Modifying the black-Scholes model to valuate preemption right

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ABSTRACT

In this paper, we try and valuate preemption rights by modifying the Black-Scholes model, which is widely used to valuate options and other derivatives. Here we first present the basics of the Black-Scholes model and then we discus modification of the model to be fit for preemption right valuation. At the end, we valuate four of the preemptive rights using the proposed model

Keywords:

Modified Black-Scholes Model, valuation, preemptive right, derivatives

1. Introduction

Derivatives valuation is one of the crucial topics in financial math. In the past few years, lots of models have been presented for assets and derivatives valuation.

In 1900 Bachelier proposed the first model of option valuation [1]. Bachelier described the price of any security as a Brownian motion. The main impediment to this model was the possibility of negative prices. In 1959 Osborne improved Bachelier's model assuming that return rates follow geometric Brownian motion (e^{B_t}) [2]. He claimed that stock prices pursue a long-normal distribution rather than a normal distribution. Samuelson (1965) criticized the previous option valuation models and claimed that compared with a stock, an option has a different level of risk [3]. Mandelbrot [4] substituted Brownian motion with a stable Levy process of $-\alpha$ with $\alpha > 2$ in his model. Press (1967) [5] and Madan and Seneta (1987) [6] presented models trying to cover deficiencies of Mandelbrot's model.

In 1973 a paper was published by Black and Scholes [7] which was completed a few months later by Merton. This paper presented the Black-Scholes model and caused a profound change in the world of financial mathematics and financial markets. It was a starting point for the exponential growth of derivatives markets.

But the model had its own defects. Some of the inconsistencies are as follows [8]:

1. The model must have the ability to show big random jumps but the price path obtained from the model is continuous.
2. Log distribution of the return should be more stretched.
3. Log distribution of the return should be crooked asymmetrically.

There has been abundant research regarding the Black-Scholes model. [9], [10], [11], [8] are only a small part of the reviews and modified models.

After this one, other models such as the Barndorff-Nielsen hyperbolic distribution model in 1977 [14], Eberlein and Hammerstein's model in 1995 [15], the generalized hyperbolic distribution of Schoutens in 2003 [16], etc. were presented. But still, the Black-Scholes model and its generalized and modified models are used the most in derivatives markets.

2. Literature Review

2.1. Preemption right

In addition to the right to vote and dividends, shareholders have the preemption right. Usually it is mentioned in the articles of association that if stocks are issued by the firm, current shareholders take precedence if they want to buy. This right allows all the current shareholders to underwrite the new shares of the firm. They have the option of using or not using their right. If a shareholder does not want to have more shares, they can go to their brokerage and sell their preemption right. Preemption rights have a short life. If some shareholders refuse to sell or underwrite the new shares by the announced deadline, the company sells the remainder of the rights in the market and after the deduction of the associated costs, the sum will be paid to the shareholders. [15]

Companies use these resources to implement a new project or pay their debts. Publication of preemption rights means issuing new stocks to the current shareholders so that they can keep their proportion of the company. The goal of this right is to maintain current shareholders' control of the company and to protect their rights against value reduction caused by issuing new stocks. [15]

Generally speaking, to convert a preemption right into a stock, one should pay the nominal value of the stock. The stock's nominal value of a listed company in Tehran Security and Exchange Commission equals 1,000 Rials and there is a waiting time between 2 and 5 months for the conversion to happen.

2.2. Black-Scholes Model

The assumptions of this model are as below [18]:

1. Stock price behavior corresponds with normal log function (mean μ and standard deviation σ).
2. There is no cost of transaction or tax and all the securities are perfectly devisable.
3. There is no arbitrage opportunity.
4. Security trading is available any time.
5. Investors can give or take loans at an equal rate (risk-free interest rate).
6. Short-term risk-free interest rate is fixed.
7. The underlying stock does not pay any dividends on the option's life time.

The following are Black-Scholes equations for a European call option with no dividends for the stock [18]:

Equation 1

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Equation 2

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

Equation 3

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Equation 4

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

In the above equations, $N(x)$ is the cumulative probability distribution function, c and p are prices of call option and of put option respectively, S_0 is the current price of the stock, K is the exercise price, r is the risk-free interest rate, T is the time left until the strike date, and σ is the volatility of the stock price.

Theoretically, the Black-Scholes model can be used for the short term where r is fixed. In practice, we assume r equals the risk-free rate of return for the time left until maturity.

