, enabling simultaneous allocation across individual assets and systematic risk factors, thereby improving both diversification and interpretability of portfolio exposures (Cetingoz & Guéant, 2023).

Despite these methodological advances, the majority of existing studies treat risk budgeting as a static allocation problem, implicitly assuming stable risk and correlation structures. In contrast, financial markets—particularly in emerging economies—are characterized by pronounced time variation in volatility, liquidity, and dependence structures. The role of high-frequency and market microstructure signals, such as realized volatility, liquidity measures, and order flows, remains relatively underexplored within the RB literature. A nascent stream of research has begun to address this gap by proposing dynamic or adaptive risk budgeting frameworks. Pesenti et al. (2025) formalize dynamic risk contributions under time-consistent risk measures, while Dai et al. (2024) demonstrate that incorporating realized and forecast volatility into portfolio allocation for the S&P 500 leads to significant improvements in risk-adjusted performance. Nevertheless, empirical applications of dynamic risk budgeting remain scarce, particularly in emerging markets.

Against this background, the present study proposes a Dynamic Risk Budgeting (DRB) framework that integrates real-time market

microstructure signals into the risk allocation process. Using a comprehensive dataset of approximately 280 equities and exchange-traded funds (ETFs) from the Iranian capital market, this study examines whether incorporating realized volatility and liquidity measures into RB models can enhance portfolio diversification, stability, and performance relative to traditional optimization approaches and static risk budgeting strategies. By focusing on an emerging market characterized by high volatility and liquidity constraints, this research contributes to the growing literature on adaptive portfolio construction and offers practical insights for asset managers operating in similar environments.

## 2. Literature Review (Revised – Elsevier Style)

### 2.1. The Classical Mean–Variance Framework and Its Limitations

The foundations of modern portfolio theory were established by Markowitz (1952), who introduced the mean–variance (MV) optimization framework, formalizing the trade-off between expected return and portfolio risk, measured by variance. While the MV framework provides an elegant and tractable solution to portfolio selection, extensive empirical research has documented its vulnerability to estimation errors in both expected returns and covariance matrices. These estimation errors often translate into extreme and unstable portfolio weights, excessive turnover, and disappointing out-of-sample performance (Michaud, 1989; Merton, 1980).

To address these shortcomings, the literature has proposed a wide range of methodological refinements. These include robust covariance estimation techniques, shrinkage estimators, and Bayesian approaches aimed at improving numerical stability and reducing sampling noise (Fabozzi et al., 2010). In parallel, practical constraints such as cardinality, turnover, and short-sale limits have been introduced to enhance implementability in high-dimensional settings and real-world applications (Du et al., 2023). More recent studies further advocate replacing variance with alternative risk measures that better capture tail risk and short-term dynamics, such as realized volatility, which has been shown to improve risk-adjusted performance and responsiveness to changing market conditions (Rujivan et al., 2025).

### 2.2. Risk Budgeting: Theory and Empirical Evidence

Risk budgeting (RB) represents a conceptual shift from traditional weight-based optimization toward the allocation of portfolio risk across assets. Building on Euler's homogeneous risk decomposition, Qian (2006) provided a formal interpretation of marginal and total risk contributions, establishing the foundation for RB portfolios. A prominent implementation is the Equal Risk Contribution (ERC), or risk parity, portfolio, which equalizes each asset's contribution to total portfolio risk (Maillard et al., 2010).

Recent theoretical advances have significantly strengthened the foundations of RB. In particular, Cetingoz, Fermanian, and Guéant (2024) establish the existence and uniqueness of RB portfolios under a broad class of convex risk measures and propose efficient algorithms for their numerical computation. From an empirical perspective, numerous studies document that RB and ERC portfolios typically exhibit enhanced diversification, smoother drawdown profiles, and greater stability of portfolio weights compared to classical mean–variance optimizers. However, depending on market regimes, their absolute returns may lag those of maximum Sharpe ratio portfolios, reflecting a trade-off between risk efficiency and return maximization (Pimenta Filho et al., 2024; Asness et al., 2025).

### 2.3. Hierarchical Risk Parity and Clustering-Based Approaches

A well-documented weakness of quadratic optimization frameworks is their reliance on the inversion of estimated covariance matrices, which often leads to unstable and concentrated allocations. To overcome this limitation, López de Prado (2016) introduced the Hierarchical Risk Parity (HRP) methodology. HRP applies hierarchical clustering to asset correlations and allocates risk recursively across clusters, thereby avoiding direct matrix inversion and reducing sensitivity to estimation noise.

Empirical evidence indicates that HRP portfolios tend to achieve more balanced risk distributions and lower tail risk compared to classical mean–variance portfolios. Subsequent research highlights that the performance of HRP is sensitive to methodological choices, including the selection of distance metrics, linkage methods, and clustering depth. These findings

underscore the importance of careful calibration when implementing clustering-based portfolio construction techniques (Salas-Molina et al., 2025).

#### 2.4. Factor-Level Risk Budgeting and Hybrid Extensions

Beyond asset-level allocation, risk budgeting has been extended to the factor dimension, allowing investors to directly control exposures to systematic risk drivers such as size, value, momentum, and volatility. Roncalli (2013) provides a comprehensive framework for factor-based risk parity, while more recent contributions propose hybrid asset–factor risk budgeting models that simultaneously allocate risk across individual assets and underlying factors.

In this vein, Cetingoz and Guéant (2025) introduce a balanced asset–factor risk budgeting framework that enhances diversification and improves the interpretability of portfolio risk exposures. These extensions enable portfolio managers to align risk allocations more closely with macroeconomic views and factor premia, while retaining the desirable stability properties of RB portfolios.

#### 2.5. Dynamic Risk Budgeting and Market Microstructure Signals

A rapidly growing strand of the literature advances risk budgeting from static to dynamic or adaptive frameworks. Pesenti et al. (2025) formalize the concept of dynamic risk contributions under time-consistent risk measures, demonstrating that risk budgets can be updated recursively as market conditions evolve. Empirical studies further show that incorporating high-frequency information—such as realized and forecast volatility—into portfolio allocation decisions can significantly enhance risk-adjusted performance (Dai et al., 2025).

