Using the Theory of Network in Finance

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ABSTRACT

It is very important for managers, investors and financial policy-makers to detect and analyze factors affecting financial markets to obtain optimal decision and reduce risks. The importance of market analysis and attempt to improve its behavior understanding, has led analysts to use the experiences of other professionals in the fields such as social sciences and mathematics to examine the interaction of market in a different way. This article reviews the use of networks and graph theory to analyze the behavior of social and financial phenomena that in recent years has been expanded. First, the original of this theory that donate from discrete mathematics, is introduced and then some details are given about the characteristics of a network, such as power law property, scale-free networks and minimum spanning tree. The results show that financial markets dynamics have caused the dynamically development of the approaches, methods and models of market analysis, so the effect of investment opportunities on each other was evaluated to identify market behavior

Keywords:

Graph Theories, Scale-Free Networks, Minimum Spanning Tree, Behavior of Financial Markets

1. Introduction

In today's world, financial markets activities lead to the production of massive amounts of data which causes many computational challenges. The use of intuitive techniques and using graphs with simple and comprehensive description of their features will increase understanding of data and extract their information (Ziegler, 2006). Small world models and scale-free networks that can be observed frequently in social phenomena have been widely used in areas such as technology sciences, physics, biology and economics. These two models of networks play a vital role in the study of complex phenomena. Networks have entered many discrete mathematics concepts and social sciences into financial sciences and have challenged the basic models.

One of the simple descriptions in many of today's systems is free-scale networks degree distribution, which is especially considered. These networks follow power law[1], while the distribution tail declines much faster in the random networks. Financial markets are considered complex networks due to the interaction between investors and companies and also influential elements, such as banks and financial institutions.

The studies of Boginsky et al. (2005) are among research conducted to describe capital market, in which threshold level method was used to create price correlation network in the U.S. market. The market structure in terms of components and independent sets using data mining techniques for classification of financial documentation were investigated in China's financial markets by Hyang et al. (2009). During the past decade, a significant proportion of theoretical and experimental studies described the behavior of financial markets using graph theories. Other studies in this area include the correlations between stock price in the U.S. stock market and analysis of market graph structure by Chi and Li (2010), the analysis of South Korea market structure by Kim and Ha (2007) and free-scale structure of the Italian stock market compared to the U.S. capital markets (Garlaschelli et al, 2005).

This paper aims to provide a new solution for network behavior modeling of financial markets which has been introduced and developed during the last 10 years in the finance literature. In this model, unlike classical models based on cost-benefit, price changes are assumed affected by group behaviors and therefore price behavior of each financial input is affected by the group behavior. Discussion in graph theory and discrete mathematics are the background of this model.

This article is organized as follows. In section 2 a detailed explanation of the research methodology is provided. Section 3 discusses scientific foundations of the study including a review of topology of networks and basic concepts used, such as the effect of small-world and scale-free networks to better understand the market structure, behavior and changes. Section 4 will show how scale-free networks can be identified and also how homogeneous communities with almost the same behavior can be determined. Section 5 reviews experimental results and performance of complex networks, and finally, conclusions are provided in Section 6.

2. Research methodology

Aiming to promote knowledge in the field of finance, this study was conducted using scientific foundations and previous research and by library research method within a descriptive method framework based on a historical knowledge approach.

3. Scientific foundations of the study

Most economic and social systems etc. have complex indicators associated with their topology structure and the relationship between their elements is relatively regular. Many of these systems can be explained through complex networks and their related rules. During the 1970s, when scientists learned more about the behavior of advanced systems they found that the behavior is not constant in construct elements of these systems, but they endeavor to adapt to their surroundings with an intelligent behavior. In the 80s and 90s, researchers began to investigate models of economic phenomena which had fundamental differences with traditional models. These models introduced economic as a dynamic interactive system rather than a static balance system. New models provided new patterns which simulated the interaction of system factors like what occurs in reality. Financial markets were not an exception as well and used new interactive models such as complex networks to describe the behavior of their internal factors. Researchers have introduced two important features for complex networks that the ability to predict the network behavior depends on them. These two features include the effect of small-world and scale-free networks. In the following we will briefly overview the basic concepts and features of complex networks, such as the effect of small-world, scale-free networks and minimum spanning tree.

3.1. Network

Network consists of a set of nodes that communicate with each other through edges (Figure 1). Networks which generally follow the rules of graph theory are considered as main issues of discrete mathematics. Euler (1735) donated to a solution of the bridge problem, which the first problem solved by networks is considered as a guide for today's knowledge of graph theory. Social sciences have been among pioneers in using networks that with using interactive questionnaires, social sciences have tried to find the relationships between people and modeling them by networks (Figure 2). Networks in which people are defined as node and interaction between individuals constitutes the edges, mainly aim to find the centrality, i.e. person or people who are in the center of interactions or have the most interactions with others and communication means how people interact in a group.

Random graphs are one of the most important types of graphs in which the edges are distributed randomly and similarly, therefore they do not have distinctive and unique models. For example, suppose that there is a binomial distribution for the number of neighbors in a network, therefore most nodes have similar degrees. Most networks in the real world are not random graphs and have clear and distinct structures.

More recently, the focus has changed from small network analysis, edges and nodes to statistical properties of large graphs. Network theory follows three goals. The first goal is to find statistical properties such as networks degree distribution or path length distribution which determine structural and behavioral characteristics of the network and also to find appropriate ways to measure these characteristics. The second goal is to establish a network model to understand the characteristics of the network, their nature and how nodes interact with each other. Finally, the third goal is to identify rules of the network and predict its behavior, especially small communities' behavior.