If the underlier of the derivative (stock) pays a specific return of q, Black Scholes equations will change as follows [18]:

Equation 5

$$c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$$

Equation 6

$$p = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

2.3. Modified Black-Scholes model

In this paper, we try to valuate preemption rights by modifying the Black-Scholes model. A preemption right has some similarity with a European call option issued on a stock. Just like a call option, a preemption right is a derivative with its value related to the underlying stock. The owner of the preemption stock has the option of converting their right into a stock, thus gaining profit or rejecting the conversion, if not economical, and selling the right and thereby avoiding loss; just like the owner of the call option.

In fact the time structure of a preemption right consists of 2 parts. The first part or T_1 is the length of time between the present and its maturity day. The second part or T_2 is the length of time between the maturity day and the time when conversion is done and the right is changed into a stock. We can use a modified Black-Scholes model for the second part (T_2).

In the light of the "Bird in the Hand" Theory, we assume that the investors who want to exercise their rights pay the nominal price of the stock on the maturity day and not sooner. The Black-Scholes model is modified as below:

Equation 7

$$P_{PST_1} = P_{ST_1} N(d_1) - (P_{ST_2} - P_N e^{rT_2}) e^{-rT_2} N(d_2)$$

Equation 8

$$P_{PS_0} = P_{PST_1} e^{-rT_1}$$

Equation 9

$$d_1 = \frac{\ln\left(\frac{S_0}{(P_{ST_2} - P_N e^{rT_2})}\right) + \left(r + \frac{\sigma^2}{2}\right)T_2}{\sigma\sqrt{T_2}}$$

Equation 10

$$d_2 = \frac{\ln\left(\frac{S_0}{(P_{ST_2} - P_N e^{rT_2})}\right) + \left(r - \frac{\sigma^2}{2}\right)T_2}{\sigma\sqrt{T_2}} = d_1 - \sigma\sqrt{T_2}$$

In Equations 7 to 10, P_{PST_1} is the value of the right at the end of the first period or T_1, P_{ST_1} is the underlying stock price estimation at the end of T_1, P_{ST_2} is the estimated price of the underlying stock price at the end of T_2, P_N is the nominal value of the stock, r is the continuous risk-free interest rate, P_{PS_0} is the current value of the right, and $N(x)$ is the cumulative probability distribution function.

It is worth noting again that the prices of the stock at the end of the first and second periods are only estimations. Further, in the proposed equation, $P_{ST_2} - P_N e^{rT_2}$ is homological to the strike price (K) in the original Black-Scholes model.

To use the proposed modified model, we still work under the first six assumptions of the original model mentioned earlier. We can make the seventh assumption that the company which publishes preemption rights with the purpose of gathering more resources generally does not pay any dividends at the same time. In any case, if it happens, we can use the equation below.

Equation 11

$$P_{PST_1} = P_{ST_1}e^{-qT_2}N(d_1) - (P_{ST_2} - P_N e^{rT_2})e^{-rT_2}N(d_2)$$

In this equation, q is the cash return of the stock during T_2.

Additionally, if the risk-free interest rate is discrete, it can become continuous with the help of the following equation.

Equation 12

$$r = m\ln(1 + \frac{r_c}{m})$$

In this equation, r is the continuous risk-free interest rate, m is the frequency of interest payment, and r_c is the discrete risk-free interest rate.

Table 1 shows four examples of preemption rights being valuated using equations 7 to 12 on July 7, 2015.

Data related to the stock and preemption right prices were obtained from the Internet (http://www.tsetmc.com). The maturity date of each right was determined from another website (http://www.codal.ir). T_2 is assumed to be 120 days by researchers. The price of the stock at the end of T_1 and T_2 and the volatility of each stock is estimated by researchers. The nominal price of each stock and the cash return of each stock between T_1 and T_2 is considered 1,000 rials and 0 respectively. Risk-free interest rate is considered the equivalent of the one-year deposit account interest rate of Iranian banks (22%) and then turned into continuous rate using Equation 12. d_1 and d_2 are calculated with the help of Equations 9 and 10. Then using Equation 7 (or alternatively Equation 11, in case the cash return of the underlying stock is 0) the preemption right is valuated at the beginning of T_2 and finally, the present value of the right is calculated using Equation 8.

3. Methodology

When using the proposed model, one should notice that it takes a collection of estimations as input. Obviously, changing the inputs such as risk-free interest rate, estimated time until the conversion of the right into stock, and stock price on the maturity date, would lead to different results. In this paper, considering the estimated inputs, the values of preemption rights of Industry and Mine Leasing and Sarma Afarin are estimated more than the their present market prices and preemption rights of Iran Argham and Kesh O Sanat Piazar less than their present market prices.