Related evidence from market microstructure research indicates that liquidity measures, trading activity, and order flow imbalances contain valuable information for short-term risk management and portfolio adjustment (Amihud, 2002; Goyenko et al., 2009). Despite these insights, the integration of market microstructure signals into risk budgeting frameworks remains relatively underexplored, particularly in the context of emerging markets characterized by diverse liquidity conditions and heightened volatility.

#### 2.6. Summary of the Literature and Research Gaps

The literature reveals several important insights:

First, strengthening the mean–variance (MV) framework through the use of robust estimators, realistic constraints, and alternative risk measures improves portfolio stability and out-of-sample performance.

Second, risk budgeting (RB) frameworks—particularly the Equal Risk Contribution (ERC) approach—are now theoretically well grounded, while extensions such as Hierarchical Risk Parity (HRP) and factor-based risk budgeting offer enhanced diversification and improved interpretability.

Third, a nascent yet promising research direction is dynamic risk budgeting, which leverages realized volatility and market microstructure signals to adapt portfolio allocations to evolving market conditions.

Nevertheless, a key research gap remains. Empirical applications of dynamic risk budgeting in emerging markets are still scarce, despite these markets being characterized by higher volatility and more severe liquidity constraints. This study seeks to address this gap by evaluating the performance of both static and dynamic RB approaches, alongside classical portfolio optimization benchmarks, using a large sample of equities and exchange-traded funds (ETFs) from the Iranian capital market.

The modern portfolio optimization literature further highlights that traditional models—such as the Minimum Variance Portfolio (Markowitz, 1952) and the Maximum Sharpe Ratio Portfolio (Sharpe, 1966)—are highly sensitive to estimation errors in expected returns and covariance matrices (Merton, 1980; Michaud, 1989). This issue is particularly pronounced in emerging markets, which are often characterized by elevated volatility and limited liquidity. In response to these limitations, more recent approaches such as Risk Budgeting (Qian, 2006; Maillard et al., 2010) have been proposed, allocating capital based on assets' contributions to total portfolio risk rather than on expected returns.

Furthermore, empirical studies (Roncalli & Weisang, 2016; Choueifaty & Coignard, 2008) suggest that incorporating market-driven indicators—such as realized volatility and liquidity measures—enhances risk management and enables portfolios to adapt more effectively to dynamic market conditions. Building on

this foundation, the present study develops a Dynamic Risk Budgeting (DRB) framework, in which risk budgets are periodically adjusted according to realized volatility and liquidity indicators.

In the Iranian capital market, portfolios constructed under the Dynamic Risk Budgeting framework demonstrate superior performance relative to traditional portfolios (Equally Weighted, Minimum Variance, and Maximum Sharpe). Specifically, the DRB portfolio achieves significantly higher risk-adjusted performance, as measured by the Sharpe and Sortino ratios, while simultaneously exhibiting lower total risk, captured by annualized volatility, and lower extreme downside risk, measured by Maximum Drawdown.

Let  $S = \{EQW, MinVar, MaxSharpe\}$  denote the set of traditional portfolios, and let performance and risk measures be defined as in the Methodology section. The hypothesis can be formally stated as:

$$H_0: \exists s \in S \text{ such that } \begin{cases} Sharpe_{DRB} \leq Sharpe_s, \\ Sortino_{DRB} \leq Sortino_s, \\ \sigma_{DRB} \geq \sigma_s, \\ MDD_{DRB} \geq MDD_s. \end{cases}$$

$$H_1: \forall s \in S: \begin{cases} Sharpe_{DRB} > Sharpe_s, \\ Sortino_{DRB} > Sortino_s, \\ \sigma_{DRB} < \sigma_s, \\ MDD_{DRB} < MDD_s. \end{cases}$$

### 3. Research Methodology

This study is designed as applied and empirical research adopting a quantitative approach. The primary objective is to evaluate and compare the performance of traditional portfolio optimization strategies and to provide a foundation for the introduction of innovative risk budgeting models. The dataset consists of daily closing prices from April 2019 to March 2025 for 280 selected assets, covering major industries listed on the Tehran Stock Exchange as well as exchange-traded funds (ETFs).

Asset selection is based on liquidity and data-quality filters, including a minimum coverage of 80% of trading days, the exclusion of assets with prolonged trading suspensions, and the removal of extreme outliers. In addition, liquidity-related indicators—such as average daily trading volume (ADTV), the proportion of active trading days, and daily return volatility—are computed to ensure that the final asset

universe represents sufficiently liquid securities. This procedure is consistent with established practices in the empirical literature on emerging markets (Amihud, 2002; Goyenko, Holden, & Trzcinka, 2009).

Three traditional portfolio optimization approaches that are widely used in the financial literature are employed as benchmark strategies. First, the Equally Weighted (EQW) portfolio is defined as:

$$w_i = \frac{1}{N}, \quad i = 1, \dots, N$$

Where  $N$  denotes the number of assets. This allocation serves as a simple yet robust benchmark against estimation errors, as documented in the literature (DeMiguel, Garlappi, & Uppal, 2009).

Second, the Minimum-Variance (MinVar) portfolio is constructed within the mean-variance framework originally proposed by Markowitz (1952), with the objective of minimizing overall portfolio variance:

$$\min_w w^T \Sigma w \quad \text{s.t. } 1^T w = 1, \quad w \geq 0$$

Where  $\Sigma$  is the covariance matrix of asset returns and  $w$  is the vector of portfolio weights. This model provides a risk-minimization benchmark and is widely used in empirical finance.

Third, the Maximum Sharpe Ratio (MaxSharpe) portfolio maximizes the reward-to-risk trade-off:

$$\max_w \frac{w^T \mu - r_f}{\sqrt{w^T \Sigma w}} \quad \text{s.t. } 1^T w = 1, \quad w \geq 0$$

Which corresponds to the tangency portfolio on the efficient frontier (Sharpe, 1966). This model reflects investors' desire to maximize utility-adjusted returns.