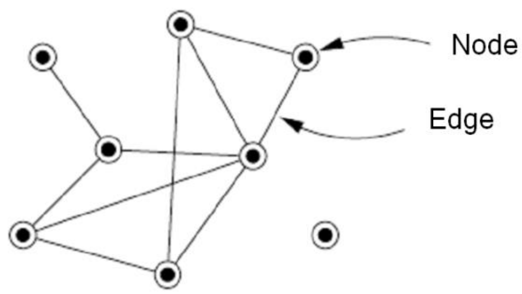


Figure 1- A small network with 8 nodes and 10 edges

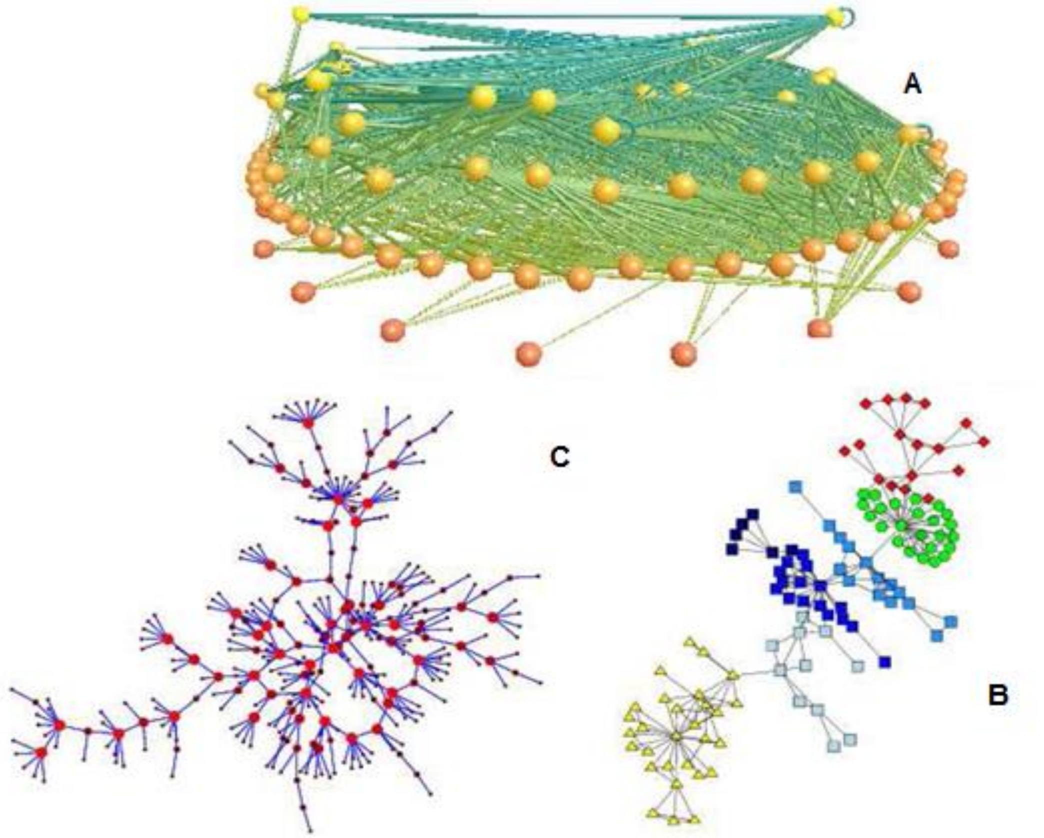


Figure 2- Three examples of real-world network: (a) Prey - hunter network in a lake, (b) Researchers' interactions network in a private research institute, (c) Network of sexual relationships between individuals

3.2. Effect of small world

Stanley Milgram (1960) in a study sent several letters through person to person for reaching the destination. Although many of the letters were missing, a quarter of them reached their destination by an average six people. According to this study, the majority of nodes in a large network communicate to each other through a short distance. This feature has been investigated in large networks. This effect justifies the consequence of dynamic phenomena in networks. In fact, this effect leads the spreading speed of events to be more than what was predicted in large networks. For example, rumor, on average, needs only six steps to reach anyone in the community.

The average shortest Euclidean distance between pairs of nodes in a network without direction is defined as follows:

$$l = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}$$

where d_{ij} is the Euclidean distance between nodes i and j. If the number of nodes connected to a central node increases exponentially, the average distance between network nodes increases at a logarithmic rate. This means that a network has this effect in which with the increased number of nodes and network size, the average distance of its nodes increases at a logarithmic rate. Bollobas and Riordan (2002) showed that the average distance of network nodes with power law distribution increases at a rate less than logarithmic rate.

3.3. Scale-free networks

Suppose that the degree of a node in a network is equal to the number of edges connected to it (k). In this case, the degree distribution of the network is a histogram drawn with k that the distribution in random graphs will be a Poisson distribution or a binomial distribution. Real-world networks have a quite different distribution compared to random graphs distribution and usually have kurtosis to the right, i.e. they have a tail and therefore have data that are significantly far from the mean. If there is a α, by which distribution function can be estimated in the form of an exponential function $P^k \sim K^{-\alpha}$, the distribution follows power law. Here, P_k is the probability that a node has in-degree k.

Networks that their distribution function follows a power law are called scale-free networks. In these networks Pareto principle (also known as the 80-20 rule) is also true i.e. 80% of the edges pass 20% nodes. Complex networks can be described by fitness model. Fitness index is the ability of each node in relation to other nodes. According to this model, the preference of each node in connection with another node depends on the fitness of a product and the number of its connections. If a network follows this model, it follows power law and therefore is a scale-free network.

One of the important properties of scale-free networks is that they are destroyed when all of their nodes are eliminated. Computer viruses use this property for their spread and stability.

3.4. Minimum spanning tree

Minimum spanning tree is defined in graphs which have weighted edges. It is a subset of graph edges which forms a tree containing all vertices and total weight of their edges is the lowest possible amount among all such trees. In fact, the problem is to find a subset of graph edges with the lowest total weight that there is still a path between both vertices of this subgraph.

Problems in which the aim is to create a network that cost should be paid for linking two members, minimum spanning tree represents the lowest cost network. Prim's and Kruskal's algorithms are algorithms used to achieve the minimum spanning tree.