Table 1- 4 Practical Example

Sarma Afarin	**Kesht O Sanat Piazar**	**Industry and Mine Leasing**	**Iran Argham**	**Underlying stocks name**
1932	2730	1422	2542	Underlying stocks price on July 7, 2015 (Rials)
July 21, 2015	July 10, 2015	July 23, 2015	July 22, 2015	Maturity date
14	3	16	15	T_1
1740	2535	1450	2342	P_{ST_1}
120	120	120	120	T_2
1858	2706	1548	2811	P_{ST_2}
1000	1000	1000	1000	P_N
22%	22%	22%	22%	r_c
20%	20%	20%	20%	r
50%	30%	35%	30%	σ
-	-	-	-	q
1.55	2.30	1.50	2.35	d_1
0.43	1.49	0.78	1.52	d_2
0.94	0.99	0.93	0.99	$N(d_1)$
0.66	0.93	0.78	0.94	$N(d_2)$

Sarma Afarin	Kesht O Sanat Piazar	Industry and Mine Leasing	Iran Argham	Underlying stocks name
1143	1078	1001	792	P_{PST_1}
1135	1076	993	786	P_{PS_0}
800	1230	517	1268	Present market value of the preemption right on July 7, 2015

4. Conclusions

The goal of this paper is to modify the Black-Scholes model so that we could valuate derivatives of Tehran Security market. Here we used our proposed model to valuate preemption rights.

First we discussed the Black-Scholes model and the preemption rights and then due to the similarity between European call options and preemption rights, we presented our modified Black-Scholes model. At the end, as an empirical example, we valuated four of the preemption rights traded in the market.

It is hoped that this paper can open new horizons in the field of valuating preemption rights as well as other derivatives.

References

1) L. Bachelier, "Théorie de la speculation," Gauthier-Villars, 1900
2) M. F. M. Osborne,, "Brownian motion in the stock market," Operations Research, vol. 7, pp. 145-173, 1959.
3) P. Samuelson, "Rational theory of warrant pricing," Industrial Management Review, vol. 6, pp. 13-32, 1965.
4) B. Mandelbrot, "The variation of certain speculative prices," The Journal of Business, vol. 36, p. 394–419, 1963.
5) S. J. Press, "A compound events model for security prices," Journal of Business, vol. 40, pp. 317-335, 1967.
6) D. B. Madan and E. Seneta, "Chebyshev polynomial approximations and characteristic function estimation," Journal of the Royal Statistical Society Series B, vol. 49, no. 2, pp. 163-169, 1987.
7) F. Black and M. Scholes, "The pricing of options and corporate liabilities," Journal of Political Economy, vol. 81, no. 3, pp. 637-654, 1973.
8) R. Poortaheri and M. Karimi Khorami, "Generalized Hyperbolic Distribution and its Application in Finanical Mathematics- in Persian," Journal of Financial Enginiering and Security Management, pp. 119-134, 2012.
9) A. Samih and T. Annie, "The Black-Scholes Currency Option Pricing Model: Evidence for Unbiasedness from Three Currencies against the US Dollar," International Journal of Economics and Finance, vol. 5, no. 8, pp. 1916-1927, 2013.
10) M. Chesney and L. Scott, "Pricing European Currency Options: A Comparison of the Modified Black-Scholes Model and a Random Variance Model," Journal of Financial and Quantitative Analysis, vol. 24, no. 3, pp. 267-284, 2009.
11) B. LAUTERBACH and P. SCHULTZ, "Pricing Warrants: An Empirical Study of the Black-Scholes Model and Its Alternatives," The Journal of Finance, vol. 45, no. 4, pp. 1181-1209, 1990.
12) O. E. Barndorff-Nielsen, "Exponentially decreasing distributions for the logarithm of particle Size," Proceedings of the Royal Society London, pp. 401-419, 1977.
13) E. Eberlein and E. A. Hammerstein, "Generalized Hyperbolic and Inverse Gaussian Distributions: Limiting Cases and Approximation of Processes," FDM Preprint 80, University of Freiburg., 2002.
14) W. Schoutens, "Lévy processes in Finance: Pricing Financial Derivatives," Wiley, 2003.
15) Tehran Securities and Exchange Organization, "Tehran Securities and Exchange Organization," [Online]. Available: http://new.tse.ir/cms/page577.html.
16) J. Hull, Fundamentals of futures and options markets, Forth ed., 2002.