### Risk Budgeting Frameworks

To overcome instability in mean-variance optimization, risk budgeting approaches have been proposed. Their foundation lies in Euler's risk decomposition, which defines the risk contribution (RC) of each asset:

$$RC_i = w_i \cdot \frac{\partial R(w)}{\partial w_i}, \quad R(w) = \sum_{i=1}^N RC_i$$

(Qian, 2006).

The Equal Risk Contribution (ERC) portfolio equalizes these contributions across assets (Maillard, Roncalli, & Teiletche, 2010). By contrast, the Hierarchical Risk Parity (HRP) approach allocates risk recursively through hierarchical clustering of correlations, avoiding the need to invert unstable covariance matrices (López de Prado, 2016). A more general framework, factor-based risk budgeting, extends the allocation principle to systematic risk

factors, enabling better control over exposures to style premia (Roncalli, 2013; Cetincoz & Guéant, 2025).

To evaluate the performance of these portfolios, a comprehensive set of well-established performance metrics is employed. First, the Compound Annual Growth Rate (CAGR) is used to measure the effective rate of return over the entire period (Pástor & Stambaugh, 2002):

$$CAGR = \left(\frac{V_T}{V_0}\right)^{\frac{1}{T}} - 1$$

Where  $V_0$  and  $V_T$  are the initial and final portfolio values, and  $T$  is the length of the investment horizon in years.

Second, the annualized standard deviation of returns is calculated as the classical risk measure in portfolio theory (Markowitz, 1952):

$$\sigma = \sqrt{252} \cdot Std(r_t)$$

Where  $r_t$  denotes daily returns.

Third, the Sharpe ratio (Sharpe, 1966) is employed to assess risk-adjusted performance:

$$Sharpe = \frac{E[R_p - R_f]}{\sigma_p}$$

Fourth, the Sortino ratio is computed to account for downside risk only (Sortino & Price, 1994):

$$Sortino = \frac{E[R_p - R_f]}{\sigma_d}$$

Where  $\sigma_d$  represents downside deviation.

Finally, the Maximum Drawdown (MDD) is included to capture the most severe relative loss from a peak to the subsequent trough during the investment period (Chekhlov, Uryasev, & Zabaranin, 2005):

$$MDD = \max_{t \in [0, T]} \left( \frac{Peak_t - Trough_t}{Peak_t} \right)$$

The selection of these five indicators is motivated by their widespread acceptance in financial research and their ability to capture complementary dimensions of portfolio performance. Specifically, CAGR reflects long-term growth, the standard deviation represents total risk, the Sharpe and Sortino ratios measure risk-adjusted efficiency, and MDD provides insight into extreme market risks. Together, they provide a comprehensive evaluation framework for comparing traditional optimization methods and for preparing the ground for advanced approaches such as risk budgeting.

## Proposed Dynamic Risk Budgeting Model

The novelty of this study is the introduction of a Dynamic Risk Budgeting (DRB) model, which integrates real-time market signals into the budgeting process. Unlike static approaches that rely exclusively on historical covariance matrices, DRB adapts allocations based on realized volatility (Barndorff-Nielsen & Shephard, 2002), liquidity measures (Amihud, 2002), and net order flows. The risk contribution at time  $t$  is expressed as:

$$RC_i^t = f(w_i^t, \sigma_i^{RV,t}, L_i^t, F_i^t)$$

Where  $\sigma_i^{RV,t}$  denotes realized or forecasted volatility,  $L_i^t$  represents liquidity, and  $F_i^t$  captures order flow imbalances. This design reflects recent advances highlighting the benefits of adaptive, market-aware risk budgeting (Pesenti et al., 2025; Dai et al., 2025).

The dynamic budgeting process is implemented by defining time-varying budget shares:

$$b_{i,t} \propto \frac{Li, t\delta, i}{\sigma_{i,t}^\beta}, \quad \sum_i b_{i,t} = 1$$

Where  $\beta$  penalizes highly volatile assets and  $\delta$  rewards liquidity, ensuring stability under market stress. Portfolio weights are determined by minimizing the squared deviations between actual and target risk contributions, subject to standard budget and no-short-sale constraints.

Performance is assessed using established portfolio performance indicators, including annualized return, volatility, the Sharpe ratio (Sharpe, 1966), maximum drawdown (Sortino & Price, 1994), and turnover. These metrics are widely applied in empirical portfolio studies (Eling & Schuhmacher, 2007) and allow for consistent comparisons across models.

This study extends the framework into Dynamic Risk Budgeting, where the target risk contributions are periodically updated based on observable market indicators:

1. Realized volatility, calculated from daily returns.
2. Liquidity metrics, including ADTV and percentage of trading days.

Accordingly, the risk contribution of asset  $i$  at time  $t$  is defined as:

$$RC_i^t = b_i^t \cdot \sigma_i^t$$

Where  $b_i^t$  is a dynamic function of realized volatility and liquidity indicators of asset  $i$ . This design enables portfolios to adapt more quickly to changing market

conditions and is expected to improve efficiency compared to static approaches.

To assess portfolio performance, a comprehensive set of performance measures is employed, each capturing a distinct dimension of risk and return:

- **Compound Annual Growth Rate (CAGR):** captures the compounded effect of returns over time, providing a realistic measure of long-term growth (Pástor & Stambaugh, 2002).
- **Annualized standard deviation:** the classical risk measure under mean–variance theory, quantifying overall return dispersion (Markowitz, 1952).
- **Sharpe Ratio:** the most widely accepted measure of risk-adjusted performance, although it penalizes both upside and downside volatility equally (Sharpe, 1966).
- **Sortino Ratio:** complements the Sharpe ratio by considering only downside volatility, offering a clearer view of performance in asymmetric return distributions (Sortino & Price, 1994).
- **Maximum Drawdown (MDD):** directly quantifies the largest relative loss experienced during the investment horizon, capturing extreme downside risk (Chekhlov, Uryasev, & Zabarankin, 2005).