Prim's algorithm

1. Starting from a node,
2. Finding the minimum path connected to the node,
3. Finding the minimum path connected to the network created,
4. Continuing the algorithm to visit all nodes.

Kruskal's algorithm

1. Ordering edges based on their weight,
2. Starting from the edge with minimum weight and adding two nodes of it to the tree,
3. Choosing the next edge and repeating the previous step,
4. Continuing the algorithm to visit all the nodes.

4. Community recognition in the network

Community or a cohesive subgroup is a set of nodes in a graph or network that has a strong, direct with a high intensity relationship. Also, connection of two nodes is the minimum number of edges that should be removed, so that there is no connection between two nodes, i.e. it is the number of paths that do not share any edge. The most important issue in network analysis is to identify models or communities that are repeatable and have similar behavior. In the following some of the most important ways of identifying these models are presented. It is worth noting that networks can be described in the form of a two-dimensional matrix as follows:

	Node 1	**Node 2**	**Node 3**	**……….**	**Node n**
Node 1	0	r_{12}	r_{13}		r_{1n}
Node 2	r_{21}	0	r_{23}		r_{2n}
Node 3	r_{31}	r_{32}	0		r_{3n}
……….					
Node n	r_{n1}	r_{n2}	r_{n3}		0

4.1. Johnson's hierarchical clustering

In this method, first the nearest two nodes are identified (the smallest number in the distance matrix). These two nodes are identified as a primary community and are also merged in the distance matrix that in this case the minimum distance in each community with other nodes is considered as the distance. This process continues until to reach a single community (Figure 3). Cluster selection depends on the desired level, and this algorithm has the flexibility to select communities.

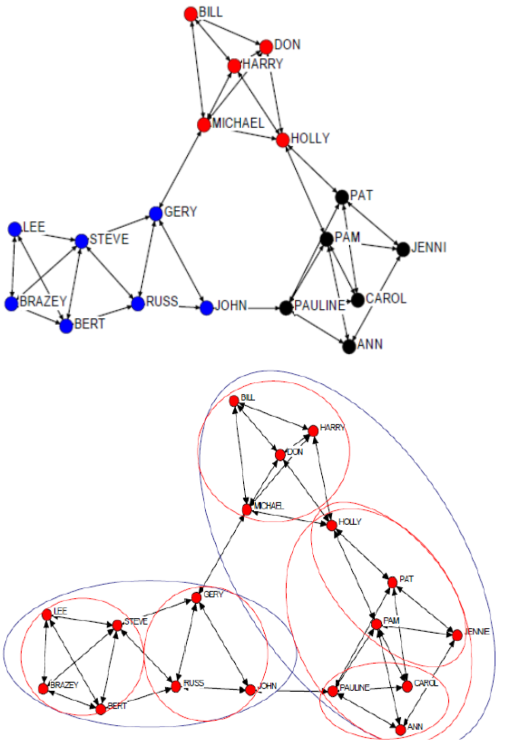


Figure 3- An example of Johnson's hierarchical clustering

4.2. Matrix permutation

In this way, the nodes that have stronger connections are placed close together in rows and columns (Figure 4). In this method the objective function is:

$$\text{Min} \sum_i \sum_j a_{ij}(i-j)^2$$

-	n_1	n_2	n_3	n_4	n_5
n_1	-	0	1	0	1
n_2	0	-	0	1	0
n_3	1	0	-	0	1
n_4	0	1	0	-	0
n_5	1	0	1	0	-

-	n_5	n_1	n_3	n_2	n_4
n_5	-	1	1	0	0
n_1	1	-	1	0	0
n_3	1	1	-	0	0
n_2	0	0	0	-	1
n_4	0	0	0	1	-

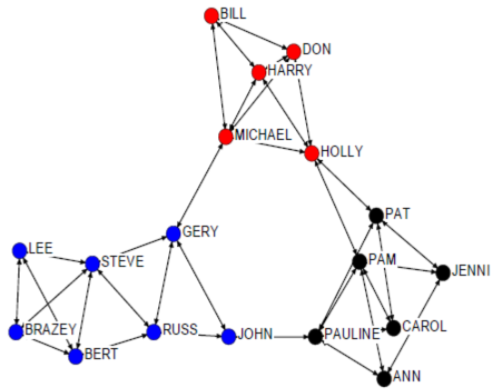
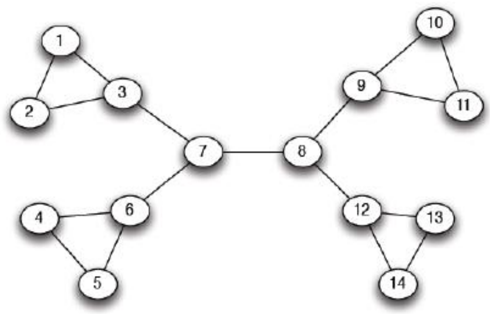


Figure 4- An example of permutation matrix

4.3. Girvan-Newman hierarchical clustering

Local bridge is the weakest tie in the network. This method is based on the belief that the removal of local bridge does not change a whole network; therefore, during consecutive steps local bridges are removed until nodes become completely single (Figure 5). Social selection also depends on the desired level.