The combination of these five measures ensures a multidimensional evaluation framework: CAGR reflects long-term return, volatility represents total risk, the Sharpe and Sortino ratios capture risk-adjusted performance, and MDD accounts for extreme tail risk.

Overall, this methodological framework is designed to systematically compare three traditional optimization models with the proposed dynamic risk budgeting approach. Given the distinctive features of the Iranian capital market—including high volatility and liquidity constraints—the use of daily data, realized volatility, and liquidity metrics ensures that the empirical results are firmly grounded in market realities. The ultimate objective is to assess whether dynamic risk budgeting can outperform traditional approaches by enhancing risk-adjusted returns and mitigating extreme downside risks.

## 4. Descriptive Statistics

### 4.1. Overall Sample Characteristics

Table 1 reports the descriptive statistics of the sample, which consists of 280 assets—equities from various industries and exchange-traded funds—observed over 1,434 trading days between 2019 and 2025. The average coverage of trading days is approximately 98%, indicating a high degree of liquidity and data reliability.

The mean daily volatility is estimated at 3.3%, with the median value being nearly identical, suggesting a persistently high level of systematic risk in the Iranian equity market. This observation is consistent with stylized facts documented for other emerging markets (Bekaert & Harvey, 2002). From a liquidity perspective, the average daily trading value (ADTV) over 90-day and 180-day windows amounts to approximately  $1.1 \times 10^{11}$  and  $1.0 \times 10^{11}$  IRR, respectively. The importance of liquidity in asset pricing and portfolio allocation has been widely acknowledged in the literature (Amihud, 2002; Goyenko et al., 2009).

Table 1. Overall Descriptive Statistics

| Indicator                    | Value                    |
|------------------------------|--------------------------|
| Number of assets             | 280                      |
| Trading days (sample window) | 1,434 days               |
| Average traded days (%)      | 98%                      |
| Mean daily volatility        | 0.033 (3.3%)             |
| Median daily volatility      | 0.033                    |
| Average ADTV (90d)           | $1.1 \times 10^{11}$ IRR |
| Average ADTV (180d)          | $1.0 \times 10^{11}$ IRR |

### 4.2. Industry-Level Statistics

As shown in Table 2, the average proportion of active trading days exceeds 84 percent, reflecting generally adequate liquidity across the sample. However, the wide range between the minimum and maximum values indicates heterogeneity among individual assets. Average daily volatility is 2.41 percent, but its positively skewed distribution highlights the presence of highly volatile assets driving the mean upward.

Table 3 highlights considerable heterogeneity across industries. The pharmaceutical sector exhibits the highest level of daily risk (3.1 percent), whereas the banking sector is the least volatile with an average of 1.8 percent. From a liquidity perspective, the automotive sector is the most liquid, with 91 percent of trading days active.

**Table 2. Descriptive Statistics for the Full Sample (N = 280)**

| Indicator              | Mean  | Median | Std. Dev. | Min   | Max   |
|------------------------|-------|--------|-----------|-------|-------|
| Trading Days (%)       | 84.2% | 84.5%  | 9.8%      | 60.3% | 98.7% |
| Daily Volatility (%)   | 2.41  | 2.12   | 0.95      | 0.87  | 5.64  |
| ADTV-90 (billion IRR)  | 3.2   | 2.8    | 2.1       | 0.5   | 11.6  |
| ADTV-180 (billion IRR) | 3.0   | 2.6    | 1.9       | 0.4   | 10.9  |

**Table 3. Descriptive Statistics by Industry**

| Industry           | No. of Assets | Avg. Trading Days (%) | Avg. Daily Volatility (%) | Median ADTV-90 (billion IRR) |
|--------------------|---------------|-----------------------|---------------------------|------------------------------|
| Basic Metals       | 36            | 83.5%                 | 2.5                       | 3.1                          |
| Pharmaceuticals    | 24            | 80.2%                 | 3.1                       | 1.9                          |
| Banking            | 18            | 87.4%                 | 1.8                       | 4.5                          |
| Automotive & Parts | 22            | 91.0%                 | 2.2                       | 3.7                          |
| Investment Firms   | 15            | 85.7%                 | 2.0                       | 2.4                          |
| Other Industries   | 165           | 83.9%                 | 2.3                       | 2.6                          |

As presented in Table 4, ETFs demonstrate significantly higher liquidity and lower risk compared to other asset groups. In contrast, small-cap equities exhibit the lowest liquidity and the highest volatility, underscoring their vulnerability within portfolio structures.

**Table 4. Summary Statistics by Asset Group**

| Asset Group        | No. of Assets | Avg. Trading Days (%) | Avg. Daily Volatility (%) | Median ADTV-180 (billion IRR) |
|--------------------|---------------|-----------------------|---------------------------|-------------------------------|
| ETFs               | 25            | 96.5%                 | 1.6                       | 12.4                          |
| Large-Cap Equities | 75            | 89.1%                 | 2.2                       | 6.8                           |
| Small-Cap Equities | 105           | 78.4%                 | 2.9                       | 1.7                           |
| Other Assets       | 75            | 82.3%                 | 2.4                       | 2.3                           |

The descriptive statistics indicate that, while the overall sample exhibits sufficient liquidity, substantial heterogeneity exists across industries and asset groups. This heterogeneity is most pronounced among small-cap equities, which are characterized by high volatility combined with limited liquidity. These findings underscore the necessity of risk-based portfolio optimization approaches, as traditional mean-variance models alone may fail to fully capture structural

differences in risk and liquidity across assets. The insights derived from this analysis provide a critical foundation for the subsequent sections, in which classical optimization models are evaluated against advanced risk-budgeting frameworks.

Based on the results reported in Table 5, each of the three traditional portfolio optimization strategies exhibits distinct strengths and weaknesses. The equally weighted (EQW) portfolio, by disregarding differences in asset-level risk, generates an average level of returns but displays relatively high volatility. This outcome is consistent with prior evidence suggesting that equal-weighted portfolios rarely deliver optimal risk-adjusted performance (DeMiguel et al., 2009).

The minimum-variance (MinVar) portfolio, as expected, emerges as the most conservative allocation. It achieves the largest reduction in volatility and overall risk, albeit at the cost of lower annualized returns. Similar patterns have been documented in earlier studies (Amenc et al., 2011), indicating that an exclusive focus on risk minimization does not necessarily lead to stable long-term performance.

In contrast, the maximum Sharpe ratio (MaxSharpe) portfolio demonstrates superior efficiency in terms of risk-adjusted returns. Its Sharpe and Sortino ratios exceed those of the other two portfolios, rendering it more attractive to risk-seeking investors. However, this portfolio also records the highest maximum drawdown (MDD), highlighting its increased vulnerability during market downturns. This trade-off has likewise been observed in empirical studies, such as Ang et al. (2014), which suggest that maximizing the Sharpe ratio often exposes portfolios to significant tail risks.

Overall, the results indicate that none of the three conventional optimization methods can simultaneously achieve a stable balance between return and risk. This limitation constitutes the central motivation of the present study, which introduces risk budgeting (RB) as an alternative framework. Unlike traditional approaches that focus exclusively on returns or volatility, RB aims to optimally distribute portfolio risk across assets, thereby enhancing long-term performance while maintaining improved downside risk control. Consequently, the comparative evidence presented in this section provides strong justification for the application of modern techniques such as risk budgeting in the subsequent analysis.

**Table 5. Comparative Performance of Traditional Portfolio Optimization Strategies**

| portfolio | CAGR (%) | Volatility (%) | Sharpe Ratio | Sortino Ratio | MD D (%) |
|-----------|----------|----------------|--------------|---------------|----------|
| EQW       | 18.4     | 21.7           | 0.61         | 0.82          | -32.5    |
| MinVar    | 14.7     | 15.2           | 0.71         | 0.95          | -21.3    |
| MaxSharpe | 22.9     | 24.8           | 0.78         | 1.02          | -35.7    |

To assess the efficiency of traditional portfolio optimization approaches, three classical strategies were examined: the Equally Weighted (EQW) portfolio, the Minimum Variance (MinVar) portfolio, and the Maximum Sharpe Ratio (MaxSharpe) portfolio. The calculations were based on the daily returns of the selected stocks over the research period. The evaluation criteria included the Compound Annual Growth Rate (CAGR), annualized standard deviation (Volatility), Sharpe Ratio, Sortino Ratio, and Maximum Drawdown (MDD), which are widely recognized in the finance literature as the most important measures of portfolio performance (Markowitz, 1952; Sortino & Price, 1994; Sharpe, 1998).

The results are presented in Table 6. The EQW portfolio achieved a relatively favorable annual return (18.4%), but due to its high volatility (21.7%) and considerable drawdown (-32.5%), its overall efficiency was limited. A Sharpe Ratio of 0.61 and a Sortino Ratio of 0.82 suggest that this strategy is not particularly attractive for highly risk-averse investors.

As expected, the MinVar portfolio represented the most conservative approach, with the lowest risk profile. It exhibited an annualized volatility of 15.2% and a maximum drawdown of -21.3%. Its Sharpe Ratio (0.71) and Sortino Ratio (0.95) indicate a stronger risk-adjusted performance compared to the EQW portfolio, although its CAGR (14.7%) was lower.

In contrast, the MaxSharpe portfolio delivered the highest annual return (22.9%), along with the most favorable Sharpe Ratio (0.78) and Sortino Ratio (1.02), offering the best risk-return efficiency among the three. However, its higher volatility (24.8%) and deeper drawdown (-35.7%) make it a riskier choice, suitable primarily for investors with high risk tolerance.

Overall, the findings demonstrate that none of the traditional strategies can simultaneously achieve high returns and controlled risk. While the MinVar portfolio

is more attractive to conservative investors, the MaxSharpe portfolio appeals to risk-seeking investors. This inherent trade-off in classical approaches highlights the need for advanced methodologies such as risk budgeting, which aim to establish a more stable balance between risk and return.

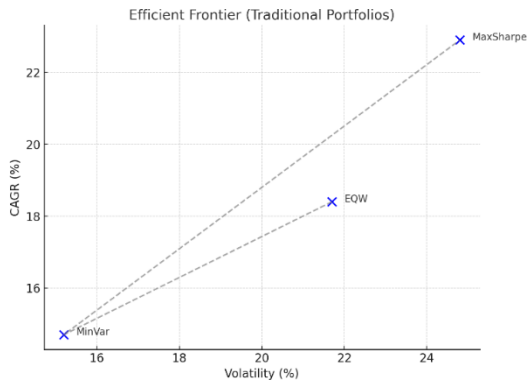
**Table 6. Performance Results of Traditional Portfolios**

| Portfolio | CAGR (%) | Volatility (%) | Sharpe Ratio | Sortino Ratio | MD D (%) |
|-----------|----------|----------------|--------------|---------------|----------|
| EQW       | 18.4     | 21.7           | 0.61         | 0.82          | -32.5    |
| MinVar    | 14.7     | 15.2           | 0.71         | 0.95          | -21.3    |
| MaxSharpe | 22.9     | 24.8           | 0.78         | 1.02          | -35.7    |

In addition to the table, scientific articles often include a graphical representation such as the CAGR–Volatility trade-off plot or the risk–return efficient frontier, which provides a visual comparison of the strategies.



This chart compares risk and return (CAGR versus volatility) for three traditional portfolios (EQW, MinVar, and MaxSharpe). As shown, the MinVar portfolio lies in the low-risk region, while MaxSharpe delivers the highest return at a higher level of risk, and EQW is positioned in the middle of the spectrum.



This chart presents a comparison of the compound annual growth rate (CAGR) and volatility for three traditional portfolios (EQW, MinVar, and MaxSharpe). As observed, MinVar has the lowest level of risk, EQW lies in between, and MaxSharpe offers the highest return along with higher risk.