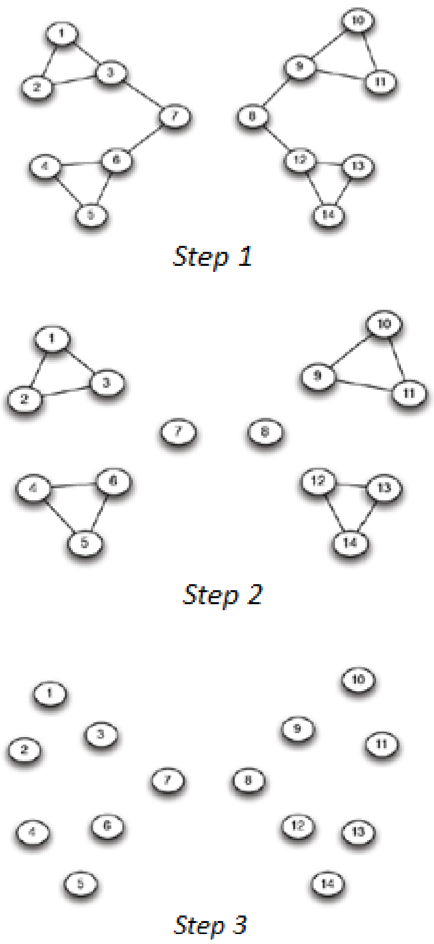


Figure 5- An example of Girvan-Newman hierarchical clustering

4.4 Modularity maximization

Module is a utility function that measures the quality of network division into communities and its value is between 1 and -1.

$$Q = \frac{1}{2m}\sum_{ij}\left[A_{ij} - \frac{K_i K_j}{2m}\right]\delta(c_i, c_j)$$

where m represents the number of expected edges between two nodes in a random graph.

$$\delta(c_i, c_j) = \begin{cases} 0 & \text{if } i \neq i \\ 1 & \text{if } i = i \end{cases}$$

Communities are identified according to possible modules in the network. Heuristic methods and algorithms, such as greedy algorithm or retrofitting algorithm are also used to identify modules (Figure 6).

4.5 Heuristic technique

This method has been inspired by models used in natural phenomena, such as the clustering of insect larvae or separation of carcasses. Here, the distance between two nodes is defined as follows:

$$d(v_i, v_j) = \frac{\left|D\left(\rho(v_i), \rho(v_j)\right)\right|}{|\rho(v_i)| + |\rho(v_j)|}$$

$$\rho(v_i) = \{v_j \in V; a_{ij} \neq 0\} \cup \{v_i\}$$

$$D(A, B) = (A \cup B) - (A \cap B)$$

Local density is also defined as follows:

$$f(v_i) = \begin{cases} \frac{1}{S^2} \sum_{v_i \in Neigh(S\times S)(r)} \left[1 - \frac{d(v_i, v_j)}{\alpha}\right] & \text{if } f > 0 \\ 0 & \text{otherwise} \end{cases}$$

where S×S is the node's neighbors in two directions, r is the current status, α is heterogeneity and $f(v_1)$ is the average distance to neighboring nodes. In this method the probability of selecting a node is as follows:

$$p_p(v_i) = \left(\frac{K_1}{K_1 + f(v_i)}\right)^2$$

In this method the probability of not selecting a node is as follows:

$$p_d(v_i) = \left(\frac{f(v_i)}{K_2 + f(v_i)}\right)^2$$

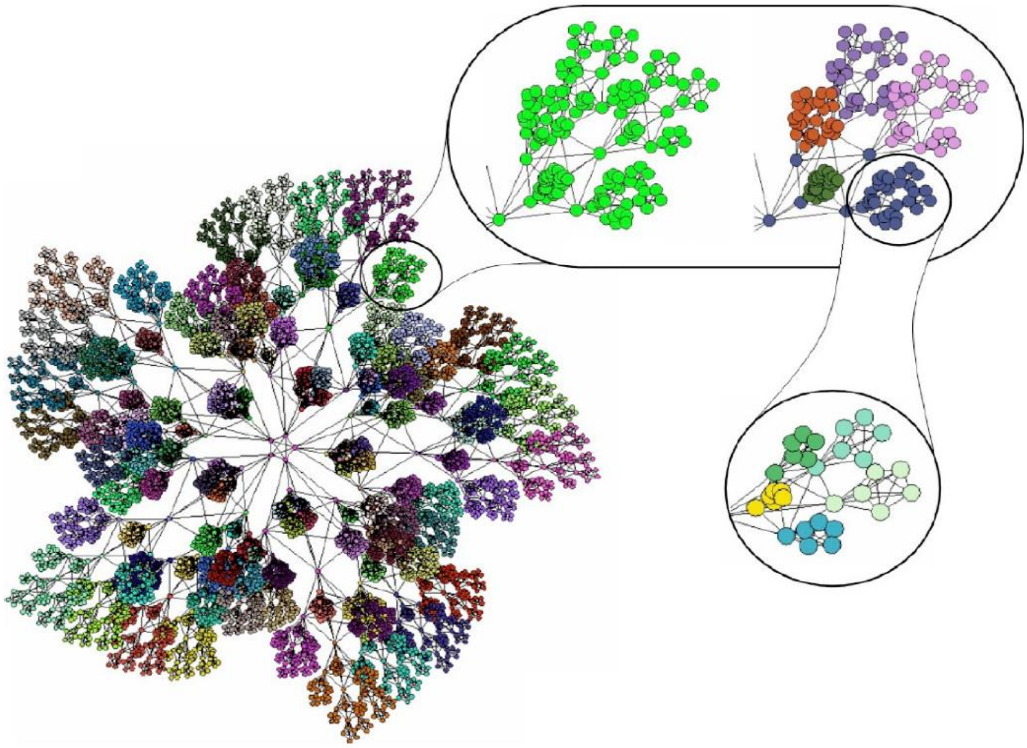


Figure 6- An example of modularity maximization

5. Research on experimental application of complex networks in financial markets

Hierarchical structure was used by Mantegna (1999), Bonanno et al. (2001), Vandewalle et al. (2000) and Bonanno et al. (2003) in their studies to achieve clear classifications of market portfolio based on time series data. Using time series data of stock prices, Mantegna examined Dow Jones and S&P 500 indices from 1989 to 1995. The first step in this research was to propose a measure to calculate the distance of stock network nodes, thus correlation coefficient was used as follows:

$$\rho_{ij}(\Delta t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}$$

so that $P_i(t)$ is the price of the stock i at time t and $r_i = \text{Ln } P_i(t) - \text{Ln } P_i(t-\Delta t)$. The distance between two stocks is defined as follows:

$$d_{i,j}(\Delta t) = \sqrt{2\,(1 - \rho_{ij}(\Delta t))}$$

For this distance three conditions are defined:

1. $d_{i,j}(\Delta t) = 0$ if $i = j$
2. $d_{i,j}(\Delta t) = d_{j,i}(\Delta t)$
3. $d_{i,j}(\Delta t) \leq d_{i,k}(\Delta t) + d_{k,j}(\Delta t)$

This research showed that it is possible to find the structure of a minimum spanning tree and hierarchical structure in the stocks distance matrix (Figures 7 and 8).

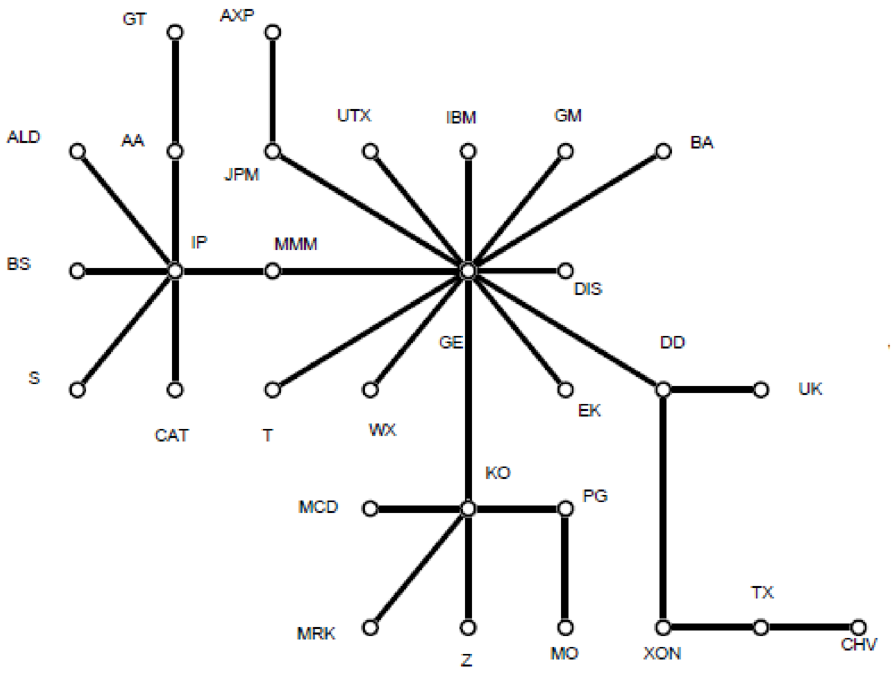


Figure 7- Minimum spanning tree including 30 shares used in the Dow Jones index

Onnela et al. (2003) examined mean distribution and variance of the New York stock exchange from January 2, 1980 to December 31, 1999 and the dynamics of assets tree in the portfolio analysis. They used statistical concepts such as mean, variance, skewness and kurtosis. Onella et al. showed that the minimum spanning tree has a well-connected structure and follows a free-size structure. They also concluded that stocks with the lowest risk tend to be on the margin of the tree, or, in other words, have a maximum distance from the central node. This study showed that the risk of the portfolio is directly related to the normality of the minimum spanning tree, thus, diversification in the portfolio should be achieved according to normalization of the tree.

Bonanno et al. (2004) examined covariance network in New York Stock Exchange (NYSE) and using different portfolios at different times tried to show how significant economic data can be extracted from communication matrix of nodes. Using time series, they calculated the correlation coefficient between stocks and created a completely connected graph, i.e. spanning tree (a type of complete graph without loop), then using the minimum spanning tree (MST) they tried to examine the behavior of the stock. Using this method, Bonanno et al. could show the connection between the stocks geometrically in a classified form. They used the formula defined by Mantegna (1999) to calculate the correlation coefficient and used the obtained distance matrix to create the minimum spanning tree. The change of stocks behavior can be evaluated in various periods of time with change of Δt. The decrease of stock return correlations by changing the time horizon is called Epps effect. Using data of the New York Stock Exchange, assuming $\Delta t = d$ was six hour and a half and dividing Δt into smaller times, the researchers drew the minimum spanning tree of stocks (Figure 9). They eventually concluded that using networks theory is a useful technique for filtering economic data and identifying stocks' behavior in different periods.

Considering shares and shareholders as nodes and connecting them based on the partnership, Garlaschelli et al. (2005) created a large network of investors. In this network the in-degree of each investor (k_{in}) is the sum of weighted edges (v) connected to v that indicates the stock in the portfolio v (portfolio diversification) and also the value of investment. To conduct their study, they used the data of the New York Stock Exchange and the National Association of Securities Dealers Automated Quotations (NASDAQ) in 2000 and Borsa Italiana in 2002. Distribution of P > K_{in} is the number of nodes whose in-degree is greater than or equal to the specific value k_{in}. Distribution of P > V is also the number of portfolios that their value is greater than or equal to v. The findings of these two researchers showed that both distributions follow the power law, so we will have:

$$P > (K_{in}) \propto K_{in}^{1-\gamma} \quad \rightarrow \quad P(K_{in}) \propto K_{in}^{-\gamma}$$

$$P > (v) \propto v^{1-\alpha} \quad \rightarrow \quad P(v) \propto v^{-\alpha}$$