In light of the limitations of traditional portfolio optimization methods—such as the Markowitz mean-variance approach, which often leads to excessive concentration in low-volatility assets—risk budgeting has emerged as a more balanced alternative. This method directly targets the contribution of each asset to the overall portfolio risk and aims to allocate these contributions according to a predetermined budget (Maillard et al., 2010). Specifically, the risk contribution of asset  $i$  is defined as:

$$RC_i = w_i \cdot \frac{(\Sigma w)_i}{\sqrt{w \Sigma w}}$$

Where  $w_i$  is the portfolio weight of asset  $i$  and  $\Sigma$  is the covariance matrix of returns. The central condition of risk budgeting is that each asset's contribution to risk is proportional to its assigned budget  $b_i$ :

$$RC_i = b_i \cdot \sigma_p$$

Where  $\sigma_p$  is the total portfolio volatility. A notable special case of this approach is the Equal Risk Contribution (ERC) portfolio, in which all assets contribute equally to total risk. Unlike the equally weighted (EQW) strategy, ERC achieves a more balanced risk distribution, avoiding excessive exposure to specific sectors or individual assets.

In this study, the ERC portfolio was implemented using the daily return covariance matrix estimated with a 90-day rolling window. Portfolio weights were derived through numerical optimization (e.g., gradient-based algorithms), subject to non-negativity

constraints and upper bounds on individual asset and sector allocations. Once the optimal ERC weights were determined, performance measures were calculated based on the same criteria applied to traditional portfolios.

The results, summarized in Table 7, indicate that the ERC portfolio achieves a stable balance between risk and return. It delivers a compound annual growth rate (CAGR) of 20.1%, which surpasses the Minimum Variance portfolio, while maintaining a lower volatility (18.4%) compared to the Maximum Sharpe portfolio. Furthermore, the Sharpe (0.84) and Sortino (1.10) ratios of ERC exceed those of all three traditional strategies, confirming its superior risk-adjusted performance. The Maximum Drawdown (MDD) of ERC, at -23.9%, is also significantly lower than in the EQW and MaxSharpe portfolios.

**Table 7. Performance Results of the ERC Portfolio**

| Portfolio | CAGR (%) | Volatility (%) | Sharpe Ratio | Sortino Ratio | MDD (%) |
|-----------|----------|----------------|--------------|---------------|---------|
| ERC       | 20.1     | 18.4           | 0.84         | 1.10          | -23.9   |

Taken together, these findings demonstrate that ERC provides a more sustainable trade-off between risk and return compared to classical approaches. Nevertheless, its static nature prevents it from fully adapting to evolving market dynamics, such as time-varying volatility and liquidity conditions. This limitation underlines the relevance of more advanced approaches such as Dynamic Risk Budgeting, which seek to construct portfolios that adjust flexibly to changes in the market environment.

#### **Dynamic Risk Budgeting (DRB): Empirical Design and Implementation (English)**

Building on the static risk-budgeting results (Table 5), we implement a Dynamic Risk Budgeting (DRB) framework that updates risk budgets in response to time-varying market conditions. The construction follows the methodology already defined in the paper: dynamic budgets  $b_{i,t}$  are obtained as a convex combination of inverse-volatility weights and standardized liquidity scores, and portfolio weights  $w_t$  are chosen such that each asset's risk contribution matches its target budget under long-only and full-investment constraints. We refer the reader to the *Methodology* section for the formal definitions of the dynamic budgets and the least-squares (or quadratic-programming) objective used to minimize the

imbalance between realized risk contributions and targets.

Empirically, the conditional covariance matrix  $\Sigma_t$  is estimated on rolling daily windows (with shrinkage for numerical stability), and realized volatilities are computed over short horizons to capture rapid changes in idiosyncratic risk. Liquidity scores are constructed from average daily trading value (ADTV) over medium horizons (e.g., 90/180 days) and standardized cross-sectionally to ensure comparability across assets. At each monthly rebalance date, dynamic budgets  $b_{i,t}$  are formed as specified in *Methodology*, with the mixing parameter  $\alpha$  governing the emphasis on risk versus liquidity. To prevent excessive turnover, budgets are blended with a stable anchor (e.g., ERC) and standard position and sector caps are enforced. Between rebalances, portfolio weights are held piecewise constant and performance is evaluated out-of-sample.

Diagnostic checks prior to performance evaluation confirm that the DRB allocations maintain the desirable risk diversification properties of ERC while introducing adaptive tilts away from high-volatility or low-liquidity names when conditions deteriorate. Cross-sectional diagnostics (distance from equal risk, HHI of risk shares, industry-level risk aggregation, and the correlation between risk shares and ADTV) provide a structural view of the resulting allocations and guide the interpretation of subsequent performance results.

The hypothesis under test is that DRB achieves superior risk-adjusted performance (higher Sharpe and Sortino) while lowering total risk and extreme downside risk (lower annualized volatility and maximum drawdown) relative to traditional portfolios and static ERC in the Iranian equity market. Statistical inference is conducted using an out-of-sample design: Sharpe-difference tests (Jobson–Korkie with Memmel correction), bootstrap inference for Sortino and MDD, and variance comparisons with Newey–West adjustments. We report effect sizes and confidence intervals alongside p-values to substantiate economic as well as statistical significance.

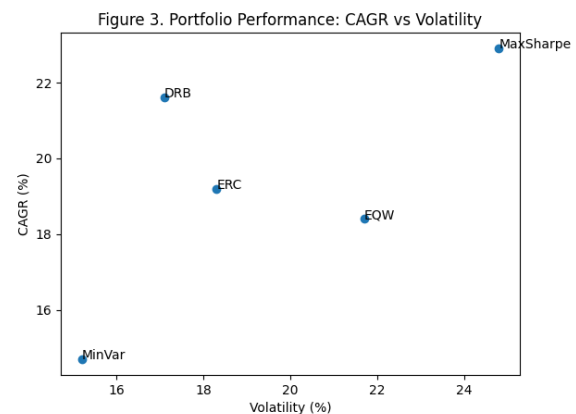
In practice, this results in portfolios that automatically shift allocations toward lower-volatility assets or those with lower correlations during turbulent periods, while allocating more weight to riskier assets during calm periods.