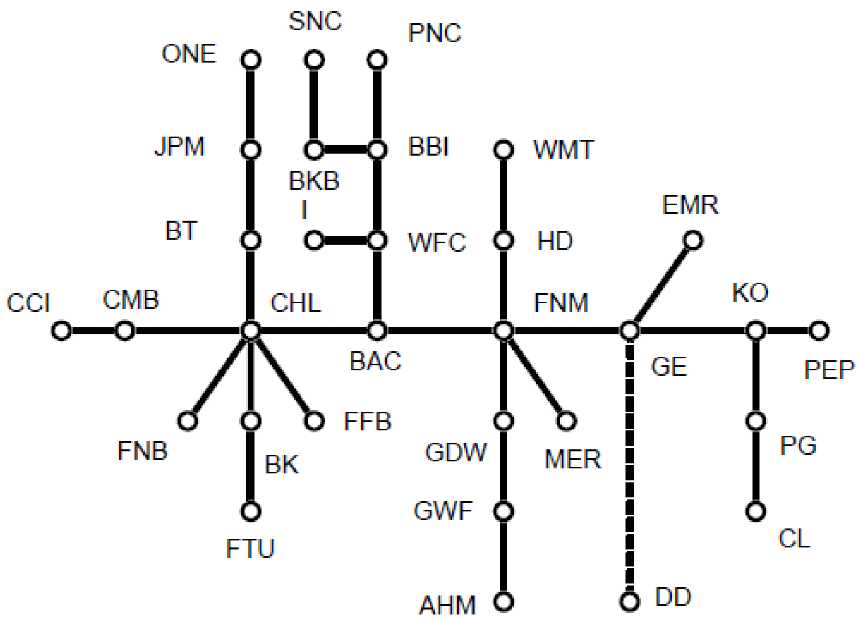


Figure 8- A part of minimum spanning tree including shares used in the S&P 500

The question raised here is that whether there is a relationship between portfolio diversification and its value? Garlaschelli et al. found that the portfolio value is an increasing function based on changes in its in-degree and their logarithmic function is a linear function. One of the statistical characteristics of the graph is node's fitness. In the graph created by these researchers, there are two nodes including N nodes as investor with fitness variable x and asset M or company with variable y and obviously the total number of nodes is M + N. Fitness variable x_i is asset value in investor's portfolio and y_j is long-term expected dividend of asset j. Here, $f(x_I,y_i)$ is the possibility of a connection between i and j and $f(x,y)\neq f(y,x)$ that come from the direction of the graph. This function can be defined as follows:

$$F(x,y) = g(x)\, h(y)$$

So that g(x) is an increasing function of x, which indicates capital increase will result in greater access to information and payment of higher transaction costs and thus portfolio diversification increases, h(x) is a function of investor's information processing strategy. If nodes x have a distribution of p(x), the in-degree distribution function of nodes is as follows:

$$P(K_{in}) = p[x(K_{in})]\,\frac{dx(K_{in})}{dK_{in}} \quad (1)$$

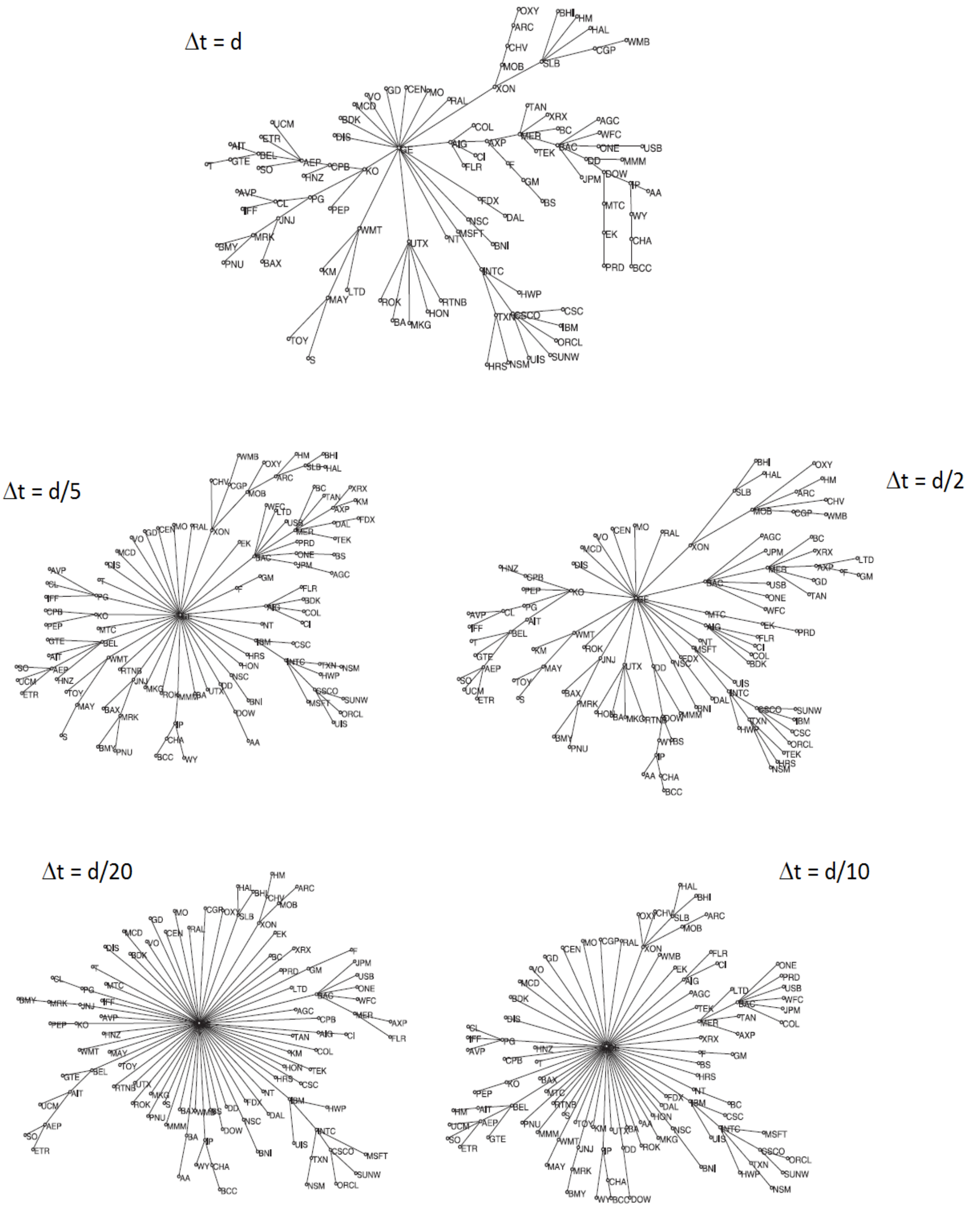


Figure 9- Minimum spanning tree for 100 shares has the highest working capital in New York stock exchange with different Δts

This research showed $g(x) = cx^\beta$, $\beta > 0$ that c is defined to ensure $0 \leq g(x) \leq 1$; therefore, assuming that the distribution p(x) in the large graph is $p(x) \propto x^\alpha$, we will have:

$$K_{in}(x) \propto x^\beta \quad (2)$$
$$P(K_{in}) \propto K_{in}^{(1-\alpha-\beta)/\beta} \quad (3)$$

These observations indicate that, p(x), $K_{in}(v)$ and $P(K_{in})$ all follow power law. The following relation is obtained by combining equations (2) and (3):

$$\beta = (1-\alpha) / (1-\gamma)$$

Kyungsik et al. (2007) examined price changes of all joint-stock companies in Karachi Stock Exchange in 2003 to identify the node degree distribution, edges' density and size of communities. Suppose that $r_i(t)$ is the stock price return, which is defined as follows:

$$r_i(t) = \ln [p_i(t+\Delta t) / p_i(t)]$$

where $p_i(t)$ is the stock price at time t. Also suppose that matrix C contains the covariance between the stock price return, in which:

$$C_{ij} = \frac{\overline{r_i r_j} - \overline{r_i}\,\overline{r_j}}{\sqrt{(\overline{r_i^2} - \overline{r_i}^2)(\overline{r_j^2} - \overline{r_j}^2)}}$$

These researchers showed that the stock network based on correlation coefficient follows the power law. Then, a threshold level θ was defined in such a way that all edges with less weight were excluded (Figures 10 and 11).

Wei Jiang et al. (2009) used the threshold level to create the correlation network of Chinese stock and to study stock structure based on it. They showed that this network follows the power law and therefore specific communities can be identified among the stocks that this contributes to stock clustering for creating an optimal portfolio. This study also indicated that China's stock network is resistant to stock random failure, but is fragile in the face of deliberate sudden changes.

Chi et al. (2010) in their study showed that the United States stock network has free-size property, so that the nodes are stocks and edges are the covariance between stocks. These researchers used transactions data from 1 July 2005 to 31 August 2007 and created stock network with threshold of 0.9 using calculated covariance by the following formula (Figure 12).

$$r_i(t) = \ln [p_i(t+\Delta t) / p_i(t)]$$

$$C_{ij} = \frac{\overline{r_i r_j} - \overline{r_i}\,\overline{r_j}}{\sqrt{(\overline{r_i^2} - \overline{r_i}^2)(\overline{r_j^2} - \overline{r_j}^2)}}$$

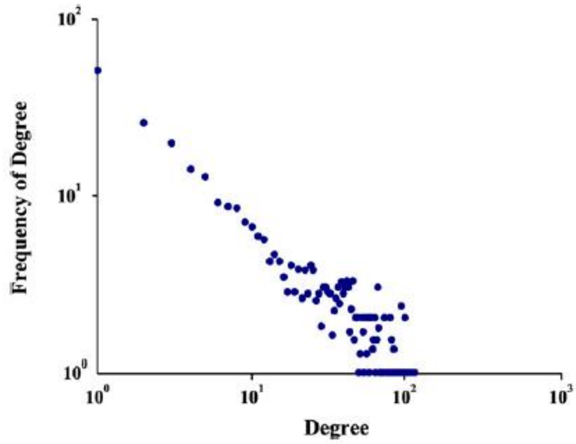


Figure 10- Distribution of edges for $\theta = 0.5$ **and** $\beta = 0.91$

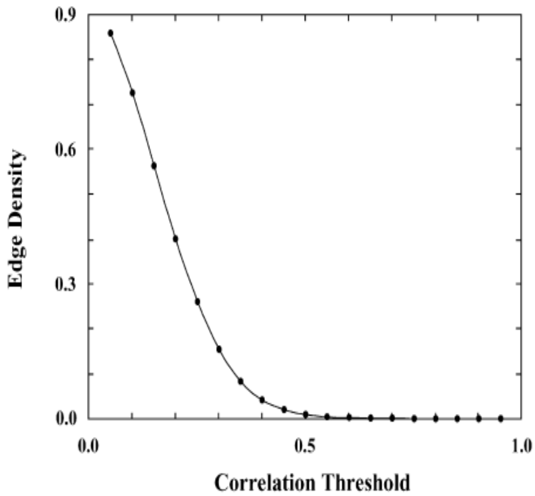


Figure 11- Edges density distribution

Kheyrkhah et al. (2015) created correlation network of Tehran Stock Exchange to detect homogenous communities and introduced them as a cluster. In fact, using stock data from 2010 to 2013, this study has provided a new method for clustering by considering the behavior of dividends. Examining pairwise correlation distribution of stocks, these researchers initially showed that this is a free-scale distribution, and thus it follows power law. Then, using thresholds of 0.75, 0.90, 0.95 and 0.5 they created four correlation graphs and calculated their amount of modularity. Then, using visualization techniques, they draw a graph with a threshold of 0.9 and 0.75 and determined the communities within them (Figure 13).