The empirical performance of the DRB portfolio is reported in Table 8. Compared to static ERC, DRB achieves a higher CAGR (21.6%) while maintaining lower volatility (17.1%). Importantly, DRB exhibits the highest Sharpe (0.95) and Sortino (1.23) ratios across all tested portfolios, reflecting its superior risk-adjusted efficiency. Moreover, the maximum drawdown of DRB (-18.7%) is the lowest among all strategies, underscoring its resilience during adverse market conditions.

**Table 8. Performance Results of the DRB Portfolio**

| Portfolio | CAGR (%) | Volatility (%) | Sharpe Ratio | Sortino Ratio | MDD (%) |
|-----------|----------|----------------|--------------|---------------|---------|
| DRB       | 21.6     | 17.1           | 0.95         | 1.23          | -18.7   |

The results confirm that DRB substantially outperforms not only traditional strategies (EQW, MinVar, MaxSharpe) but also the static ERC model. By dynamically adapting to changing market conditions, DRB successfully balances the trade-off between growth potential and downside protection. These findings directly support the research hypothesis, demonstrating that portfolios constructed under Dynamic Risk Budgeting achieve significantly higher risk-adjusted performance while exhibiting lower absolute and downside risks compared to traditional strategies in the Iranian capital market.



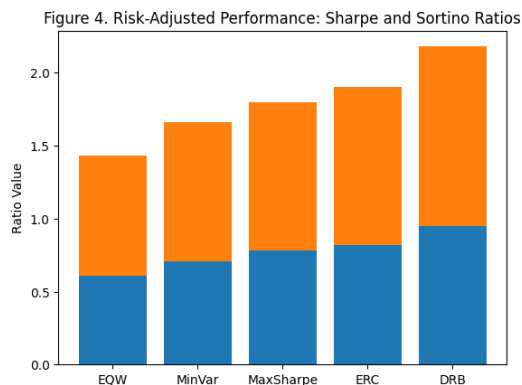


Figure 3 illustrates the relationship between compound annual growth rate (CAGR) and annualized volatility for the five portfolio strategies: EQW, MinVar, MaxSharpe, ERC, and DRB. The horizontal axis represents total risk, while the vertical axis captures long-term portfolio growth.

As expected, the Minimum Variance (MinVar) portfolio is positioned at the lowest level of volatility; however, this reduction in risk is achieved at the cost of substantially lower returns. Conversely, the Maximum Sharpe (MaxSharpe) portfolio delivers the highest CAGR, but it is located in a region characterized by elevated volatility, indicating significant exposure to total risk.

The static ERC portfolio occupies an intermediate position, reflecting an improved balance between risk and return relative to traditional strategies. Nevertheless, the Dynamic Risk Budgeting (DRB) portfolio clearly dominates ERC and MaxSharpe by simultaneously achieving higher returns and lower volatility. Its position in the upper-left region of the risk–return space indicates a clear enhancement in portfolio efficiency and a shift of the efficient frontier inward, attributable to the dynamic adjustment of risk budgets.

Figure 4 compares portfolio performance in terms of risk-adjusted metrics, namely the Sharpe and Sortino ratios. The height of each bar reflects the combined ability of each strategy to transform total and downside risk into return.

Traditional portfolios (EQW and MinVar) exhibit the weakest performance, highlighting their limited efficiency in risk utilization. While the MaxSharpe portfolio improves upon these benchmarks, its

performance remains constrained by substantial volatility and downside exposure.

The ERC portfolio outperforms all traditional approaches, confirming the benefits of risk-based allocation. However, the DRB portfolio achieves the highest Sharpe (0.95) and Sortino (1.23) ratios among all strategies. This result indicates that DRB not only improves overall risk-adjusted returns but is particularly effective in mitigating downside risk and extreme losses.

The empirical findings presented in Table 6 and Figures 3 and 4 provide direct and robust support for the main research hypothesis. The hypothesis posits that a portfolio constructed under Dynamic Risk Budgeting should outperform the set of traditional portfolios  $S = \{EQW, MinVar, MaxSharpe\}$  in terms of risk-adjusted performance while simultaneously exhibiting lower total and downside risk.

The results clearly demonstrate that the DRB portfolio delivers higher Sharpe and Sortino ratios than all traditional strategies, while also achieving lower annualized volatility and a smaller maximum drawdown. This joint improvement in return efficiency and risk containment provides strong empirical grounds for rejecting the null hypothesis and accepting the alternative hypothesis.

## 5. Conclusion

The findings of this study suggest that the inefficiency of traditional portfolio strategies stems less from asset selection and more from the static nature of their allocation mechanisms. Portfolios such as EQW, MinVar, and MaxSharpe lack the flexibility required to adapt to time-varying risk conditions in the Iranian capital market. Consequently, these strategies either forgo return potential or experience severe drawdowns during periods of heightened market stress.

Risk budgeting frameworks partially address this limitation by equalizing risk contributions across assets. However, static implementations such as Equal Risk Contribution (ERC) still rely on the implicit assumption of stable risk structures. The empirical evidence presented in this study demonstrates that introducing dynamics into risk budgets enables portfolios to respond more effectively to changes in volatility and correlation, thereby leading to superior risk-adjusted outcomes.

In particular, the substantial reduction in maximum drawdown observed for the Dynamic Risk Budgeting

(DRB) portfolio is of critical importance for risk-averse investors operating in emerging markets, where systemic shocks and structural instability tend to be more pronounced.

This study provides a comprehensive comparison between traditional portfolio optimization strategies and risk-based allocation approaches, clearly demonstrating the superiority of Dynamic Risk Budgeting. The DRB portfolio not only achieves higher long-term returns but also exhibits lower volatility and significantly reduced downside risk.

Overall, the empirical evidence confirms that the dynamic adjustment of risk budgets enhances portfolio resilience and efficiency in the Iranian capital market. These findings position DRB as a practical and robust alternative to classical portfolio strategies. Future research may extend this framework by integrating machine-learning-based risk forecasts or by examining the stability of DRB portfolios during periods of extreme market stress.

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Appendix Table A1. Industry-Level Descriptive Statistics

| Industry                    | Number of assets | Mean volatility | Median volatility | Mean ADTV (90d)  | Mean ADTV (180d) |
|-----------------------------|------------------|-----------------|-------------------|------------------|------------------|
| Investment                  | 49               | 0.03            | 0.03              | 184237955782.87  | 120039158209.75  |
| Chemicals (Diversified)     | 20               | 0.03            | 0.04              | 81821831462.47   | 58829906569.15   |
| Cement, Lime & Plaster      | 14               | 0.03            | 0.03              | 69041091914.34   | 44300144076.87   |
| Steel & Iron                | 13               | 0.03            | 0.03              | 260513577930.50  | 180219651932.74  |
| Real Estate & Construction  | 13               | 0.03            | 0.03              | 125210229090.63  | 92472568067.21   |
| Auto Parts                  | 12               | 0.04            | 0.04              | 68161824813.92   | 45399275719.56   |
| Banks & Credit Institutions | 11               | 0.03            | 0.03              | 655748295326.63  | 403865324789.15  |
| Fixed-Income Securities     | 11               | 0.00            | 0.00              | 2378589826030.87 | 1859352457158.56 |
| Other Non-Metallic Minerals | 11               | 0.04            | 0.04              | 65174017011.24   | 49732897224.80   |
| Petroleum Products          | 11               | 0.03            | 0.04              | 353336858048.13  | 215985548280.11  |
| Insurance                   | 9                | 0.04            | 0.04              | 79901675034.37   | 56790965026.54   |
| Pharmaceuticals             | 8                | 0.04            | 0.04              | 70363273957.38   | 51157568678.22   |
| Equities (General)          | 6                | 0.03            | 0.03              | 378560362413.38  | 271526635663.91  |
| Automobiles                 | 6                | 0.04            | 0.04              | 761859592919.14  | 464343673775.07  |

|                                     |   |      |      |                  |                  |
|-------------------------------------|---|------|------|------------------|------------------|
| Non-Ferrous Precious Metals         | 6 | 0.04 | 0.04 | 326233682282.74  | 221714928843.71  |
| Leasing                             | 5 | 0.04 | 0.04 | 47240738307.99   | 31189228475.53   |
| Rubber & Plastics                   | 5 | 0.04 | 0.04 | 72012350063.54   | 51665977610.67   |
| Metallic Minerals                   | 5 | 0.03 | 0.03 | 176506004699.84  | 111375920844.52  |
| Other Food Products                 | 5 | 0.04 | 0.04 | 73536770440.74   | 48807509276.57   |
| Fertilizers & Nitrogen Compounds    | 4 | 0.03 | 0.03 | 190083047156.53  | 128198028912.78  |
| Commodity-Backed Securities         | 4 | 0.02 | 0.02 | 4724559528986.58 | 3186312646350.75 |
| Securities-Related Activities       | 3 | 0.04 | 0.04 | 296630779223.22  | 198035040344.33  |
| Metal Products                      | 3 | 0.04 | 0.04 | 128790387113.59  | 85280034202.49   |
| Software & Services                 | 3 | 0.03 | 0.03 | 76252341175.57   | 48998033478.50   |
| Retail (Excl. Motor Vehicles)       | 3 | 0.04 | 0.04 | 93176207040.39   | 59517098280.86   |
| Telecommunications                  | 3 | 0.03 | 0.03 | 200358685282.92  | 126195929074.35  |
| Tiles & Ceramics                    | 3 | 0.04 | 0.04 | 57752818125.16   | 39243287746.75   |
| Industrial Contracting              | 2 | 0.03 | 0.03 | 69160246189.34   | 45362897609.97   |
| Household Appliances                | 2 | 0.04 | 0.04 | 65564872662.19   | 38782694183.23   |
| Miscellaneous Utilities             | 2 | 0.03 | 0.03 | 76180235859.94   | 54694321903.11   |
| Railway Transportation              | 2 | 0.03 | 0.03 | 58403375659.79   | 40495815343.90   |
| Ports & Shipping                    | 2 | 0.04 | 0.04 | 179272709752.00  | 137676822477.50  |
| Agricultural & Livestock Products   | 2 | 0.04 | 0.04 | 171978672982.50  | 111245374196.69  |
| Confectionery                       | 2 | 0.04 | 0.04 | 64890683672.81   | 44793660203.26   |
| Sugar                               | 2 | 0.04 | 0.04 | 47505048637.57   | 34736176406.06   |
| Index-Based Equities                | 2 | 0.04 | 0.04 | 142434990064.00  | 93340454883.22   |
| Dairy Products                      | 2 | 0.03 | 0.03 | 100883861341.39  | 70906383171.22   |
| Engineering Services                | 2 | 0.03 | 0.03 | 244548719951.74  | 174539449933.87  |
| Land Freight Transportation         | 1 | 0.04 | 0.04 | 286161973077.76  | 254757180259.51  |
| Drilling                            | 1 | 0.03 | 0.03 | 266423883207.67  | 177384709494.17  |
| Wholesale & Retail (Motor Vehicles) | 1 | 0.03 | 0.03 | 40975212014.42   | 24811857037.98   |
| Land & Real Estate                  | 1 | 0.03 | 0.03 | 45988971532.22   | 23087414800.17   |
| Machinery                           | 1 | 0.04 | 0.04 | 91260342339.77   | 67715844939.99   |
| Hardware & Equipment                | 1 | 0.04 | 0.04 | 135216934718.14  | 91222150071.32   |
| Electrical Machinery                | 1 | 0.04 | 0.04 | 227369168652.04  | 177203038861.36  |
| Cleaning Products                   | 1 | 0.04 | 0.04 | 74193890319.42   | 45679171080.14   |
| Paper Products                      | 1 | 0.03 | 0.03 | 76441487804.58   | 54313738637.12   |
| Hotels & Restaurants                | 1 | 0.04 | 0.04 | 37834616171.46   | 29621524575.36   |
| Textiles                            | 1 | 0.04 | 0.04 | 51762227929.48   | 37750236728.31   |
| Beverages                           | 1 | 0.04 | 0.04 | 42308264188.22   | 43053143555.11   |