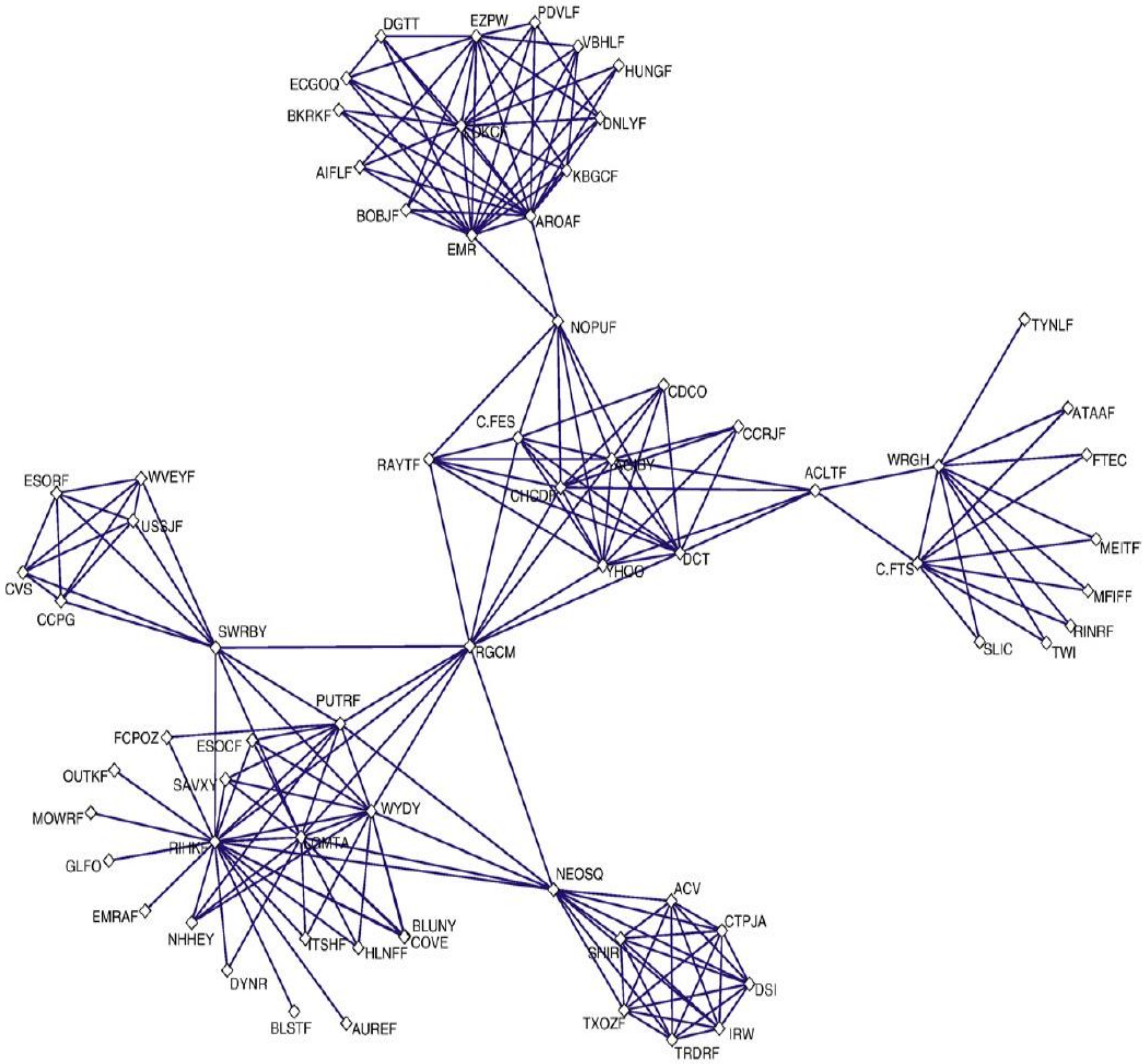
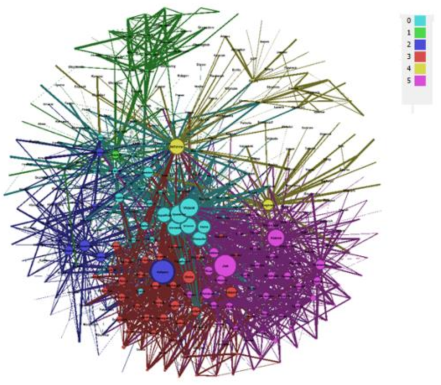
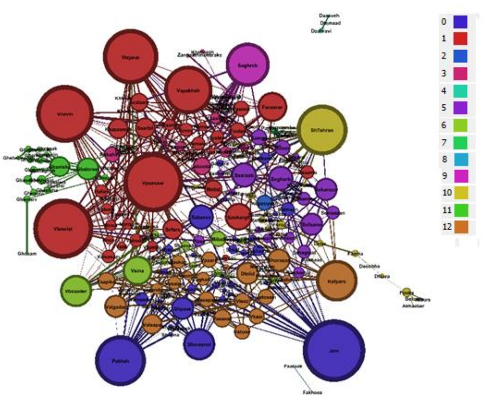


Figure 12- Correlation network of U.S. stock exchange from 1 July 2005 to 31 August 2007 with threshold of 0.9

ρ > 0.75



ρ >



0.90

Figure 13- Stocks correlation network

6. Conclusions and discussion

Different components of financial markets do not interact unilaterally and affect each other quite extensively. Given the complexity of business environment, and also advancement of decision-making techniques, traditional linear analysis methods or prediction algorithms based on identifying how a dependent variable is influenced by several independent variables do not satisfy the needs of investors and managers. The study of the application of networks and graph theories explains the behavior of financial markets and the interaction of market factors together. The review of the above applications showed that each variable depends on other factors, therefore, no variable can be assumed as independent. It is clear that the introduction of network communication in the financial area does not reject previous methods and models in this area, but rather challenges the assumption of considering variables as independent. In this approach, the relationship between variables is determined through available methods and models.

In this paper, the relatively short history of using graph theory and network in finance, features and methodology used are described. There are many areas that require further study in this regard, such as economic factors or financial marketing.

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Not’s

[1]- $p(k) \sim k^{-\gamma}$, where p(k) is network degree distribution